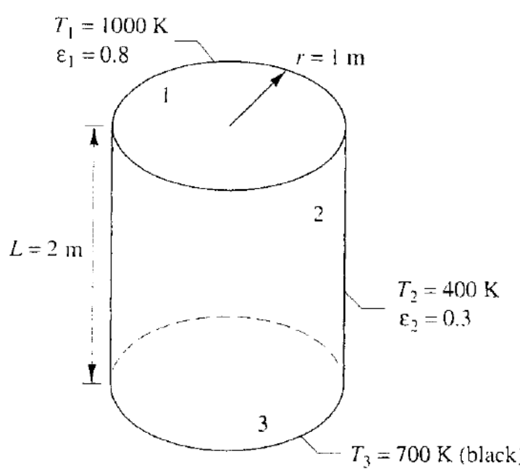
ME 331 Homework Assignment #8

Problem 1

Statement: A cylindrical enclosure has a length and radius of 2.0 m and 1.0 m, respectively, as shown in the figure. The top surface is maintained at 1000 K and has an emissivity of 0.80. The curved surface is maintained at 400 K and has an emissivity of 0.30. The bottom surface approximates a blackbody and is maintained at 700 K. Find the net radiation for each surface.

Find: Net radiation for each surface.

Schematic:



Assumptions:

1. Steady state
2. Diffuse, gray surfaces.

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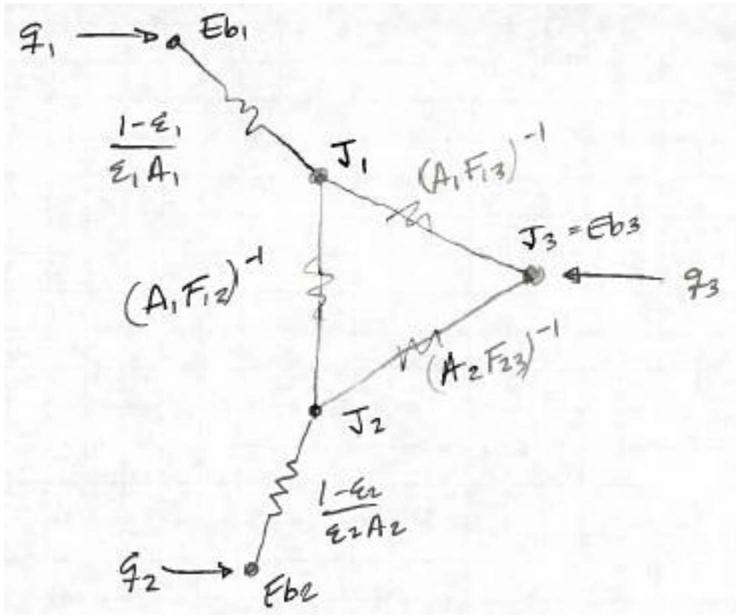
Analysis:

The resistance network looks like:



Summation at each J-node

(1) $q_1 = (J_1 - J_2)(A_1F_{12}) + (J_1 - J_3)(A_1F_{13}) = \left(E_{b_1} - J_1\right)\left(\frac{\epsilon_1 A_1}{1-\epsilon_1}\right)$

(2) $q_2 = (J_2 - J_1)(A_2F_{21}) + (J_2 - J_3)(A_2F_{23}) = \left(E_{b_2} - J_2\right)\left(\frac{\epsilon_2 A_2}{1-\epsilon_2}\right)$

(3) $q_3 = (J_3 - J_1)(A_3F_{31}) + (J_3 - J_2)(A_1F_{32})$

Surface 3 is black $J_3 = E_{b_3} = \sigma T_3^4$

To find view factors, we look at table 13.2, fig13.5

$R_i = \frac{r_i}{L} = \frac{1m}{2m} = 0.5$

$R_j = \frac{r_j}{L} = \frac{1m}{2m} = 0.5$

$S = 1 + \frac{1+R_j^2}{R_i^2} = 1 + \frac{1+0.25}{0.25} = 6$

$F_{ij} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2\right]^{0.5}\right\}$

$F_{13} = \frac{1}{2}\{6 - [6^2 - 4(1)^2]^{0.5}\} = 0.172$

$F_{13} = F_{31}$ from symmetry

Equation (1):

$$(J_1 - J_2)(A_1F_{12}) + (J_1 - J_3)(A_1F_{13}) = \left(E_{b_1} - J_1\right)\left(\frac{\epsilon_1 A_1}{1-\epsilon_1}\right)$$

$$(J_1 - J_2)(F_{12}) + (J_1 - J_3)(F_{13}) = E_{b_1}\left(\frac{\epsilon_1}{1-\epsilon_1}\right) - J_1\left(\frac{\epsilon_1}{1-\epsilon_1}\right)$$

$$J_1\left[F_{12} + F_{13} + \frac{\epsilon_1}{1-\epsilon_1}\right] + J_2[-F_{12}] = \sigma T_1^4\left(\frac{\epsilon_1}{1-\epsilon_1}\right) + \sigma T_3^4 F_{13}$$

$$\left[1 + \frac{0.8}{0.2}\right]J_1 - 0.828J_2 = \left(5.67\times10^{-8}\,\frac{W}{m^2K^4}\right)\left[(1000K)^4\left(\frac{0.8}{0.2}\right) + (700K)^4(0.172)\right]$$

Equation (4): $5J_1 - 0.828J_2 = 229{,}142\,\frac{W}{m^2}$

Equation (2)

$$\frac{\left((J_2-J_1)(A_2F_{21}) + (J_2-J_3)(A_2F_{23})\right)}{F_{21}} = \frac{\left(\left(E_{b_2} - J_2\right)\left(\frac{\epsilon_2 A_2}{1-\epsilon_2}\right)\right)}{F_{21}}$$

From symmetry $F_{23} = F_{21}$

$$J_2 - J_1 + (J_2 - J_3)\left(\frac{F_{23}}{F_{21}}\right) = \frac{E_{b2} - J_2}{F_{21}}\left(\frac{\epsilon_2}{1-\epsilon_2}\right)$$

$$-J_1 + 2J_2 - J_3 = \frac{E_{b2}}{F_{21}}\left(\frac{\epsilon_2}{1-\epsilon_2}\right) - \frac{J_2}{F_{21}}\left(\frac{\epsilon_2}{1-\epsilon_2}\right)$$

$$A_2F_{21} = A_1F_{12} \Longrightarrow F_{21} = \frac{A_1}{A_2}F_{12} = \frac{\pi R^2}{2\pi RL}F_{12} \Longrightarrow F_{21} = \frac{R}{2L}F_{12}$$

$$-J_1 + \left[2 + \frac{2L}{RF_{12}}\left(\frac{\epsilon_2}{1-\epsilon_2}\right)\right]J_2 = \frac{2L}{R}\frac{\sigma T_2^4}{F_{12}}\left(\frac{\epsilon_2}{1-\epsilon_2}\right) + \sigma T_3^4$$

$$-J_1 + \left[2 + \left(\frac{0.3}{0.7}\right)\left(\frac{2\times 2m}{1m\times 0.828}\right)\right]J_2 = \left(5.67\times10^{-8}\,\frac{W}{m^2K^4}\right)\left[\frac{2(2m)}{1m(0.828)}(400K)^4\left(\frac{0.3}{0.7}\right) + (700K)^4\right]$$

$$-J_1 + 4.0704J_2 = 16619\ W/m^2$$

Equation (5): $-5J_1 + 20.352J_2 = 83{,}095\ W/m^2$

Equation (4): $5J_1 - 0.828J_2 = 229{,}142\ W/m^2$

$$19.524J_2 = 312{,}237\ W/m^2$$

$$J_2 = 15992.5\ W/m^2$$

Equation (4): $5J_1 - 0.828\times15992.5\,\frac{W}{m^2} = 229{,}142\,\frac{W}{m^2}$

$$J_1 = 48{,}476.8\,\frac{W}{m^2}$$

Equation (1) $q_1 = \left(E_{b_1} - J_1\right)\left(\frac{\epsilon_1 A_1}{1-\epsilon_1}\right) = (\sigma T_1^4 - J_1)\left(\frac{\epsilon_1 \pi R^2}{1-\epsilon_1}\right))$

$$q_1 = \left[\left(5.67\times10^{-8}\frac{W}{m^2K^4}\right)(1000K)^4 - 48{,}476.8\frac{W}{m^2}\right]\left[\frac{(0.8)(\pi)(1m)^2}{0.2}\right] = 103{,}336\ W$$

$\boxed{q_1 = 1.03\times10^5 W}$ **ANSWER**

Eq. (2) $q_2 = \left(E_{b_2} - J_2\right)\left(\frac{\epsilon_2 A_2}{1-\epsilon_2}\right) = \left(\sigma T_2^4 - J_2\right)\left(\frac{\epsilon_2 2\pi RL}{1-\epsilon_2}\right)$

$$q_2 = \left[\left(5.67\times10^{-8}\frac{W}{m^2K^4}\right)(400K)^4 - 15{,}992.5\frac{W}{m^2}\right]\left[\frac{(0.3)(2\pi)(1m)(2m)}{0.7}\right] = -78{,}312\ W$$

$\boxed{q_2 = -7.8\times10^4 W}$ **ANSWER**

Eq. (3) $q_3 = (J_3 - J_1)(A_3F_{31}) + (J_3 - J_2)(A_1F_{13}) = (\sigma T_3^4 - J_1)(\pi R^2)(F_{31}) + (\sigma T_3^4 - J_2)(A_3F_{32})$

$$q_3 = \left[\left(5.67\times10^{-8}\frac{W}{m^2K^4}\right)(700K)^4 - 48{,}476.8\frac{W}{m^2}\right][\pi(1m)^2(0.172)] +$$
$$\left[\left(5.67\times10^{-8}\frac{W}{m^2K^4}\right)(700K)^4 - 15{,}992.5\frac{W}{m^2}\right][\pi(1m)^2(0.828)] = -25{,}026.3\ W$$

$\boxed{q_3 = -2.5\times10^4\ W}$ **ANSWER**

Comments:

We confirm that

$q_1 + q_2 + q_3 = 0$

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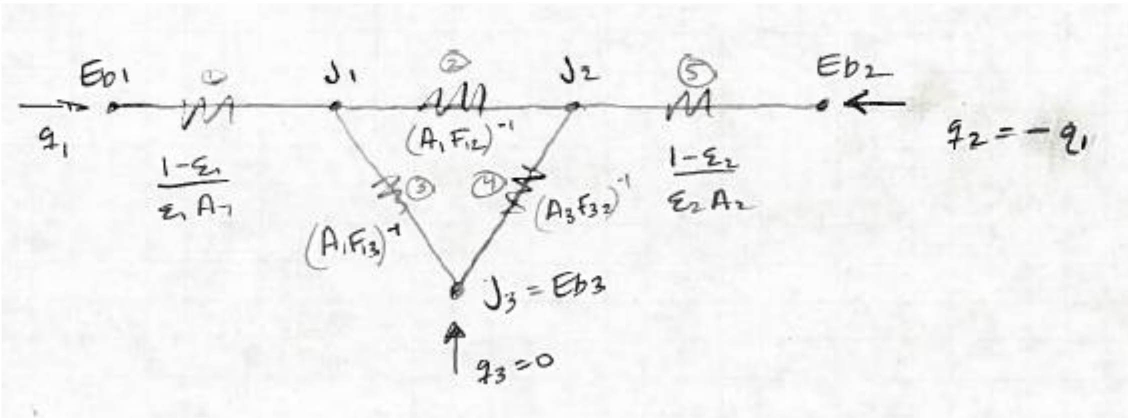
Problem 2

Statement: Repeat problem 2, replacing the black surface with a reradiating surface.

Find: net radiation for each surface.

Schematic:



Assumptions:

1. Same as in problem 1

Analysis:

$$q_1 = -q_2 = \frac{E_{b_1} - E_{b_2}}{R_{TOT}}$$

$$R_{TOT} = R_1 + R_{234} + R_5$$

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3 + R_4}$$

$$R_{TOT} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3 + R_4}\right)^{-1} + R_5$$

$$R_{TOT} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3 + R_4}\right)^{-1} + R_5$$

$$R_{TOT} = \frac{1-\epsilon_1}{\epsilon_1 A_1} + \left(\frac{1}{(A_1 F_{12})^{-1}} + \frac{1}{(A_1 F_{13})^{-1} + (A_3 F_{32})^{-1}}\right)^{-1} + \frac{1-\epsilon_2}{\epsilon_2 A_2}$$

We know all the quantities from problem 1

$$R_{TOT} = \frac{0.5933}{m^2}$$

$$q_1 = -q_2 = \frac{E_{b_1} - E_{b_2}}{R_{TOT}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{TOT}} = 93{,}120.6\ W$$

$$\boxed{\begin{aligned} q_1 &= -q_2 = 9.3\times 10^4 W \\ q_3 &= 0 \end{aligned}}$$

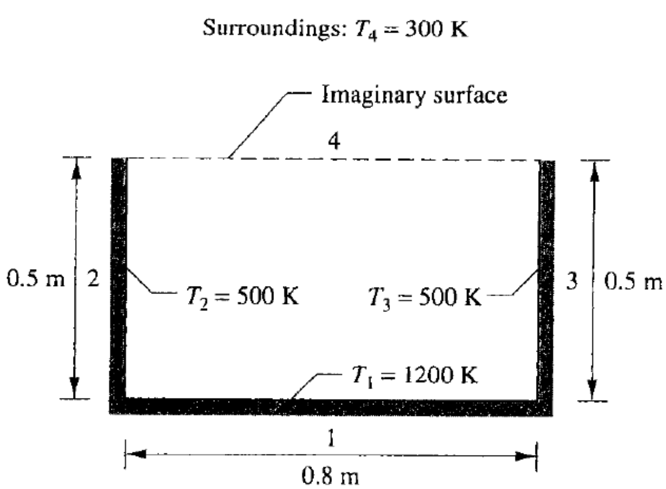
ANSWER

Problem 3

Statement: The long channel shown approximates a blackbody. The temperature of surface 1 is maintained at 1200 K, and the temperatures of surfaces 2 and 3 are maintained at 500 K. The channel is open to 300 K surroundings, which may also be approximated as black. Find the radiant heat loss to the surroundings.

Find: Radiant heat loss to surroundings.

Schematic:



Assumptions:

1. Steady state
2. All surfaces behave like black bodies
3. Surroundings are much larger than channel

Analysis:

$$q'_{lost} = q'_4$$

$$q_4 = A_2F_{24}\sigma(T_2^4 - T_4^4) + A_1F_{14}\sigma(T_1^4 - T_4^4) + A_3F_{34}\sigma(T_3^4 - T_4^4)$$

We find view factors from table 13.1, perpendicular plates with a common edge

$$F_{ij} = \frac{1 + \left(\frac{w_j}{w_i}\right) - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{0.5}}{2}$$

$$F_{12} = \frac{1 + \left(\frac{0.5}{0.8}\right) - \left[1 + \left(\frac{0.5}{0.8}\right)^2\right]^{0.5}}{2}$$

$$F_{12} = 0.223$$

$F_{12} = F_{13} = 0.223$ from symmetry

$$F_{14} = 1 - F_{12} - F_{13} = 1 - 2(0.223) = 0.554$$

$$q_4' = \frac{q_4}{L} = \frac{\sigma}{L}[A_2F_{24}(T_2^4 - T_4^4) + A_1F_{14}(T_1^4 - T_4^4) + A_3F_{34}(T_3^4 - T_4^4)]$$

$A_2F_{24} = A_2F_{21}$ from symmetry

$A_2F_{21} = A_1F_{12}$ from reciprocity

$A_3F_{34} = A_3F_{31}$ from symmetry

$A_3F_{31} = A_1F_{13}$ from reciprocity

$$q_4' = \frac{q_4}{L} = \frac{\sigma}{L}[A_1F_{12}(T_2^4 - T_4^4) + A_1F_{14}(T_1^4 - T_4^4) + A_1F_{13}(T_3^4 - T_4^4)]$$

$F_{12} = F_{13}$ from symmetry

$$T_2 = T_3 = 500\ K$$

$$q_4' = \frac{A_1\sigma}{L}[2F_{12}(T_2^4 - T_4^4) + F_{14}(T_1^4 - T_4^4)] = \frac{\sigma(w \cdot L)}{L}[2F_{12}(T_2^4 - T_4^4) + F_{14}(T_1^4 - T_4^4)]$$

$$q_4' = \left(5.67\times10^{-8}\frac{W}{m^2K^4}\right)(0.8m)[2(0.223)((500K)^4 - (300K)^4) + 0.554((1200K)^4 - (300K)^4)]$$

$$q_4' = 53{,}009.8\frac{W}{m}$$

$\boxed{q'_{lost} = q_4' = 53\,\frac{kW}{m}}$

ANSWER

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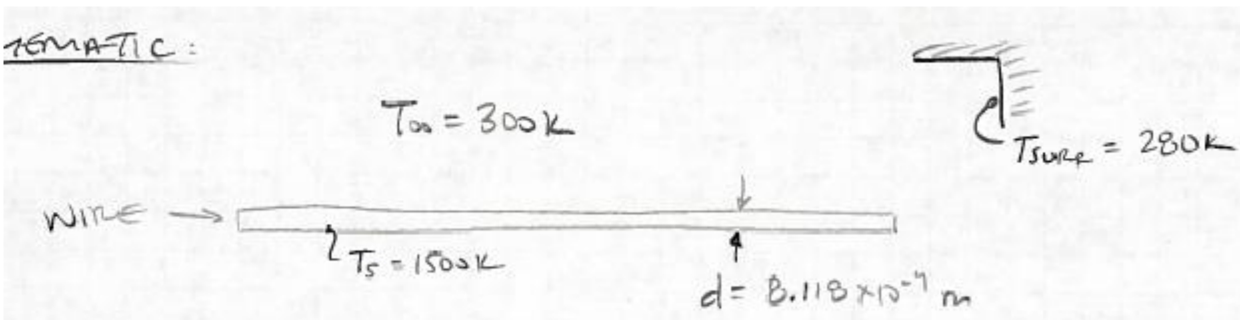
Problem 4

Statement: A long, bare 20-gage nichrome wire (R = 2.162 Ω/m, d = 8.118x10^{-4} m) runs horizontally through a 280K enclosure containing atmospheric air at 300 K. If the temperature of the wire is not to exceed 1500 K, find the maximum electrical current the wire can carry for the cases of (a) natural convection, and (b) forced convection across the wire at 20 m/s. Find the convection and radiation heat loss from the wire for both cases. Treat the nichrome wire as diffuse, gray with emissivity of 0.30.

Find: maximum electrical current (I_{max}), convection and radiation heat loss $(q_{conv}, q_{rad})$for: (a) natural convection; (b) forced convection

Schematic:



Assumptions:

1. Steady state
2. Wire has uniform temperature throughout
3. Treat as infinitely long cylinder
4. Radiation exchange between small object (wire) and large surroundings

Analysis:

(a) Natural Convection

Consider the energy balance for the wire

$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \frac{dE}{dt} = 0$ steady state assumption

$\dot{E}_{gen} = \dot{E}_{out}$ no energy input

$\dot{E}_{gen} = \dot{E}_{elec} = I^2R = q_{conv} + q_{rad}$

$$I^2R = \bar{h}A_s(T_s - T_\infty) + \epsilon A_s\sigma(T_s^4 - T_{surr}^4)$$

We have to find $\bar{h}$ for natural convection

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

Properties at $T_f = \frac{T_s+T_\infty}{2} = \frac{1500+300}{2}K = 900K$

From Table A-4

$\nu = 102.9\times10^{-6}m^2/s$ $\alpha = 143\times10^{-6}\ m^2/s$ $\beta = 1/T_f$

$k = 62.0\times10^{-3}\frac{W}{m\cdot K}$ $P_r = 0.720$

$$Ra_D = \frac{\left(9.81\frac{m}{s^2}\right)\left(\frac{1}{900K}\right)(1500-300)K(8.118\times10^{-4}m)^3}{(102.9\times10^{-6}\frac{m^2}{s})(143\times10^{-6}\frac{m^2}{s})}$$

$$Ra_D = 0.4756$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387Ra_D^{1/6}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$ eq. 9.34 for long horizontal cylinder

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387(0.4756)^{1/6}}{\left[1+\left(\frac{0.559}{0.720}\right)^{9/16}\right]^{8/27}}\right\}^2 = 0.7817$$

$$\bar{h} = \overline{Nu}_D\cdot\frac{k_f}{D} = \frac{(0.7817)\left(62.0\times\frac{10^{-3}W}{m\cdot K}\right)}{8.118\times10^{-4}m} = 59.70\frac{W}{m^2K}$$

$$I^2R' = \left(\frac{1}{L}\right)\left[\bar{h}A_s(T_s - T_\infty) + \epsilon A_s\sigma(T_s^4 - T_{surr}^4)\right]$$

$$I^2R' = \left(\frac{1}{L}\right)\left[\bar{h}\pi DL(T_s - T_\infty) + \epsilon\pi DL\sigma(T_s^4 - T_{surr}^4)\right] = \bar{h}\pi D(T_s - T_\infty) + \epsilon\pi D\sigma(T_s^4 - T_{surr}^4)$$

$$I^2R' = \bar{h}\pi D(T_s - T_\infty) + \epsilon\pi D\sigma(T_s^4 - T_{surr}^4)$$

$$q'_{conv} = \bar{h}\pi D(T_s - T_\infty)$$

$$q'_{rad} = \epsilon\pi D\sigma(T_s^4 - T_{surr}^4)$$

Substituting values

$$I^2R' = 182.71\frac{W}{m} + 219.35\frac{W}{m} = 402.06\frac{W}{m}$$

$$\boxed{q'_{conv} = 183\frac{W}{m} \quad q'_{rad} = 219\frac{W}{m}}$$

ANSWER

$$I = \sqrt{\frac{1}{R'}402.06\frac{W}{m}} = \sqrt{\frac{402.06\frac{W}{m}}{2.162\frac{\Omega}{m}}\left(\frac{\Omega}{\frac{W}{A^2}}\right)} = 13.637\ A$$

$$\boxed{I_{MAX} = 13.6\ A}$$

ANSWER

(a) Forced Convection, $U_\infty = 20\ m/s$

Cylinder in a cross flow

$$Re_D = \frac{U_\infty D}{\nu} = \frac{\left(20\frac{m}{s}\right)(8.118\times10^{-4}m)}{102.9\times10^{-6}m^2/s} = 157.8$$

From eq 7.54

$$\overline{Nu}_D = 0.3 + \frac{0.62Re_D^{0.5}P_r^{1/3}}{\left[1+\left(\frac{0.4}{P_r}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_D = 6.481$$

$$\bar{h} = \overline{Nu}_D \cdot \frac{k_f}{D} = \frac{(6.481)\left(62.0\times\frac{10^{-3}W}{m\cdot K}\right)}{8.118\times10^{-4}m} = 495.0\frac{W}{m^2K}$$

$$I^2R' = \bar{h}\pi D(T_s - T_\infty) + \epsilon\pi D\sigma(T_s^4 - T_{surr}^4)$$

$$I^2R' = 1{,}514.7\frac{W}{m} + 219.35\frac{W}{m} = 1{,}734.1\frac{W}{m}$$

$$\boxed{q'_{conv} = 1{,}510\frac{W}{m} \qquad q'_{rad} = 219\frac{W}{m}}$$

ANSWER

$$I = \sqrt{\frac{1734.1\frac{W}{m}}{2.162\frac{\Omega}{m}}\left(\frac{\Omega}{\frac{W}{A^2}}\right)} = 28.321\ A$$

$$\boxed{I_{MAX} = 28.3\ A}$$

ANSWER

Comments:

Radiation is 13% of q_{total} for the forced convection case and 54% of q_{total} for natural convection case. It is not negligible in either case.

The heat loss due to forced convection is an order of magnitude higher than for natural convection. Since the surface temperature is fixed, radiative heat transfer is the same in both cases. Forced convection allows the wire to carry a higher current (while keeping the same temperature).