HW Assignment #4

Problem #1

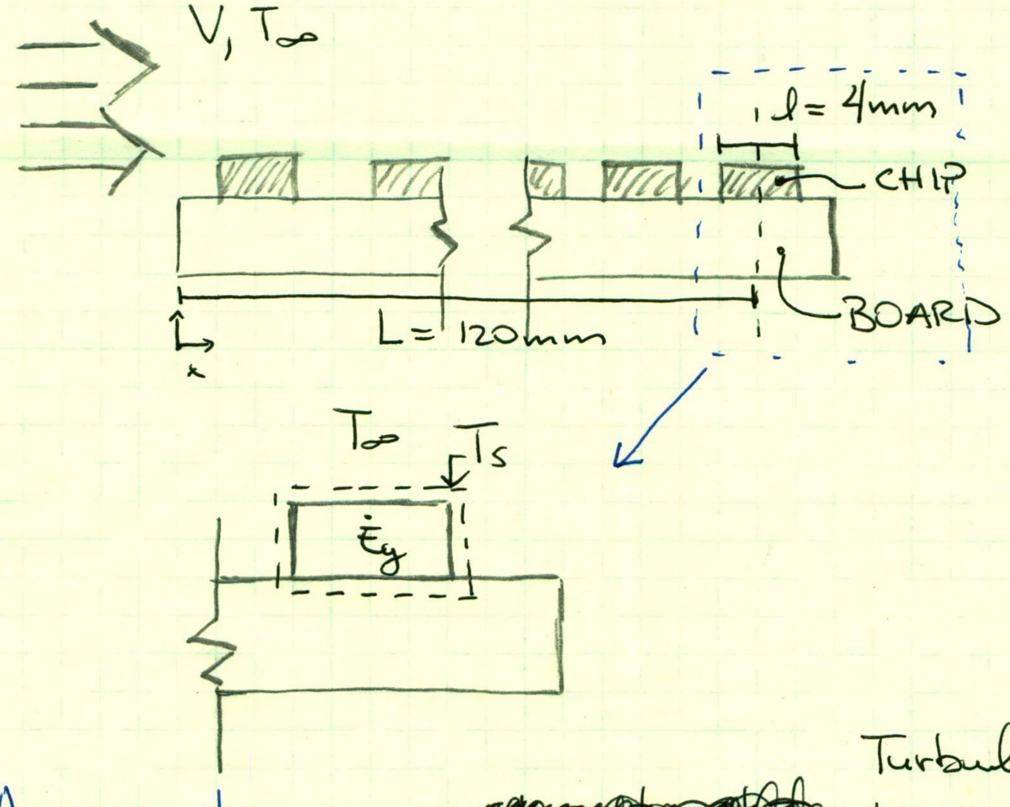
Statement: Circuit board cooled by forced convection.

$T_\infty = 20°C$ $\quad A = 4 \times 4\ mm^2$ $\quad \dot{E}_g = 40\ mW$

$V = 10\ m/s$ $\quad L = 120\ mm$ $\quad Nu_x = 0.04\ Re_x^{0.85}\ Pr^{1/3}$

Find: Chip surface temperature (T_s).

Schematic:



Assumptions:

- [redacted] Turbulent Boundry layer due to chip disturbance
- All $\dot{E}_g$ is dissipated by convection
- h_L = average heat transfer coefficient

Analysis:

We start with Energy Balance: $\dot{E}_{in}^{\,0} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}^{\,0}$ (steady state)

$\dot{E}_{out} = \dot{E}_g = 40 \times 10^{-3}\,W$

$\dot{E}_{out} = hA(T_s - T_\infty)$

Now we only need h, we will use correlations.

Start by calculating the local Reynolds Number:

$Re_x = \frac{Vx}{\nu}$, $\nu_{air} = 15.89 \times 10^{-6} \frac{m^2}{s}$ @ 300k

(from table A.4)

$$Re_x = \frac{10\frac{m}{s} \cdot 0.12\,m}{15.89 \times 10^{-6}\frac{m^2}{s}} = \underline{75,519}$$

Calculate the local Nusselt number

$Nu_x = 0.04\, Re_x^{0.85}\, Pr^{1/3}$; $Pr_{air} = 0.707$ @ 300k

(Table A.4)

$$Nu_x = 0.04\,(75,519)^{0.85}(0.707)^{1/3} = \underline{499.13}$$

Now we can calculate h at $x = 0.120\,m$

$Nu_x = \frac{hx}{k}$, $k_{air} = 26.3 \times 10^{-3} \frac{W}{m\text{-}k}$ @ 300k

(Table A.4)

$$h_x = \frac{Nu_x k}{x}$$

Problem 1 continued

$$h_x = \frac{Nu_x k}{x} = \frac{499.13 \cdot 26.3\times10^{-3} \frac{W}{m\text{-}K}}{0.12m}$$

$$\underline{h_x = 109.39} \ \frac{W}{m^2K}$$ local heat transfer coefficient

Going back to energy balance:

$hA(T_s - T_\infty) = 0.040\,W$ where $A = (0.004m)^2$

$$\Rightarrow T_s = \frac{0.040\,W}{109.39\frac{W}{m^2K}(1.6\times10^{-5}m^2)} + 20°C = \underline{42.85°C}$$

$$\boxed{T_s = 43°C}$$

Answer

Problem #2

Statement: circuit board from problem 1, different altitude.

Seattle @ sea level $p = 1$ atm

La Paz, Bolivia @ 3650m above sea level $p = 0.64$atm

Find: a) Surface temperature at same location on chip when operated in La Paz.

b) Required air velocity to maintain chip @ same surface temperature as problem 1.

Schematic: Same as problem 1.

Assumptions: Same as problem 1.

Analysis:

(a) Calculate local Re_x for $p = 0.64$atm

$$Re_x = \frac{Vx}{\nu} \quad \text{and} \quad \nu = \frac{\mu}{\rho}$$

Kinematic viscosity is dependent on density

From ideal gas law we can calculate ρ @ $p = 0.64$atm

$$p = \rho RT \Rightarrow \rho = \frac{p}{RT}$$

$$\nu_{La Paz} = \frac{\mu}{\rho_{La Paz}} \quad ; \quad \nu_{seattle} = \frac{\mu}{\rho_{seattle}}$$

Problem 2 continued.

μ does not vary with pressure.

$$\Rightarrow \mu = \rho_{LaPaz} \nu_{LaPaz} = \rho_{Seattle} \nu_{Seattle}$$

$$\Rightarrow \nu_{LaPaz} = \nu_{Seattle} \frac{\rho_{Seattle}}{\rho_{LaPaz}} = \nu_{Seattle} \frac{P_{Seattle}}{P_{LaPaz}}$$

$$= 15.89 \times 10^{-6} \frac{m^2}{s} \left(\frac{1 \text{ atm}}{0.64 \text{ atm}} \right)$$

$$= \underline{24.83 \times 10^{-5} \frac{m^2}{s}}$$

$$\Rightarrow Re_x = \frac{10 \frac{m}{s} (0.12 m)}{24.83 \times 10^{-5} \frac{m^2}{s}} = \underline{48,332}$$

lower than in Seattle!

Calculate local Nusselt Number

$$Nu_x = 0.04 Re_x^{0.85} Pr^{1/3}$$

Prandtl number Does Not depend on pressure, it only depends on temperature

$$Pr = \frac{\nu}{\alpha} = \frac{C_p \mu}{k}$$

$$\Rightarrow Nu_x = 0.04 (48332)^{0.85} (0.707)^{1/3}$$

$$Nu_x = \underline{341.56}$$

Calculate local h

$$h_x = \frac{Nu_x k}{x} = \frac{341.56 \cdot 26.3 \times 10^{-3} \frac{W}{m \cdot K}}{0.12 m} = 74.86 \frac{W}{m^2 K}$$

$h_x = 74.86 \frac{W}{m^2 K}$ again, lower than in Seattle!

Calculate surface temperature

$hA(T_s - T_\infty) = 0.040\ W$

$T_{s_{La\ Paz}} = \frac{0.040\ W}{74.86 \frac{W}{m^2K} (1.6 \cdot 10^{-3} m^2)} + 20°C = 53.40°C$

$T_{s_{La\ Paz}} = 53°C$ ANSWER

(b) From desired heat transfer coefficient and desired Reynolds number, calculate required speed.

$Re_x = 75{,}519 = \frac{Vx}{\nu}$

$V = \frac{75519\nu}{x}$

$V = \frac{75519 \cdot (24.83 \times 10^{-5} \frac{m^2}{s})}{0.12 m} = 15.625 \frac{m}{s}$

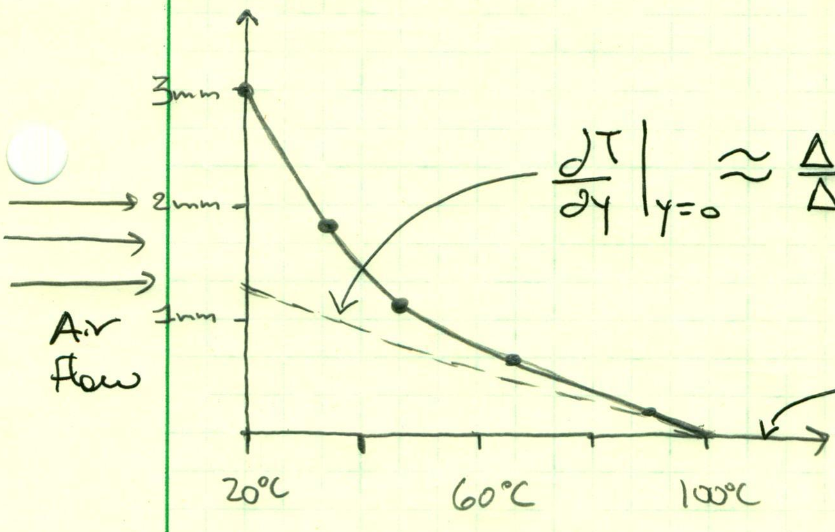
$V_{La\ Paz} = 16 \frac{m}{s}$ Answer

Problem #3

Statement: Flat plate subject to ambient air flowing at $T_\infty = 20°C$. Air temperature profile is given at the location of interest.

Find:
- heat flux q''
- local heat transfer coefficient h.
- Thermal boundary layer thickness δ_t

Schematic:



$$\left.\frac{\partial T}{\partial y}\right|_{y=0} \approx \frac{\Delta T}{\Delta y} = \frac{(20-100)K}{1.2mm} = -66.7 \frac{K}{mm}$$

$$k_{air} = 28.7 \times 10^{-3} \frac{W}{m \cdot K} \text{ evaluated at } T_{film} = \frac{T_s + T_\infty}{2} = 60°C$$
$$= 333K$$

Assumptions:
- Steady State
- Constant Properties
- Negligible Radiation

Analysis: $q'' = -k \left.\frac{\partial T}{\partial y}\right|_{y=0} = -0.0287 \frac{W}{mK} \left.\frac{\partial T}{\partial y}\right|_{y=0}$

$$q'' = -0.0287 \frac{W}{mK} \left(-66.7 \cdot 10^3 \frac{K}{m}\right) = 1914.29 \frac{W}{m^2}$$

$$\boxed{q'' = 1.9 \times 10^3 \frac{W}{m^2}}$$ Answer

To obtain the local heat transfer coefficient,

$$q'' = h(T_s - T_\infty)$$

$$\Rightarrow h = \frac{q''}{T_s - T_\infty} = \frac{1914.29 \text{ W/m}^2}{(100-20)\text{ K}} = 23.9286 \frac{W}{m^2 K}$$

$$\boxed{h = 24 \frac{W}{m^2 K}}$$ Answer

Now we estimate δ_t, boundary layer thickness.

δ_t is thickness at which $\left|\frac{T - T_s}{T_s - T_\infty}\right| = 0.99$

From the plot we estimate

$$\boxed{\delta_t = 3.0 \text{ mm}}$$ Answer