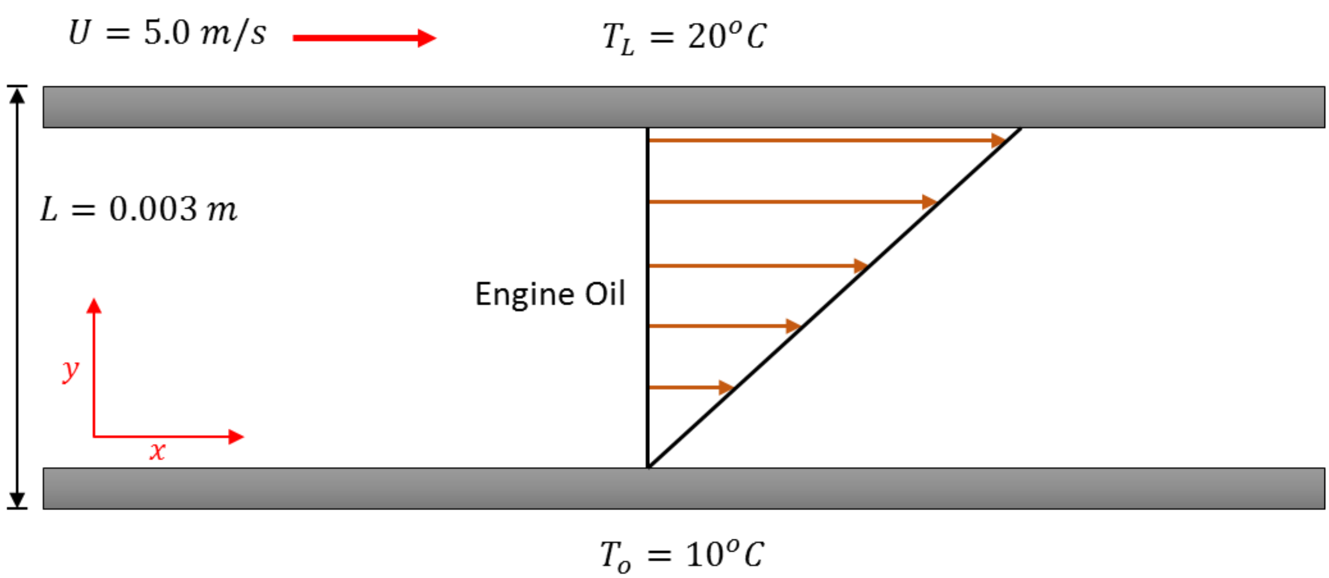
ME 331 Homework Assignment #5

Problem 1

A journal bearing can be idealized as a stationary flat plate and a moving flat plate that moves parallel to it. Consider such a bearing in which the stationary and moving plates are at 10°C and 20°C, respectively, the distance between them is 3.0 mm, the speed of the moving plate is 5.0 m/s, and there is engine oil between the plates. Calculate the heat flux to the upper and lower plates. Determine the maximum temperature of the oil.

Find: **heat flux at upper and lower plates ($q''|_{y=0}, q''|_{y=L}$) and maximum oil temperature (T_{MAX}).**

Schematic:



Assumptions:

1. Couette flow
2. Fully developed hydrodynamically and thermally
3. Steady State
4. Engine oil properties at mean temperature of $T_f = \frac{T_o+T_L}{2} = \frac{20+10}{2}\ °C = 288\ K$ are (Table A.5) :

$\mu = 1.23\ \frac{N \cdot s}{m^2}$

$k = 0.145\ \frac{W}{m \cdot K}$

Analysis:

Starting with temperature profile for Couette flow:

$$T(y) = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + c_1 y + c_2$$

We determine the constants using our boundary conditions

$$T(0) = T_o = c_2$$

$$T(L) = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 L^2 + c_1 L + T_o = -\frac{\mu}{2k}U^2 + c_1 L + T_o = T_L$$

$$c_1 = \frac{T_L - T_o}{L} + \frac{\mu}{2kL}U^2$$

$$T(y) = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + \left[\frac{T_L - T_o}{L} + \frac{\mu}{2kL}U^2\right] y + T_o$$

$$T(y) = -\frac{\mu U^2}{2k}\left[\left(\frac{y}{L}\right)^2 - \left(\frac{y}{L}\right)\right] + (T_L - T_o)\left(\frac{y}{L}\right) + T_o$$

$$q'' = -k\nabla T = -k\frac{dT}{dy}$$

$$\frac{dT}{dy} = -\frac{\mu U^2}{2k}\left[\frac{2}{L^2}y - \frac{1}{L}\right] + \frac{T_L - T_o}{L}$$

At the bottom plate

$$\left.\frac{dT}{dy}\right|_{y=0} = \frac{\mu U^2}{2kL} + \frac{T_L - T_o}{L} = \frac{1.23\frac{N\cdot s}{m^2}\left(5\frac{m}{s}\right)^2}{2\cdot 0.145\frac{W}{mK}\cdot 0.003\ m} + \frac{(20-10)K}{0.003\ m} = 35{,}344.8\frac{K}{m} + 3{,}333.3\frac{K}{m}$$

$$\left.\frac{dT}{dy}\right|_{y=0} = 38{,}678\frac{K}{m}$$

At the top plate

$$\left.\frac{dT}{dy}\right|_{y=L} = -\frac{\mu U^2}{2k}\left[\frac{2}{L^2}L - \frac{1}{L}\right] + \frac{T_L - T_o}{L} = -\frac{\mu U^2}{2kL} + \frac{T_L - T_o}{L}$$

$$\left.\frac{dT}{dy}\right|_{y=L} = -\frac{1.23\frac{N\cdot s}{m^2}\left(5\frac{m}{s}\right)^2}{2\cdot 0.145\frac{W}{mK}\cdot 0.003\ m} + \frac{(20-10)K}{0.003\ m} = -35{,}344.8\frac{K}{m} + 3{,}333.3\frac{K}{m}$$

$$\left.\frac{dT}{dy}\right|_{y=L} = -32{,}011\frac{K}{m}$$

Hence the flux at the bottom is:

$$q''|_{y=0} = -k\frac{dT}{dy}\bigg|_{y=0} = -\left(0.145\frac{W}{mK}\right)38{,}678\frac{K}{m} = -5{,}608.3\frac{W}{m^2}$$

$$\boxed{q''|_{y=0} = -5{,}600\frac{W}{m^2}}$$ **ANSWER**

Here the negative sign indicates that heat is flowing into (cooler) bottom plate.

The flux at the top is:

$$q''|_{y=L} = -k\frac{dT}{dy}\bigg|_{y=L} = -\left(0.145\frac{W}{mK}\right)\left(-32{,}011\frac{K}{m}\right) = 4612\frac{W}{m^2}$$

$$\boxed{q''|_{y=L} = 4{,}600\frac{W}{m^2}}$$ **ANSWER**

The positive sign here indicates that heat is also flowing into (warmer) upper plate. This is a result of heat generation due to viscous flow.

The maximum temperature occurs where $\frac{dT}{dy} = 0$

$$\frac{dT}{dy} = -\frac{\mu U^2}{2k}\left[\frac{2}{L^2}y - \frac{1}{L}\right] + \frac{T_L - T_o}{L} = 0$$

$$\frac{\mu U^2}{2k}\left[\frac{2}{L^2}y - \frac{1}{L}\right] = \frac{T_L - T_o}{L}$$

$$y = \frac{L^2}{2}\left(\frac{2k}{\mu U^2}\frac{T_L - T_o}{L} + \frac{1}{L}\right) = \frac{kL}{\mu U^2}(T_L - T_o) + \frac{L}{2}$$

$$y = \frac{0.145\frac{W}{m\cdot K}\cdot 0.003m(20-10)K}{1.23\frac{N\cdot s}{m^2}\cdot\left(5.0\frac{m}{s}\right)^2} + \frac{0.003m}{2} = 0.00164\ m$$

$$T_{MAX} = T(y = 0.00164\ m)$$

$$= -\frac{1.23\frac{Ns}{m^2}\left(5\frac{m}{s}\right)^2}{2\left(0.145\frac{W}{mK}\right)}\left[\left(\frac{0.00164\ m}{0.003\ m}\right)^2 - \left(\frac{0.00164\ m}{0.003\ m}\right)\right] + (20-10)K\left(\frac{0.00164\ m}{0.003\ m}\right) + 10°C$$

$$= -106.03°C(-0.2478) + 5.467°C + 10°C = 41.74°C$$

$$\boxed{T_{MAX} = 42°C}$$ **ANSWER**

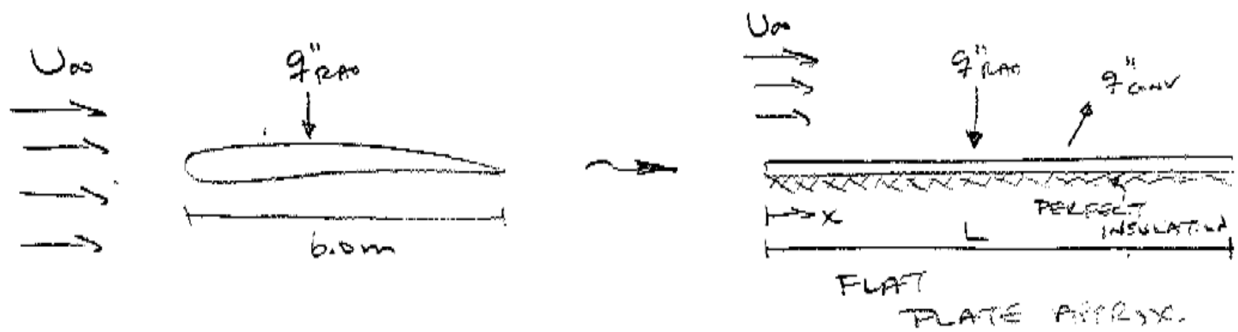
The maximum temperature occurs at $y = 1.6\ mm$, above the bottom plate.

Problem 2

The wing of an airplane has a polished aluminum skin. At an altitude of 1500 meters it absorbs 100 W/m^2 by solar radiation. Assuming that the interior surface of the wing's skin is well insulated and the wing has a chord of 6.0 m length, estimate the equilibrium temperature of the wing at a flight speed of 150 m/s at distances of 0.10 m, 1.0 m, and 5.0 m from the leading edge.

Find: **Equilibrium Temperature at three locations along the wing.**

Schematic:



Assumptions:

1. Steady State
2. Wing approximated as flat plate
3. Polished wing ($\epsilon = 0$) so no radiation emitted by wing.
4. All incoming solar radiation is absorbed by wing ($\alpha_{SOLAR} = 1.0$).
5. Perfect insulation below wing surface, so that $q''_{\underset{in}{radiation}} = q''_{\underset{out}{conv}}$
6. Transition to turbulence at $Re_{cr} = 5\times10^5$
7. Standard atmosphere conditions at an altitude of 1500 meters, so that:

   $T_\infty = 5.25°C$ $\qquad$ $P_{1500m} = 8.469\times10^4 Pa$

   see for instance

   (http://www.engineeringtoolbox.com/international-standard-atmosphere-d_985.html).

Analysis:

For a flat plate with constant surface flux we have two possible correlations to find the Nusselt number: eq. 7.45 (laminar) and eq. 7.46 (turbulent).

Since incoming solar flux and T_∞ are both low, we can assume that the film temperature is close to T_∞. So, we evaluate the air properties at $T_\infty = 278.4\ K$.

From Table A.4:

$\mu = 173.8 \times 10^{-7} \frac{kg}{sm}, k = 0.0246 \frac{W}{mK}\ ,\ \ Pr = 0.7126$

(Although these quantities are given at atmospheric pressure, they depend weakly on pressure.)

From ideal gas law

$$p = \rho RT\ \Rightarrow \rho = \frac{p}{RT} = \frac{84{,}690 Pa}{287.058 \frac{J}{kgK} 278.4K} = 1.060 \frac{kg}{m^3}$$

$$Re_{x=0.1m} = \frac{\rho U_\infty x}{\mu} = \frac{1.060 \frac{kg}{m^3} \cdot 150 \frac{m}{s} \cdot 0.1\ m}{173.8 \times 10^{-7} \frac{kg}{s \cdot m}} = 9.148 \times 10^5 > 5 \times 10^5$$

Hence, we have turbulent flow. Using equation 7.46:

$$Nu_x = 0.0308 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0308 (9.148 \times 10^5)^{4/5} (0.7126)^{1/3} = 1616.4$$

$$h_{x=0.1m} = Nu_x \cdot \frac{k}{x} = 1616.4 \cdot \frac{0.0246 \frac{W}{mK}}{0.1\ m} = 397.6 \frac{W}{m^2K}$$

To find the temperature we used the energy balance on the plate.

$$\dot{E}_{in,solar} = \dot{E}_{out,conv}$$

$$q''_{rad} = q''_{conv} = h_x \left(T_{s,x} - T_\infty\right)$$

$$T_{s,x} = \frac{q''_{rad}}{h_x} + T_\infty = \frac{100 \frac{W}{m^2}}{397.6 \frac{W}{m^2K}} + 5.25°C = 5.501°C$$

$\boxed{T_s|_{x=0.1m} = 5.5°C}$ **ANSWER**

Repeating the procedure for $x = 1.0\ m$ and $x = 5.0\ m$

$Re_{x=1.0m} = 9.148{\times}10^6$ $\quad Nu_{x=1.0m} = 10{,}199$ $\quad h_{x=1.0m} = 250.9\,\frac{W}{m^2K}$

$Re_{x=5.0m} = 5.489{\times}10^7$ $\quad Nu_{x=5.0m} = 42{,}765.5$ $\quad h_{x=5.0m} = 175.3\,\frac{W}{m^2K}$

$\boxed{T_s|_{x=1.0m} = 5.6°C}$ **ANSWER**

$\boxed{T_s|_{x=5.0m} = 5.8°C}$ **ANSWER**

Comments:

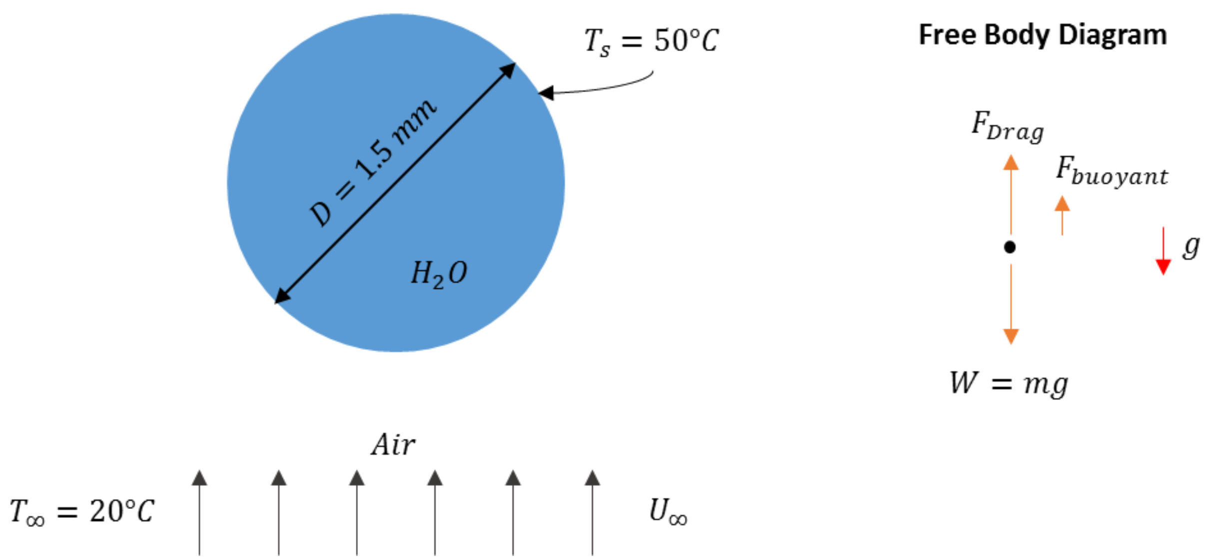
The assumption that $T_{film} \approx T_\infty$ is a very good approximation, in this case. With increasing distance along the airfoil, the heat transfer coefficient for the turbulent boundary layer is decreasing. For constant $q''_{\substack{radiation\\in}} = q''_{\substack{conv\\out}}$ this results in increasing T_s.

Problem 3

A spherical water droplet of 1.5 mm diameter is freely falling in atmospheric air. Calculate the average convection heat transfer coefficient when the droplet has reached its terminal velocity. Assume that the water is at 50°C and the air is at 20°C. Neglect mass transfer and radiation.

Find: **Average heat transfer coefficient** $\bar{h}$

Schematic:



Assumptions:

1. Steady state
2. Constant air properties
3. Neglect radiation
4. Neglect mass transfer effects
5. Neglect upward buoyant force
6. Properties of air at $T_\infty = 293\ K$:

$\rho = 1.194\ kg/m^3$ $\qquad$ $k = 0.02574\ W/mK$

$\nu = 15.27\times10^{-6}\ m^2/s$ $\qquad$ $Pr = 0.709$

Analysis:

We have a convection correlation (equation 7.49) specifically derived for freely falling liquid drops. However, we need to calculate the Reynolds number first, for which we need the drop's speed.

We get the drop's speed from the terminal velocity condition. At terminal velocity there is no acceleration. Hence:

$$\sum \boldsymbol{F} = mg - F_{drag}$$

$$mg = C_D \cdot A_f \frac{\rho_{air} U_\infty^2}{2}$$

Here we have two unknowns and only one equation, so we can write the flow velocity in terms of the Reynolds number

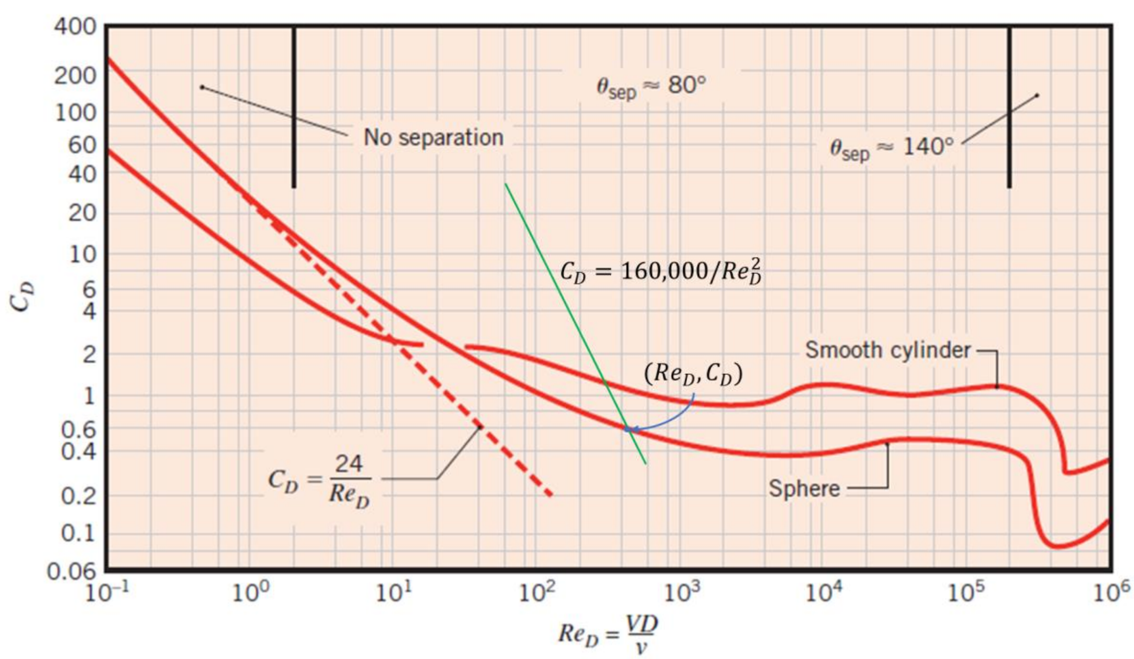
$$Re_D = \frac{U_\infty D}{\nu} \Rightarrow U_\infty = \frac{\nu \cdot Re_D}{D}$$

$$mg = C_D \cdot A_f \frac{\rho_{air}\left(\frac{\nu \cdot Re_D}{D}\right)^2}{2}$$, where A_f is the frontal area

$$C_D = \frac{2mgD^2}{A_f \rho_{air} \nu^2 Re_D^2} = \frac{2\rho_{H_2O} V g D^2}{\frac{\pi D^2}{4}\rho_{air}\nu^2 Re_D^2} = \frac{8\rho_{H_2O}\frac{\pi}{6}D^3 g D^2}{\pi D^2 \rho_{air} \nu^2 Re_D^2} = \frac{4}{3}\frac{D^3 g}{\nu^2}\frac{\rho_{H_2O}}{\rho_{air}}\frac{1}{Re_D^2}$$

$$C_D = \frac{4}{3}\frac{(0.0015\ m)^3\left(9.81\frac{m}{s^2}\right)}{\left(15.27{\times}10^{-6}\frac{m^2}{s}\right)^2}\frac{1000\ kg/m^3}{1.194\ kg/m^3}\frac{1}{Re_D^2} = \frac{158{,}562}{Re_D^2}$$

Now determine values of C_D and Re_D using figure 7.9 which relates the two:



From the figure above we can see that

$$Re_D \approx 10^{2.42} = 263$$

$$C_D \approx 0.6$$

Now we can use equation 7.49:

$$\overline{Nu}_D = 2 + 0.6\, Re_D^{1/2} Pr^{1/3} = 2 + 0.6(263)^{1/2}(0.709)^{1/3} = 10.676$$

Then use eqn. 7.44:

$$\boxed{\bar{h} = \overline{Nu}_D \frac{k}{D} = 10.676 \frac{0.02574 \frac{W}{mK}}{0.0015 m} = 183 \frac{W}{m^2 K}}$$

ANSWER

Comments:

The terminal velocity was

$$U_\infty = Re_D \cdot \frac{\nu}{D} = 263 \cdot \frac{\left(15.27 \times 10^{-6} \frac{m^2}{s}\right)}{0.0015\ m} = 2.7 \frac{m}{s}$$

Which is a reasonably good approximation compared to results from:

Gunn, R., & Kinzer, G. (1949). THE TERMINAL VELOCITY OF FALL FOR WATER DROPLETS IN STAGNANT AIR. *Journal of Meteorology, 6*(4), 243-248.

Problem 4

Neglecting radiation, consider a convective heat loss of 100 W from your body if you are standing on a windy hilltop with average wind velocity of 15 mph at T_∞ = 20°C. Without sufficient protective layers of clothing, the average surface temperature of your body under these conditions is estimated at 27°C (or 10°C lower than the core body temperature of 37°C). The surface area of a typical adult human is approximately 1.7 m^2. If the wind dies down (quiescent air), the convective heat loss from your body decreases to 80 W, and your average surface temperature increases to 30°C. Compare the heat transfer coefficients for these two cases.

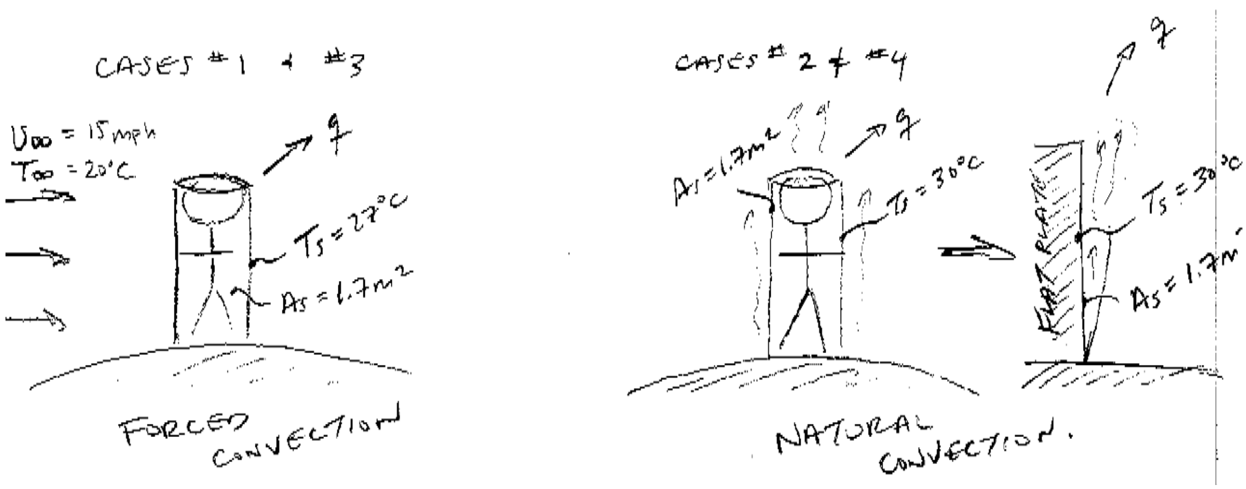
Now, compare the two rates of heat loss in air (for forced and natural convection) with the rates of heat loss that would occur in water at T_∞ = 20°C. For the natural convection case, consider standing in a lap pool with the water depth equal to your body height. Determine the rate of heat loss in quiescent water, assuming your average surface temperature is 30°C (the same as in natural convection of air).

Next, if the water flow is turned on with an average water velocity of 15 mph, and you remain standing in the lap pool, what is the rate of heat loss, assuming your average surface temperature is 27°C (the same as in forced convection of air)?

Find: **Average heat transfer coefficient $\bar{h}$ for cases 1 and 2.**

Rate of heat loss for cases 3 and 4.

Schematic:



Assumptions:

1. Steady state, constant fluid properties in each case.

2. The human body is approximated to be a cylinder in cross flow for case 3 and a vertical flat plate for case 4.

3. Assume the person is 6 ft tall

Analysis:

CASE#1

$$q = \bar{h}A(T_s - T_\infty)$$

$$\bar{h} = \frac{q}{A(T_s - T_\infty)}$$

$$\bar{h}_1 = \frac{100W}{1.7m^2(27-20)K} = 8.403\frac{W}{m^2K}$$

$$\boxed{\bar{h}_1 = 8.4\frac{W}{m^2K}}$$

Forced convection

CASE#2

$$\bar{h}_2 = \frac{80W}{1.7m^2(30-20)K} = 4.706\frac{W}{m^2K}$$

$$\boxed{\bar{h}_2 = 4.7\frac{W}{m^2K}}$$

Free convection

Forced convection coefficient is about twice as large as the free convection coefficient.

CASE#3

We will model the human body as a cylinder in cross flow for this problem, evaluating the properties at film temperature.

$$T_f = \frac{(27+20)°C}{2} = 23.5°C = 296.5K$$

Properties for saturated water at 296.5K

$\rho = 997.7\ kg/m^3$ $\quad\mu = 927.8\times10^{-6}\ N\cdot s/m^2$ $\quad k = 0.608\ W/mK$

$Pr = 6.383$ $\quad\nu = 0.930\times10^{-6}m^2/s$

$$Re_D = \frac{U_\infty D}{\nu} = \frac{6.706\frac{m}{s}0.296m}{0.930\times10^{-6}m^2/s} = 2.133\times10^6$$

Where $D_{body} = \frac{A_s}{\pi H}$

$$\overline{Nu}_D = 0.3 + \frac{0.62Re_D^{0.5}Pr^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{\frac{2}{3}}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282\times10^3}\right)\right]^{4/5} = 5{,}437$$

$$\bar{h} = \overline{Nu}_D\cdot\frac{k}{D} = 5{,}437\frac{\left(0.608\frac{W}{mK}\right)}{0.296\ m} = 11167\frac{W}{m^2K}$$

$$q = \bar{h}A(T_s - T_\infty) = 11167\frac{W}{m^2K}(1.7m^2)(27-30)K = 132{,}889\ W$$

$$\boxed{q = 133\ kW}$$

The heat transfer in moving water is three orders of magnitude higher than in air.

Notice that the skin surface temperature we used is unrealistically high. After a short exposure to such high heat transfer skin temperature would drop quickly.

CASE#4

Natural convection with flow along the human body approximated as flow along a vertical plate with the same surface area.

Properties at $T_f = \frac{(30+20)}{2}°C = 25°C = 298K$

Saturated water at 298K

$\rho = 997.4\ kg/m^3$ $\mu = 896.6\times10^{-6} Ns/m^2$ $k = 0.6102\ W/mK$

$Pr = 6.146$ $\beta = 256.7\times10^{-6}K^{-1}$ $c = 4.18\times10^3\ J/kgK$

$\alpha = 1.464\times10^{-7}m^2/s$ $\nu = 8.989\times10^{-7}m^2/s$

$$Ra_H = \frac{g\beta(T_s - T_\infty)H^3}{\nu\alpha} \quad \text{(where } H = 6ft = 1.829\ m\text{)}$$

$$Ra_H = 1.171\times10^{12} > 10^9$$

So the boundary layers transitions from laminar to turbulent.

$$\overline{Nu}_H = \left\{0.825 + \frac{0.387Ra_H^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = 1{,}451.3$$

$$\bar{h} = \overline{Nu}_H \cdot \frac{k}{L} = 482.2\frac{W}{m^2K}$$

$$q = \bar{h}A(T_s - T_\infty) = 8{,}231.3\ W$$

$$\boxed{q = 8.2\ kW}$$

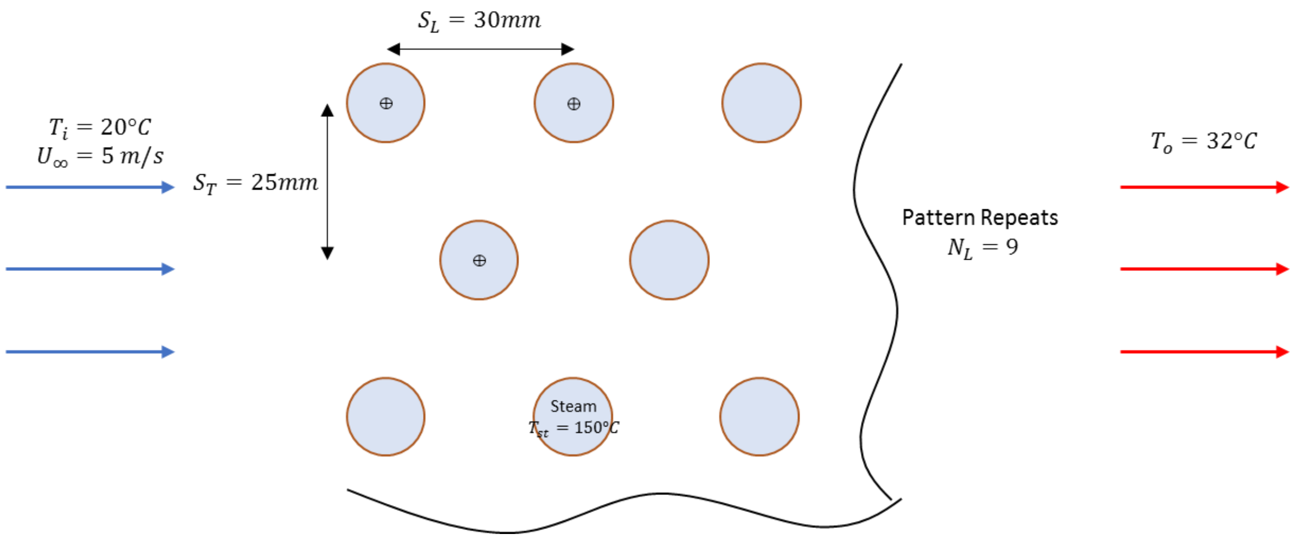
This rate of heat loss from your body in quiescent water is still very high, at ~80 times the normal rate of heat loss. Body surface temperature quickly drops to reduce the rate of heat loss.

Problem 5

Consider a heat exchanger consisting of 12.5 mm OD copper tubes in a staggered arrangement with transverse spacing of 25 mm and longitudinal spacing of 30 mm with 9 tubes in the longitudinal direction. Condensing steam at 150°C flows inside the tubes. The heat exchanger is used to heat a stream of air flowing at 5.0 m/s from 20°C to 32°C. What are the average heat transfer coefficient and pressure drop for the tube bank?

Find: Average heat transfer coefficient $\bar{h}$ and pressure drop Δp

Schematic:



Assumptions:

1. Condensing steam maintains a constant temperature inside tubes of 150°C.
2. $T_s = T_{steam}$ due to high k_{copper} and low thermal resistance of tube wall.
3. Steady state, negligible radiation, constant fluid properties.
4. $T_{\substack{avg\\air}} = \frac{(20+32)°C}{2} = 26°C = 299\ K$

Properties of air at 300 K (Table A.4)

$\rho = 1.1614\ kg/m^3$ $\quad k = 0.0263\ W/mK$ $\quad \nu = 15.89 \times 10^{-6}\ m^2/s$

$Pr = 0.707$

For air at $T_s = 150°C = 423K$, $Pr_s = 0.688$

Analysis:

We can use correlations 7.50 and 7.51 to calculate the Nusselt number. In order to get the maximum Reynolds number we need to calculate the maximum flow speed.

We check in which plane the maximum velocity occurs using the following condition (see section 7.6) :

$$S_D = \left[S_L^2 + \left(\frac{S_T}{2}\right)^2\right]^{1/2} < \frac{S_T + D}{2}$$

$$S_D = 32.5\,mm > 18.75\,mm$$

Since the above condition is not met, the maximum velocity occurs at A_1; hence the maximum velocity is given by eqn. 7.52:

$$V_{MAX} = \frac{S_T}{S_T - D}U_\infty = \frac{25mm}{25mm - 12.5mm}5.0\frac{m}{s} = 10\frac{m}{s}$$

$$Re_{D_{MAX}} = \frac{V_{MAX}D}{\nu} = \frac{10\frac{m}{s} \cdot 0.0125\,m}{15.89\times 10^{-6} m^2/s} = 7867$$

From Table 7.5, for $10^3 < Re_{D_{MAX}} < 2\times 10^5$ and $0.7 < \frac{S_T}{S_L} = \frac{5}{6} = 0.83 < 2$,

$$C_1 = 0.35\left(\frac{S_T}{S_L}\right)^{1/5} = 0.35\left(\frac{5}{6}\right)^{1/5} = 0.3375$$

And $$m = 0.60$$

Using equation 7.51, we obtain a correction for ($N_L < 20$), from Table 7.6 :

$$C_2 = 0.963 \quad \text{(interpolated)}$$

Hence

$$\overline{Nu}_D = C_2 C_1 Re_{D_{MAX}}^m Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{0.25}$$

$$\overline{Nu}_D = 0.963 \cdot 0.3375 \cdot 7867^{0.6} \cdot 0.707^{0.36}\left(\frac{0.707}{0.688}\right)^{0.25} = 62.8$$

We can compute $\bar{h}$:

$$\bar{h} = \overline{Nu}_D \cdot \frac{k_{air}}{D} = \frac{(62.8)\left(0.0263\frac{W}{mK}\right)}{0.0125\,m} = 132.18\frac{W}{m^2K}$$

$$\boxed{\bar{h} = 130\frac{W}{m^2K}}$$

ANSWER

We find the pressure drop from equation 7.57:

$$\Delta p = N_L \chi \left(\frac{\rho V_{MAX}^2}{2} \right) f$$

Use figure 7.15 to determine χ and f

$$\frac{P_T}{P_L} = \frac{\frac{S_T}{D}}{\frac{S_L}{D}} = \frac{S_T}{S_L} = \frac{25}{30} = \frac{5}{6} = 0.833$$

$$P_T = \frac{25mm}{12.5mm} = 2$$

For $P_T = 2$ and $Re_{D_{MAX}} \approx 8000$ we read from figure 7.15, $f = 0.4$ and $\chi = 1.05$

$$\Delta p = 9 \cdot 1.05 \cdot \left(1.1614 \frac{kg}{m^3} \right) \left(10 \frac{m}{s} \right)^2 \cdot \frac{1}{2} \cdot 0.4 = 219.5 \frac{kg}{m^3} \cdot \frac{m^2}{s^2}$$

$\boxed{\Delta p = 220\ Pa}$ **ANSWER**