9.2 Work-Energy Theorem and Energy Conservation

9.2.1 Koenig's Decomposition

The definition of the kinetic energy of a rigid body is

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where $\mathcal{R}$ is the region of space occupied by the body, ρ is its density, and $\bar{v}$ is the velocity of a material point in the body.

In practice, however, we use Koenig's decomposition of this expression for a rigid body:

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where m is the body's mass, $\bar{\bar{v}}$ is the velocity of the center of mass, $\bar{H}$ is the angular momentum about the center of mass, and $\bar{\omega}$ is the angular velocity.

9.2.2 The Work-Energy Theorem

Starting with the Koenig decomposition of T, the first form of the work-energy theorem for a rigid body can be derived:

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where $\bar{F}$ is the resultant force on the body and $\bar{M}$ is the resultant moment about the center of gravity. This is a natural extension of the work-energy theorem for a single particle

There is a second form of the theorem that is often useful:

$$\boxed{}\ ,$$

where $\overline{F}_i$ are the n forces applied to a body from §9.1, $\overline{v}_i$ are the velocities of the points where the $\overline{F}_i$ are applied, and $\overline{M}_p$ is the externally applied moment (not a result of the $\overline{F}_i$).

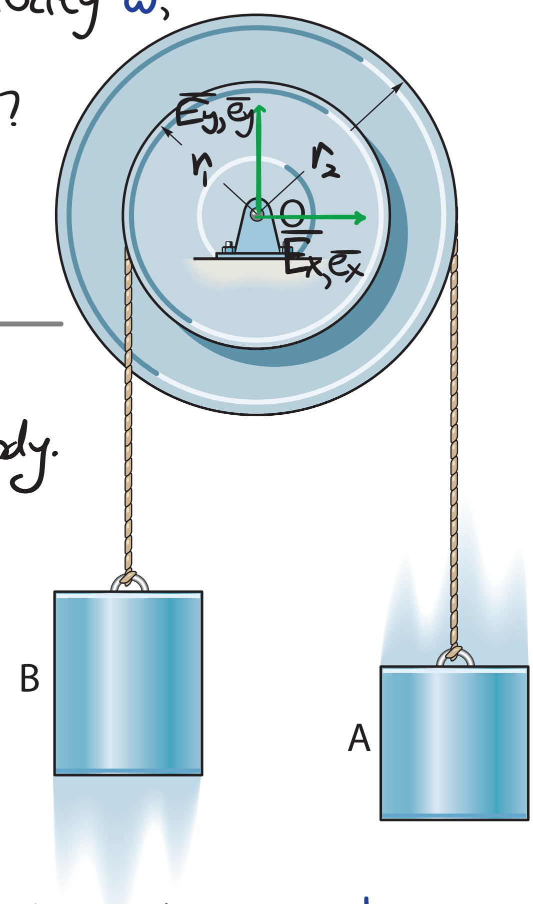
So we can see from this form of the theorem that:

I. The mechanical power of a force $\overline{P}$ applied to a body at a point X on the body is $\overline{P}\cdot\overline{v}$, where $\overline{v}$ is the velocity of point X.

II. The mechanical power of an applied moment $\overline{L}$ is $\overline{L}\cdot\overline{\omega}$.

Something like Hibbeler 18-7 (example)

Given the double spool's angular velocity ω, what is the system's kinetic energy?
Is the energy of the system conserved?
Assume that the ropes do not slip.
The moment of inertia of the spool about O in the $\overline{E}_z$-direction is I_{zz}.



The kinetic energy of the system is the sum of the kinetic energy of each body.

The kinetic energy of the spool is

Since B and A are translating and not rotating, $T_A = \frac{1}{2} m_A \overline{v}_A \cdot \overline{v}_A$ and $T_B = \frac{1}{2} m_B \overline{v}_B \cdot \overline{v}_B$.

So all we need to find are the velocities of A and B.

If we assume the ropes aren't slack, the tangential velocity of each mass is the same as the corresponding ropes at each point, which is the same as the corresponding radius of the spool's vel. Therefore, if we find the velocity of a point on each radius, we will have the velocity of each mass.

Using the important equation relating the velocities of any two points on a rigid body, using O as one of the points because it has a known (zero) velocity, we can find the velocity $\underline{v}_1$ of a point on the radius $\underline{X}_1$:

Similarly for $\underline{v}_2$ of point $\underline{X}_2$:

Finally: $T_A = \frac{1}{2} m_A \underline{v}_A \cdot \underline{v}_A = \frac{1}{2} m_A r_2^2 \omega^2$ and $T_B = \frac{1}{2} m_B r_1^2 \omega^2$. The total is:

What about energy conservation? The only forces or moments external to the system are gravity and the pin reaction, which we now assume has no friction.

The gravitational force is conservative, as it always is. The pin reaction force does no work because the point at which they are applied has zero velocity (doesn't move).

Therefore, the system's energy is conserved. ← ANS