ME331: Introduction to Heat Transfer
Spring 2017

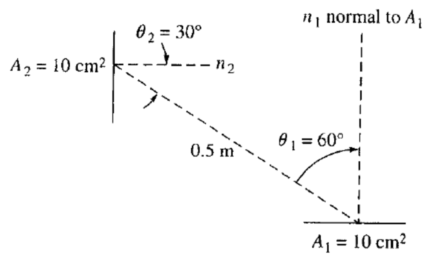
ME 331 Homework Assignment #7

Problem 1

Statement: A flat, black surface of area $A_1 = 10\ cm^2$ emits $1000\ W/(m^2\ sr)$ in the normal direction. A small surface A_2 having the same area as A_1 is placed relative to A_1 as shown in the figure, at a distance of 0.50m. Determine the solid angle subtended by A_2 and the rate at which A_2 is irradiated by A_1.

Find: **(a) Solid angle; (b) Irradiation rate G**

Schematic:



Assignments:

1. A_2 may be approximated as a differential surface, $(A_2/r^2)\ \leq 1$
2. Surface A_1 emits diffusely.

Analysis:

$$\omega_{2-1} = \frac{A_2 cos\theta_2}{r^2} = \frac{(10\times10^{-4})m^2\times \cos 30°}{(0.5m)^2} = 3.464\times10^{-3}sr$$

$\omega_{2-1} = 3.5\times10^{-3}sr$ **ANSWER**

Since A_1 is a diffuse emitter, the emitted intensity I is independent of direction: $I = I_n$

$$q_{1-2} = I \cdot A_1 cos\theta_1 \cdot \omega_{2-1} = 1000\frac{W}{m^2 sr} \cdot 10\times10^{-4}m^2\times cos60°\times3.464\times10^{-3}sr = 1.732\times10^{-3}W$$

$q_{1-2} = 1.7\ mW$ **ANSWER**

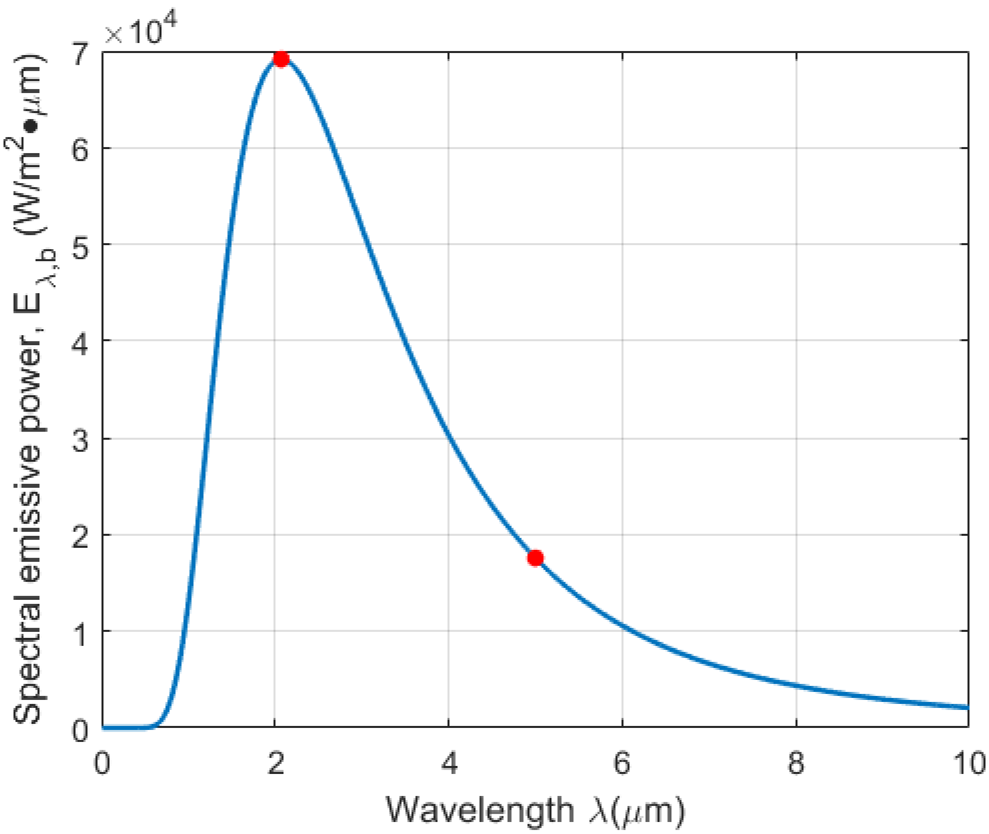
$G_{1-2} = \frac{q_{1-2}}{A_2} = 1.7\frac{W}{m^2}$

Problem 2

Statement: Determine the following: (a) the wavelength at which the spectral emissive power of a tungsten filament at 1400 K is maximized, (b) the spectral emissive power at that wavelength, and (c) the spectral emissive power at 5.0 μm

Find: **(a)** $\boldsymbol{\lambda_{max}}$ **(b) maximum** $\boldsymbol{E_{\lambda,b}}$ **(c) maximum** $\boldsymbol{E_{\lambda,b}\big|_{\lambda=5\ \mu m}}$

Schematic:



Assumptions:

1. Blackbody Radiation

Analysis:

(a) $\lambda_{max} = \frac{C_3}{T} = \frac{2898\ \mu m \cdot K}{1400\ K} = 2.07\ \mu m$ (eq 12.31)

$\lambda_{max} = 2.1\ \mu m$ **ANSWER**

(b) $E_{\lambda,b} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]} = \frac{3.742\times10^8 \frac{W\mu m^4}{m^2}}{(2.07\ \mu m)^5\left[\exp\left(\frac{1.439\times10^4\ \mu m\cdot K}{2.07\times1400\mu m\cdot K}\right)-1\right]} = 69{,}152\ \frac{W}{\mu m\cdot m^2}$ (eq. 12.30)

$$E_{\lambda,b}\big|_{\lambda_{max}} = 6.9\times10^4 \frac{W}{\mu m\cdot m^2}$$

ANSWER

(c) $\boldsymbol{E_{\lambda,b}\big|_{\lambda=5\ \mu m} = \dfrac{3.742\times10^8 \frac{W\mu m^4}{m^2}}{(5.0\ \mu m)^5\left[exp\left(\frac{1.439\times10^4 \mu m\cdot K}{5.0\times1400\mu m\cdot K}\right)-1\right]} = 17,577 \frac{W}{\mu m\cdot m^2}}$

$$E_{\lambda,b}\big|_{\lambda=5\mu m} = 1.8\times10^4 \frac{W}{\mu m\cdot m^2}$$

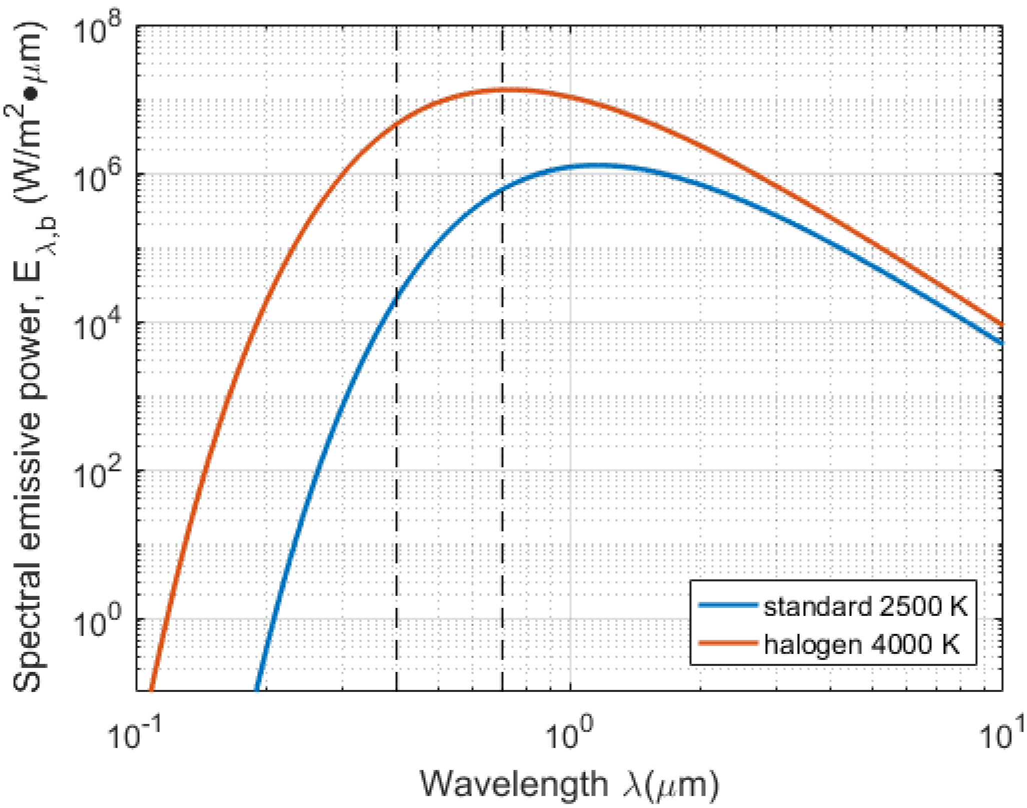
ANSWER

Problem 3

Statement: The filament of an incandescent lamp emits radiation in the visible and infrared wavelength ranges. A halogen lamp is an incandescent lamp with a halogen gas added to reduce evaporation of the filament. Halogen lamps operate at higher temperatures than standard incandescent lamps. Assuming that the filament temperatures of standard incandescent lamps and halogen lamps are 2500 K and 4000 K, respectively, and that both lamps radiate as blackbodies, find the fraction of radiation emitted in the visible range for these two lamps. Comment on the efficiencies of converting electrical energy into visible light.

Find: Fraction of radiation emitted in the visible range $F_{0.4-0.7\mu m}$ for the standard incandescent and halogen lamps.

Schematic:



The visible range is within the dashed lines, from 0.4 to 0.7 μm

Assumptions:

1. Blackbody radiation.
2. All the electrical energy is transformed to thermal energy in the form of radiation.
3. Visible range is $[0.4\ \mu m, 0.7\mu m]$

Analysis:

(a) Standard incandescent

$\lambda_1 T_1 = 0.40\mu m \cdot 2500K = 1000\ \mu m \cdot K$ $\quad F_1(0 \to \lambda_1) = 0.000321$ (Table 12.2)

$\lambda_2 T_1 = 0.70\mu m \cdot 2500K = 1750\ \mu m \cdot K$ $\quad F_1(0 \to \lambda_2) = 0.034435$

$F_1(\lambda_1 \to \lambda_2) = F_1(0 \to \lambda_2) - F_1(0 \to \lambda_1) = 0.034435 - 0.000321 = 0.034114$

$F_{1_{vis}} = 0.034$ **ANSWER**

(b) Halogen

$\lambda_1 T_2 = 0.40\mu m \cdot 4000K = 1600\ \mu m \cdot K$ $\quad F_2(0 \to \lambda_1) = 0.019718$

$\lambda_2 T_2 = 0.70\mu m \cdot 4000K = 2800\ \mu m \cdot K$ $\quad F_2(0 \to \lambda_2) = 0.227897$

$F_2(\lambda_1 \to \lambda_2) = F_2(0 \to \lambda_2) - F_2(0 \to \lambda_1) = 0.227897 - 0.019718 = 0.208179$

$F_{2_{vis}} = 0.21$ **ANSWER**

Comments:

The halogen lamp is much more efficient than the standard incandescent lamp (about six times more efficient).

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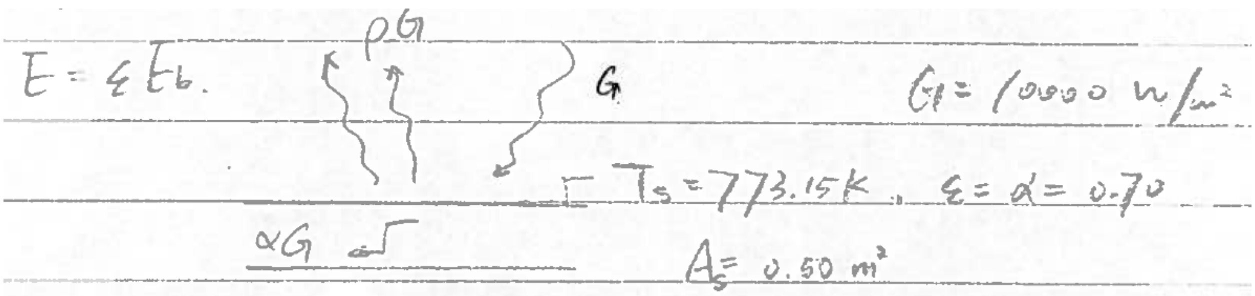
Problem 4

Statement: A plane, gray, diffuse, opaque surface with absorptivity 0.70 and a surface area of 0.50 m^2, is maintained at 500°C and receives radiant energy at a rate of 10,000 W/m^2. Determine per unit time: (a) the energy absorbed, (b) the radiant energy emitted, (c) the total energy leaving the surface per unit area, (d) the radiant energy emitted by the surface in the wave band 0.20 µm to 4.0 µm, and (e) the net radiation heat transfer from the surface.

Find: (a) the energy absorbed, (b) the radiant energy emitted, (c) the total energy leaving the surface per unit area, (d) the radiant energy emitted by the surface in the wave band 0.20 µm to 4.0 µm, and (e) the net radiation heat transfer from the surface.

Schematic:



Assumptions:

1. Gray, diffuse, opaque surface
2. $\epsilon = \alpha$

Analysis:

(a) $A_s \alpha G = 0.50 m^2 \times 0.70 \times 10{,}000 \frac{W}{m^2} = 3.5\ kW$

$A_s \alpha G = 3.5\ kW$ **ANSWER**

(b) $A_s \epsilon E_b = A_s \epsilon \sigma T_s^4 = 0.50 m^2 \times 0.70 \times 5.67 \times 10^{-8} \frac{W}{m^2 K^4} (773.15\ K)^4 = 7{,}091\ W$

$A_s \epsilon E_b = 7.1\ kW$ **ANSWER**

(c) $J = E + \rho G = E + (1-\alpha) G = \frac{7091 W}{0.5 m^2} + (1 - 0.70) 10000 \frac{W}{m^2} = 17{,}182 \frac{W}{m^2}$

$J = E + \rho G = 17\ kW$ **ANSWER**

(d)

$\lambda_1 T = 0.20\mu m \cdot 773.15K = 154.63\ \mu m \cdot K$ $\quad F(0 \to \lambda_1) = 0$ (Table 12.2)

$\lambda_2 T = 4.0\mu m \cdot 773.15K = 3092.6\ \mu m \cdot K$ $\quad F(0 \to \lambda_2) = 0.294007$

$F(\lambda_1 \to \lambda_2) = F(0 \to \lambda_2) - F(0 \to \lambda_1) = 0.294007$

$A_S \cdot E(\lambda_1 \to \lambda_2) = F(\lambda_1 \to \lambda_2) E \cdot A_s = 0.294007 \cdot 7{,}091\ W$

$A_S \cdot E(\lambda_1 \to \lambda_2) = 2.1\ kW$ **ANSWER**

(e) $q_{net} = A_s(E + \rho G - G) = A_s(E - \alpha G) = 0.50m^2\left(\frac{7091W}{0.5m^2} - 0.70 \times 10{,}000\,\frac{W}{m^2}\right) = 3591\ W$

$q_{net} = 3.6\ kW$ **ANSWER**

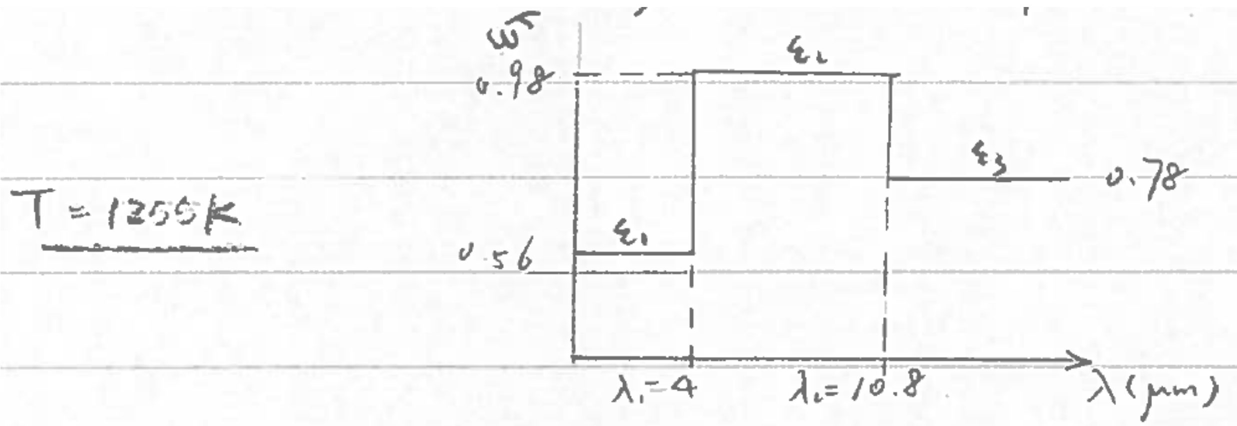
Where the positive sign indicates that the net radiation is out of the surface.

Problem 5

Statement: The spectral hemispherical emissivity of an aluminum oxide surface is shown in the figure. Determine the total hemispherical emissivity and the emissive power at 1255 K.

Find: total hemispherical emissivity and the emissive power at 1255 K.

Schematic:



Assumptions:

1. Use step function approximation.

Analysis:

$$\epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{\lambda,b} d\lambda}{E_b} = \frac{\int_0^{4\mu m} \epsilon_1 E_{\lambda,b} d\lambda}{E_b} + \frac{\int_{4\mu m}^{10.8\mu m} \epsilon_2 E_{\lambda,b} d\lambda}{E_b} + \frac{\int_{10.8\mu m}^{\infty} \epsilon_3 E_{\lambda,b} d\lambda}{E_b}$$

$$\epsilon = \epsilon_1 F(0 \to 4\mu m) + \epsilon_2 [F(0 \to 10.8\mu m) - F(0 \to 4\mu m)] + \epsilon_3 [1 - F(0 \to 10.8\mu m]$$

$\lambda_1 T = 4\mu m \times 1255K = 5020\ \mu m \cdot K$ $\quad$ $F(0 \to 4\ \mu m) = 0.636269$

$\lambda_2 T = 10.8\mu m \times 1255K = 13554\ \mu m \cdot K$ $\quad$ $F(0 \to 10.8\ \mu m) = 0.959437$

$$\epsilon = 0.56 \times 0.636269 + 0.98 \times [0.959437 - 0.636269] + 0.78 \times [1 - 0.959437]$$

$$\epsilon = 0.704654$$

$$E = \epsilon E_b = \epsilon \sigma T^4 = 0.704654 \times 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \times (1255K)^4 = 99114 \frac{W}{m^2}$$

$$\epsilon = 0.70$$

$$E = 9.9 \times 10^4 \frac{W}{m^2}$$

ANSWER

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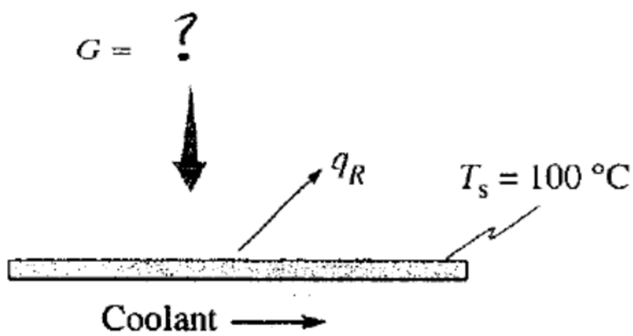
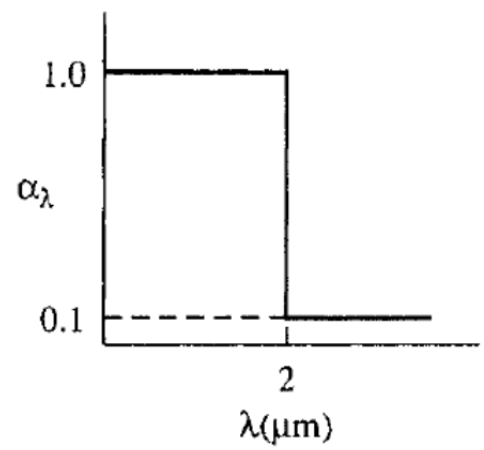
Problem 6

Statement: A specially coated diffuse, opaque surface with spectral absorptivity of 1.0 for 0<λ<2 and 0.10 for 2 μm<λ<∞ is exposed to solar radiation in the outer reaches of the atmosphere. Determine: (a) the heat flux by radiation from the surface to the surroundings if the surface is maintained at 100°C by a coolant, (b) the equilibrium temperature of the surface, if the coolant flow stops and the surface is insulated on the side that does not receive solar radiation, (c) compare the values in parts a and b if the surface is black.

Find: (a) the heat flux (b) the equilibrium temperature of the surface (c) compare the values in parts a and b if the surface is black.

Schematic:





Assumptions:

1. Diffuse, opaque surface

2. $\epsilon_\lambda = \alpha_\lambda$, since we have diffuse surface

3. Solar irradiation can be viewed as that from a blackbody at 5780 K.

Analysis:

Irradiation flux at the outer atmosphere is $S_c = \frac{\sigma T_{sun}^4 R_{sun}^2}{R_{sun\to earth}^2} = 1{,}364\,\frac{W}{m^2}$

(a) $\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{G} = \alpha_1 F(0 \to 2\mu m) + \alpha_2[1 - F(0 \to 2\mu m)]$

$\lambda_1 T_{sun} = 2\mu m \times 5780K = 11{,}560\ \mu mK$ $\qquad F(0 \to 2\mu m) = 0.940575$

$\alpha = 1.0 \times 0.940575 + 0.1 \times (1 - 0.940575) = 0.9465175$

$$G_{abs} = \alpha G = 0.9465175 \times 1{,}364\,\frac{W}{m^2} = 1291\,\frac{W}{m^2}$$

Similarly

$\epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{\lambda,b} d\lambda}{E_b} = \epsilon_1 F(0 \to 2\mu m) + \epsilon_2[1 - F(0 \to 2\mu m)]$

$\lambda_1 T_s = 2\mu m \times 373.15K = 746.3\ \mu mK$ $\qquad F(0 \to 2\mu m) = 0.000012$

$\epsilon = 1.0 \times 0.000012 + 0.1 \times (1 - 0.000012) = 0.1000108$

$$E = \epsilon\sigma T_s^4 = 0.1000108 \times 5.67 \times 10^{-8}\,\frac{W}{m^2K^4} \times (373.15K)^4 = 109.9\,\frac{W}{m^2}$$

$$q'' = E - G_{abs} = (109.9 - 1291)\,\frac{W}{m^2} = -1181\,\frac{W}{m^2}$$

$q'' = -1.2\,\frac{kW}{m^2}$ **ANSWER**

The negative sign indicates that heat must be removed from the surface to maintain it at 100°C.

(b) Still $G_{abs} = 1291\,\frac{W}{m^2}$ but E has changed so that

$q'' = E - G_{abs} = 0$

To get E we need ϵ but this will be a function of temperature as $F(0 \to 2\mu m)$ is a function of temperature, so we have to iterate to find solution. Take $T_s = 700\ K$ as initial guess, then

$\lambda_1 T_s = 2\mu m \times 700K = 1400\ \mu mK$ $\qquad F(0 \to 2\mu m) = 0.007790$

$\epsilon = 1.0 \times 0.007790 + 0.1 \times (1 - 0.007790) = 0.1070$

$$E = \epsilon\sigma T_s^4 = 0.1070 \times 5.67 \times 10^{-8}\,\frac{W}{m^2K^4} \times (700K)^4 = 1457\,\frac{W}{m^2}$$

$$q'' = E - G_{abs} = (1457 - 1291)\,\frac{W}{m^2} = 166\,\frac{W}{m^2}$$

So, we need a lower T_s. After iterations, we find that $T_s = 680.7\ K$

$$T_s = 680\ K$$

ANSWER

(c) $\epsilon = \alpha = 1$ for a black body

$G_{abs} = G = 1364\,\frac{W}{m^2}$

$E = E_b = \sigma T_s^4 = 5.67\times10^{-8}\,\frac{W}{m^2K^4}\times(373.15K)^4 = 1099.3\,\frac{W}{m^2}$

$$q'' = E - G_{abs} = (1099.3 - 1364)\,\frac{W}{m^2} = -264.7\,\frac{W}{m^2}$$

$$q'' = -260\,\frac{W}{m^2}$$

ANSWER

$$E = \sigma T_s^4 = G = 1364\,\frac{W}{m^2}$$

$$T_s = \left(\frac{G}{\sigma}\right)^{1/4} = \left(\frac{1364\,\frac{W}{m^2}}{5.67\times10^{-8}\,\frac{W}{m^2K^4}}\right)^{1/4} = 393.8\,K$$

$$T_s = 390\ K$$

ANSWER

Comments:

Notice that making the surface black results only in small increase in the absorbed radiation (about 5%) since the surface was originally “black” for the short wavelengths at which the sun emits. However, the emitted radiation increases substantially by making the surface black (390 K is about 117°C, just slightly above the temperature in part (a) where we needed the coolant) since the original emissivity at long (infrared) wavelengths was very small, and it is at these long wavelengths that a black body at around 100°C mostly radiates. In this case making the body black would actually result in huge savings in cooling (more than a fourfold decrease in the heat flux).