ME 331 Midterm Exam
April 25, 2017

1. Consider the energy loss through a single 4.0 mm thick glass window-pane, of conductivity k = 1.5 W/(m K) and surface area 1.2 m^2 on a windy and overcast day (with negligible solar influx) in Seattle. The effective heat transfer coefficient outside is 42 W/(m^2 K) with a free-stream temperature of $T_{\infty,\ outside}$ = 2.0°C. The natural convection coefficient inside the room is 7.5 W/(m^2 K), and the room air temperature is $T_{\infty,\ inside}$ = 22.0°C. Calculate and discuss the conduction and convection resistances involved and their relative magnitudes. Remember that the conductive resistance for a plane wall is L / [k*A] where L is the glass thickness, and convective resistances are 1 / [h*A]. Calculate the rate of heat loss through the window-pane (in Watts) and the inside and outside glass surface temperatures (in °C). Plot the exact temperature distribution through the glass thickness on this page, then extend the plot to show temperature changes in the gas phase near both the inner and outer surfaces of the window-pane.

2. Consider the planet Earth of diameter $D = 1.276 * 10^7$ m with average thermal conductivity of $k = 35$ W/(m K) and surface temperature $T_s = 278$ K. Express the exact ***steady-state*** temperature distribution T(r) as a function of Earth's radial coordinate for a total thermal generation rate of $28.2 * 10^{12}$ W distributed uniformly within the Earth's volume. First, calculate the heat generation rate per unit volume (in Watts / m^3). Describe the control volume you have chosen for the purpose of determining T(r). What is the maximum temperature and where does it occur? Plot the temperature distribution on this page making sure to include the surface and maximum temperatures with the correct shape of the temperature profile.

3. Consider a solid rubber sphere of radius $R = 1.2$ cm, conductivity $k = 0.145$ W/(m K), density $\rho = 1100$ kg / m^3, and specific heat $c = 1800$ J / (kg K), initially at a uniform temperature $T_i = 25°C$. Suddenly at $t = 0$ seconds, the sphere is immersed into boiling water at $T_\infty = 100°C$ with heat transfer coefficient $h \approx 5000$ W/(m^2 K). What is the surface temperature T_s (approximate) of the sphere? After $t = 420$ seconds of immersion in the boiling water, what is the temperature near the center of the sphere at $r = 0.12$ cm? What approximation did you use and why is it valid?

   Hint: The single term approximation would be valid if Fourier = $(\alpha\, t / R^2) > 0.2$.
   Recall $r^* = r / R$, $Bi = h\, R / k$, $Fo = (\alpha\, t / R^2)$ and $\alpha = k / (\rho * c)$.

4. Consider the same rubber sphere as in problem 3, but now calculate the temperature at a position 0.12 cm ***below the outer surface*** of the sphere at t = 2.5 seconds. Why can you not use the approximation you used in problem 3?

Hint: If Fourier = $(\alpha\, t / R^2) < 0.05$, consider a semi-infinite solid solution.