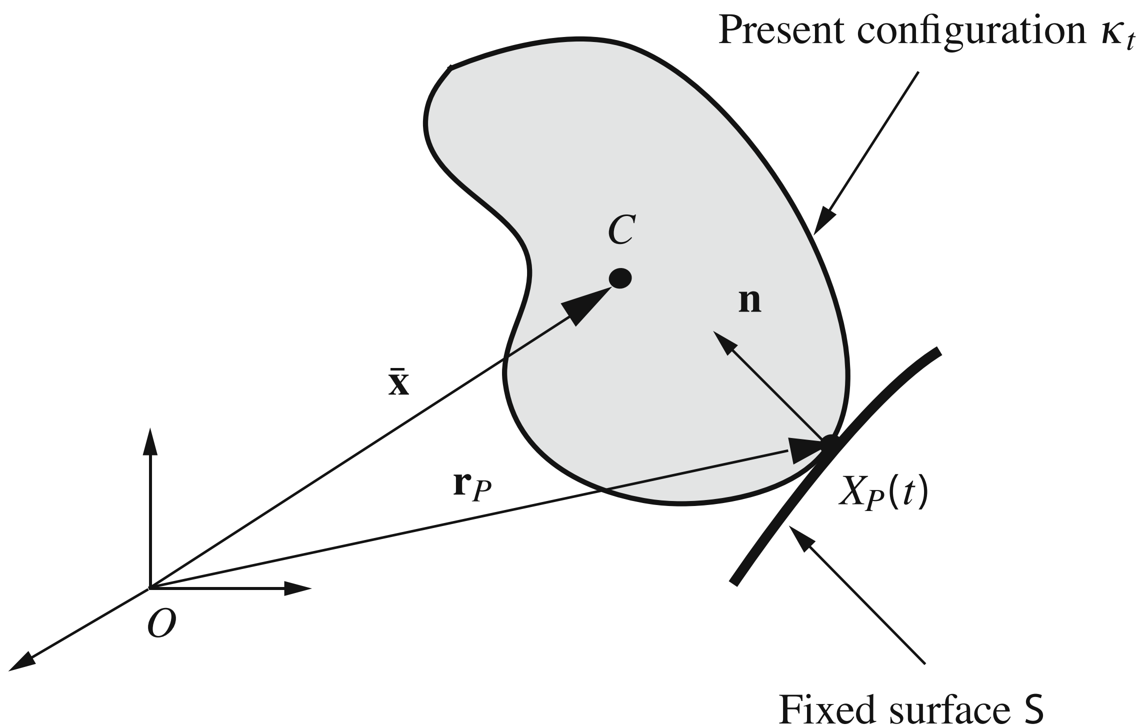
8.5 Kinematics of Rolling and Sliding

Often, we would like to analyze a body B that is in contact with another body's surface $\mathcal{S}$ at a single point. The material point $P = X_P(t)$ of B that is in contact varies with time. We denote the position vector of P at time t, $\bar{r}_P$ and the velocity, $\bar{v}_P$. The unit normal vector to $\mathcal{S}$ at P is denoted $\bar{n}$.



Because P is a material point on B, it has velocity and acceleration:

If the rigid body is sliding on the fixed surface:

which implies the sliding condition:

If the rigid body is rolling on the fixed surface:

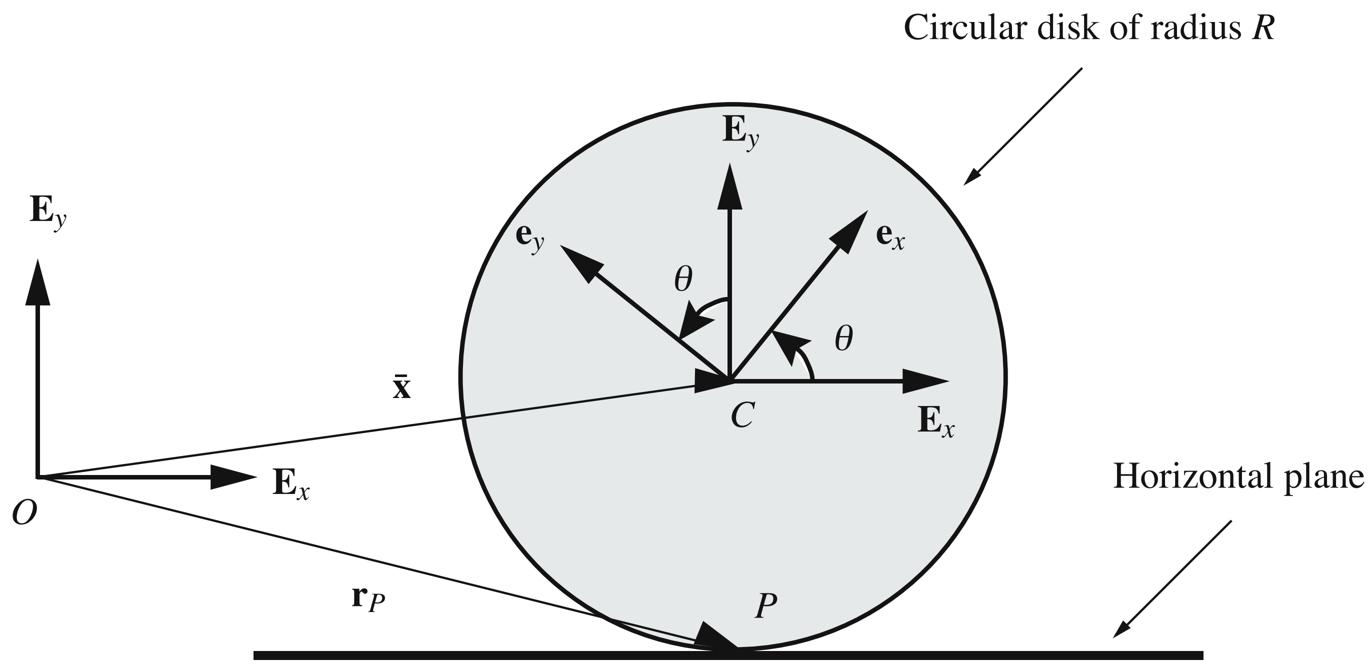
which implies the rolling condition:

Finally, we note that the acceleration of P for a rolling rigid body is not necessarily $\bar{0}$!

8.6 Kinematics of a Rolling Circular Disk

A common problem in rigid body dynamics is the rolling circular disk of radius R.

First, we define the corotational basis:



Because this is fixed-axis (planar) rotation,

The center of mass position vector is written

Using the rolling condition, we find that

The velocity of P is zero. Let's calculate its acceleration.

Wait, what? How can $\bar{v}_P = \bar{0}$ and $\bar{a}_P \neq \bar{0}$?
In fact, if we calculate $\dot{\bar{r}}_P$ and summarize what we know:

Turning to the velocity and acceleration of an arbitrary point on the body $\underline{X}$, we write:

$$\bar{X} - \bar{x} = x_1 \bar{e}_x + y_1 \bar{e}_y ,$$

where x_1 & y_1 are constants.

Velocity:
$$\bar{v} = \bar{\bar{v}} + \bar{\omega} \times (\bar{x} - \bar{\bar{x}})$$
$$= -R\dot{\theta}\bar{E}_x + \dot{\theta}\bar{E}_z \times (x_1 \bar{e}_x + y_1 \bar{e}_y)$$
$$= -R\dot{\theta}\bar{E}_x + \dot{\theta}(x_1 \bar{e}_y - y_1 \bar{e}_x)$$

Acceleration:

$$\bar{a} = \bar{\bar{a}} + \alpha \times (\bar{x} - \bar{\bar{x}}) + \bar{\omega} \times (\bar{\omega} \times (\bar{x} - \bar{\bar{x}}))$$
$$= -R\ddot{\theta}\bar{E}_x + \ddot{\theta}\bar{E}_z \times (x_1 \bar{e}_x + y_1 \bar{e}_y) + \dot{\theta}\bar{E}_z \times (\dot{\theta}E_z \times (x_1 e_x + y_1 e_y))$$
$$= -R\ddot{\theta}\bar{E}_x + \ddot{\theta}(x_1 \bar{e}_y - y_1 \bar{e}_x) - \dot{\theta}^2 (x_1 \bar{e}_x + y_1 \bar{e}_y)$$

Let's examine two points A and B on a disk rolling at constant horizontal translational velocity.