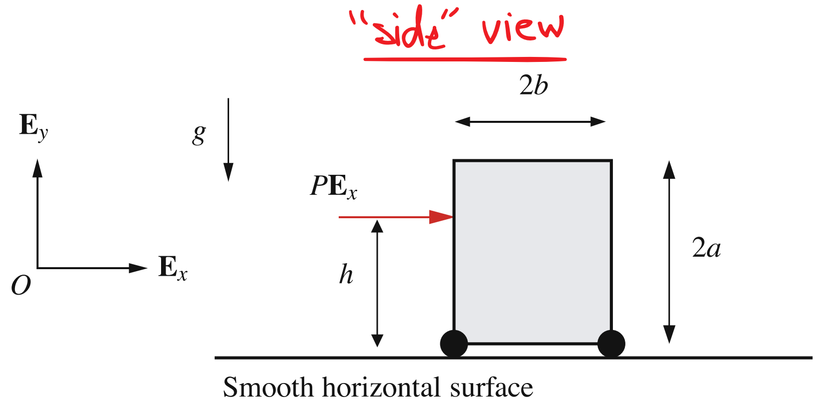
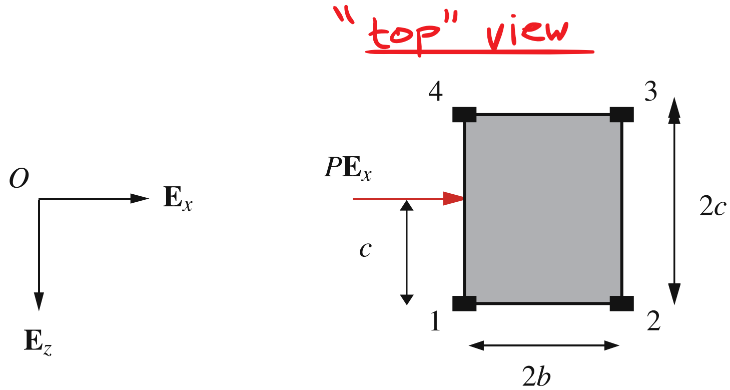
9.3.1 The Overturning Cart

The cart of mass m slides on a smooth horizontal surface.



Over what range of applied force P will the cart not tip? Ignore the mass of the wheels.



What assumptions are required to obtain the answer?

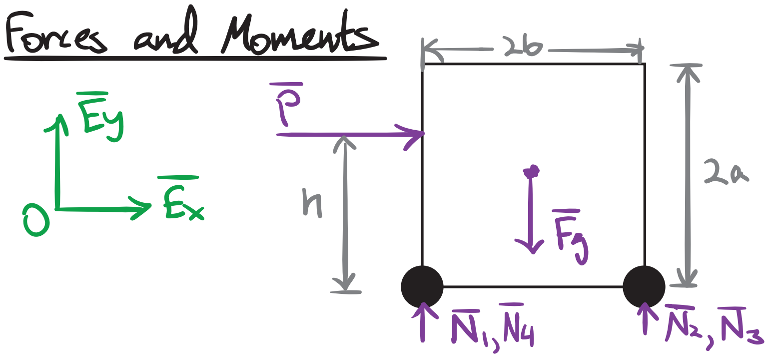
Kinematics

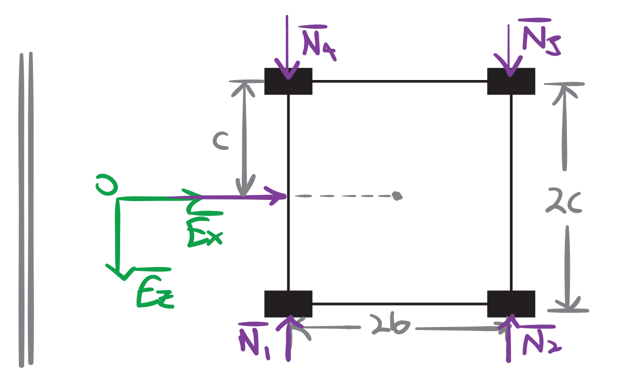
The center of mass has the following position, velocity, and acceleration:

The momenta are: $\bar{G} = m\bar{v} = m\dot{x}\bar{E}_x$ and $\bar{H} = I\bar{\omega} = \bar{0}$

The time rate of change of these momenta are: $\dot{\bar{G}} = m\ddot{x}\bar{E}_x$ and $\dot{\bar{H}} = \bar{0}$.

Forces and Moments





Gravity: force:

Reaction forces: forces:

Similarly, if we write out the position vectors of each wheel relative to the center of mass, we get

$$\bar{N}_i = N_{iy}\bar{E}_y + N_{iz}\bar{E}_z \quad \text{and}$$

$$\bar{M}_2 = (\bar{X}_2 - \bar{X}) \times \bar{N}_2 = (-cN_{2y} - aN_{2z})\bar{E}_x - bN_{2z}\bar{E}_y + bN_{2y}\bar{E}_z$$

$$\bar{M}_3 = (cN_{3y} - aN_{3z})\bar{E}_x - bN_{3z}\bar{E}_y + bN_{3y}\bar{E}_z$$

$$\bar{M}_4 = (cN_{4y} - aN_{4z})\bar{E}_x + bN_{4z}\bar{E}_y - bN_{4y}\bar{E}_z \quad .$$

Applied force: $\bar{P} = P\bar{E}_x$, moment: $\bar{M}_P = (\bar{X}_P - \bar{X}) \times \bar{P}$

$$= (-b\bar{E}_x + (h-a)\bar{E}_y + 0\bar{E}_z) \times P\bar{E}_x$$

$$= (a-h)P\bar{E}_z \, .$$

<u>Euler's Laws</u>

First Law in Cartesian coordinates:

Second Law in Cartesian coordinates: $\bar{M} = \dot{\bar{H}}$

$$\begin{bmatrix} c(-N_{1y} - N_{2y} + N_{3y} + N_{4y}) - a(N_{1z} + N_{2z} + N_{3z} + N_{4z}) \\ b(N_{1z} - N_{2z} - N_{3z} + N_{4z}) \\ b(-N_{1y} + N_{2y} + N_{3y} - N_{4y}) + (a-h)P \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We have six scalar equations.

Analysis

We have eight unknown reaction forces and the unknown acceleration $\ddot{x}$ and only six equations. This is an indeterminate system, so we must make some assumptions. We're only interested in the $\bar{E}_y$-direction because we want to know the conditions under which the wheels lose contact. Therefore, let's ignore the equations with $\bar{E}_z$-component reaction forces. Also, the $\bar{E}_x$-component of the First Law doesn't involve reaction forces, so we ignore it as well.

This leaves the $\bar{E}_y$ scalar equation of the First Law and the $\bar{E}_z$ scalar equation of the Second Law. With four unknown reaction forces in these equations, we must make two assumptions.

Let's assume that the front wheels have the same $\bar{E}_y$ reaction and the rear wheels have the same (but different than the front, in general) $\bar{E}_y$ reaction. That is:

and ← ANS

← ANS

.

Now we can easily solve for the reactions:

$$N_{1y} = N_{4y} = \frac{1}{4}\left(mg - \frac{(h-a)}{b}P\right)$$
$$N_{2y} = N_{3y} = \frac{1}{4}\left(mg + \frac{(h-a)}{b}P\right) \quad .$$

The threshold for losing contact is $N_{1y} = N_{4y} < 0$ or $N_{2y} = N_{3y} < 0$ for the rear and front wheels, respectively. This implies, for the cart not to tip,

$\Rightarrow$ The cart will not tip if $\boxed{}$.