Part III: Dynamics of a Single Rigid Body

Chapter 8 Planar Kinematics of Rigid Bodies

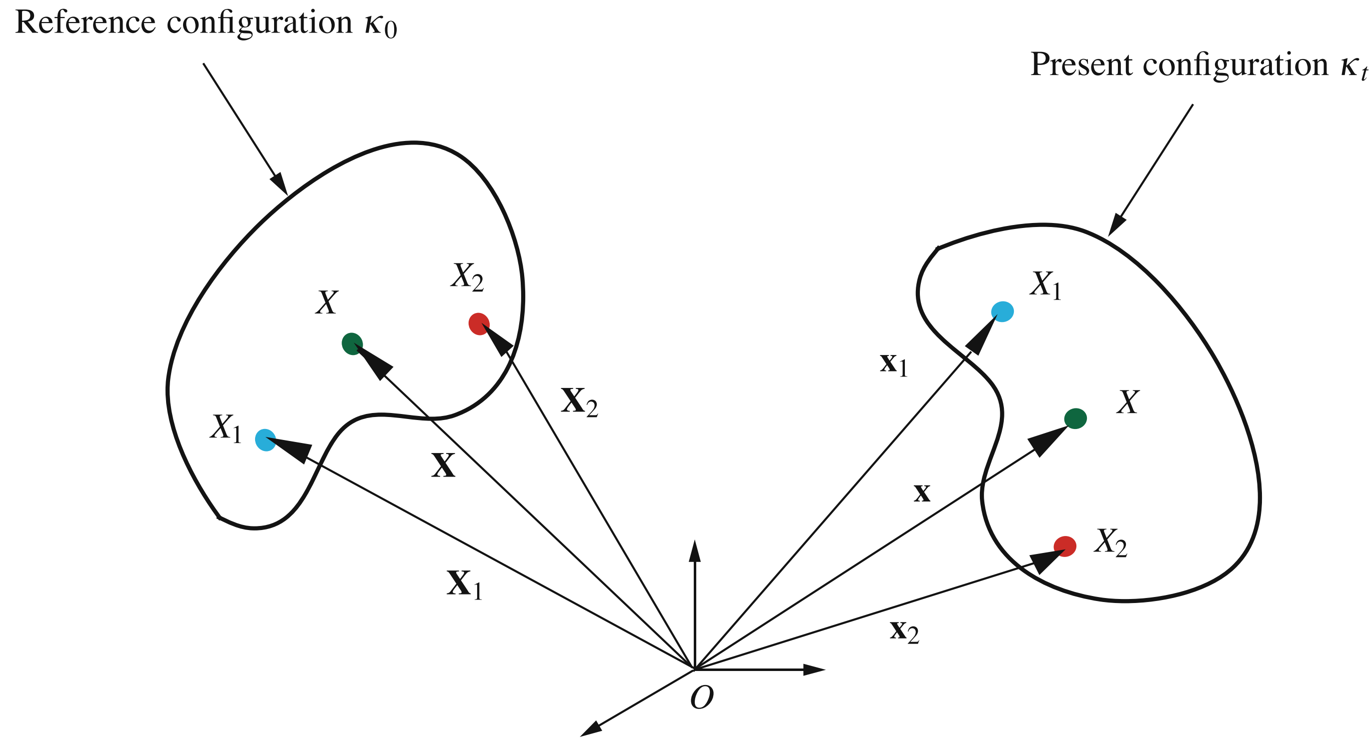
Until now, we have considered only "particle" masses as models of bodies. In Part III, we consider a new model: rigid bodies.

8.1 Motion of a Rigid Body

A body $\mathcal{B}$ is a collection of points representing particles. A point is denoted X. The vector $\bar{x}$ describes the pos. of point X at time t. The present configuration $\bar{\kappa}_t$ of $\mathcal{B}$ is a smooth bijection (function). It maps points of $\mathcal{B}$ to vectors in $\mathbb{E}^3$.

Let $\bar{\kappa}_0$ be a reference configuration and $\bar{X} = \bar{\kappa}_0(X)$ be the corresponding vector-valued function of time of the position of X in $\mathbb{E}^3$.

One can define the motion of $\mathcal{B}$ as a function of $\bar{X}$ and t:



So the position vector $\bar{x}$ of a point X is a function of the initial position vector $\bar{X}$ of X and the time t.

8.1.2 Rigidity

The above is true for any body, including bodies that deform. In the field of continuum mechanics, this is useful. We will not consider this most-general case in this course. We are concerned with rigid-body motion $\bar{x} = \chi_R(\bar{X}, t)$, which simplifies the situation.

In a rigid body, the following two physical ideas are modeled by the math:

The first is expressed mathematically, for points X_1 and X_2, as

The second is expressed by restricting motion to be such that the following linear transformation has certain properties:

The first of these restricts the nine components of Q to three independent parameters. The most common parameterization is Euler angles. We will typically consider only cases that require a single parameter (planar rotation).

Because Q is a rotation matrix,

We will use this in the next section.