Ann Mescher

HW Assignment #1

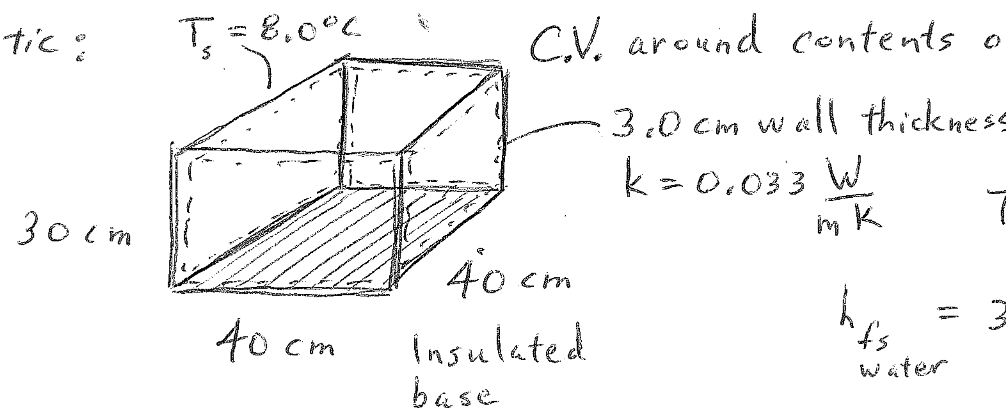
March 30, 2017

Problem #1

Statement: Ice chest (30 cm × 40 cm × 40 cm) of wall thickness 3.0 cm is filled with 40 kg H_2O, initially solid ice at 0.0 °C. Outer surface temperature of the ice chest is 8.0 °C, and the 40 cm × 40 cm base is insulated.

Find: Time for ice to completely melt.

Schematic: C.V. around contents of ice chest



$T_\infty = 30°C$

$h_{fs,\,water} = 333.7 \frac{kJ}{kg}$

Assumptions: Inner surfaces of ice chest are at uniform temperature 0.0°C. Outer surfaces of top and four side walls are at uniform temperature 8.0°C. 1-D conduction is assumed through these five walls. Base of ice chest is insulated. No sensible energy change.

Analysis: Over a period of time Δt,

$$E_{in} - \cancel{E_{out}}^{\,0} + \cancel{E_g}^{\,0} = \Delta E_{stored}$$

Energy flows from ambient environment, through ice chest walls to ice chest contents.

(No thermal generation $\dot{q}$ within H_2O)

Problem #1 continued

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$$E_{in} = q\Delta t = q''A\Delta t = m_{H_2O}\, h_{fg_{H_2O}} = \Delta E_{stored}$$

where $q'' = -k\dfrac{\Delta T}{\Delta x} = -0.033\,\dfrac{W}{mK}\cdot\dfrac{(0.0-8.0)K}{0.03m} = 8.8\,\dfrac{W}{m^2}$

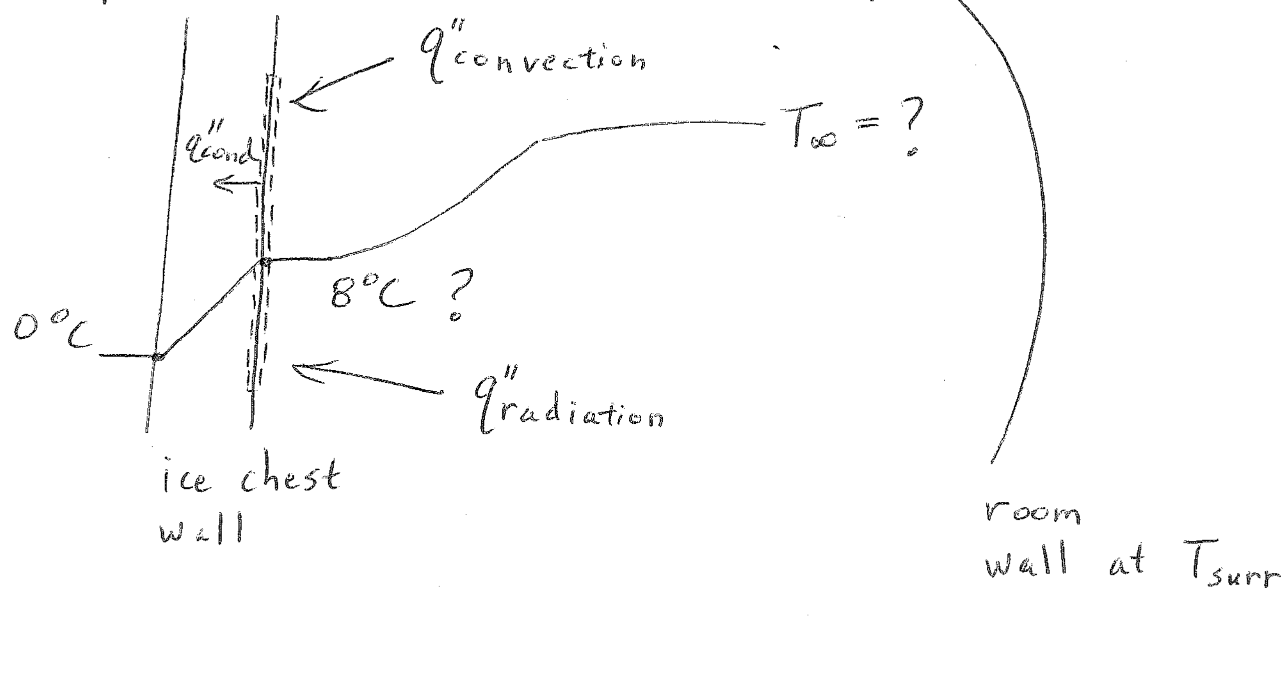
$$A = 4(0.37m\cdot 0.27m) + 0.37m\cdot 0.37m = 0.5365\ m^2$$

$$\Delta t = \frac{m_{H_2O}h_{fg_{H_2O}}}{q''A} = \frac{40kg\cdot 333.7\,\frac{kJ}{kg}}{8.8\times10^{-3}\,\frac{kW}{m^2}\cdot 0.5365\,m^2} = 2.827\times10^6 \text{ seconds}$$

$$\Delta t = 785 \text{ hours} = 32.7 \text{ days}$$

Comments: This seems a long time!

Is it realistic that the outer surface could be maintained at 8.0°C? It depends on the surrounding wall and air temperatures. Perform surface energy balance.



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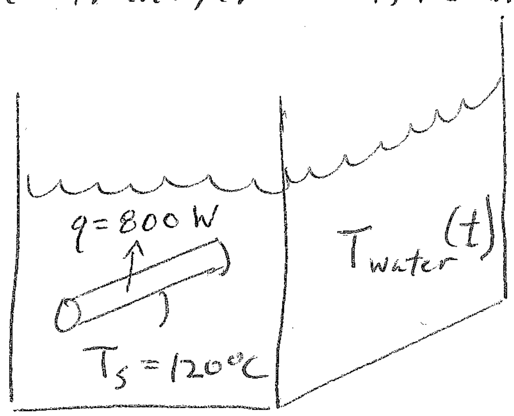
Problem #2

Statement: 800 W electric resistance heating element (cylindrical with $L = 50$ cm, $D = 0.50$ cm) has constant surface temperature $T_s = 120°C$ and is immersed in 40 kg of water at an initial temperature of 20°C.

Find: How long to raise the temperature of the water from 20°C to 80°C, and the initial and final heat transfer coefficients?

Schematic:



Control Volume is around the water, excluding the electric resistance heater.

Assumptions: The tank is insulated, so that there is negligible heat loss from the water to surroundings. Water circulates so that the water temperature is approximately uniform. The specific heat of the liquid water from 20°C to 80°C is treated as constant. Negligible radiation transfer.

Problem #2 Continued:

Analysis: $\dot{E}_{in}^{\,0} - \cancel{\dot{E}_{out}}^{\,0} + \cancel{\dot{E}_g}^{\,0} = \dot{E}_{stored}$ for: CV surrounding water

no heat loss from tank — no thermal generation in water

$$q = 800\ W = m_w c_w \frac{dT_w}{dt} = 40\ kg \cdot 4181 \frac{J}{kg\,K} \frac{dT_w}{dt}$$

where the specific heat of water $c_w = 4181 \frac{J}{kg\,K}$ is evaluated at the average water temperature of $50°C = 323\,K$.

Over a time interval Δt, $E_{in} = \Delta E_{stored}$

$$q\,\Delta t = 800\ W \cdot \Delta t = m_w c_w \Delta T_w = 40\ kg \cdot 4181 \frac{J}{kg\,K} (80-20) K$$

Answer: $\Delta t = 12{,}543$ seconds $= \boxed{3.48 \text{ hours}}$

At the initial water temperature, $T_\infty = 20°C$,

$$q = hA(T_s - T_\infty) \quad \text{where } A = 2\pi r^2 + 2\pi r L = \frac{\pi D^2}{2} + \pi D L$$

$$h = \frac{800\ W}{(120-20)\,K \cdot \pi \cdot 0.005\,m\left(0.50\,m + \frac{0.005\,m}{2}\right)}$$

Answer: $h = 1013.5 \frac{W}{m^2 K}$ $\boxed{h_{initial} = 1010 \frac{W}{m^2 K}}$

At the final water temperature, $T_\infty = 80°C$,

Answer: $h = \dfrac{800\ W}{(120-80)\,K \cdot A} = 2533.8 \frac{W}{m^2 K}$ $\boxed{h_{final} = 2530 \frac{W}{m^2 K}}$

Comments: In this transient process it is impossible for T_s to be maintained at 120°C for a constant $q = 800\,W$ from the electric heater. For forced or natural convection, $h_{initial} > h_{final}$; therefore $(T_s - T_\infty)_{initial} < (T_s - T_\infty)_{final}$. Since T_∞ is increasing, T_s must also increase.

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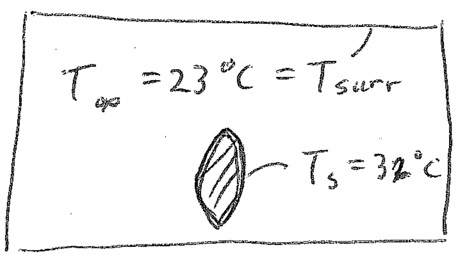
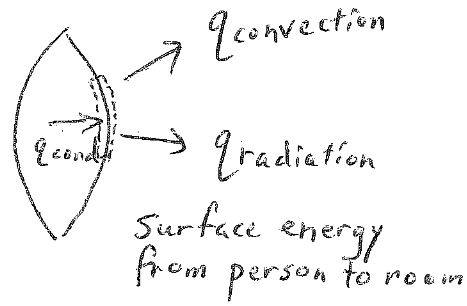
Problem #3

Statement: Person of surface area $1.7\ m^2$ at surface temperature of $32°C$ with gray emissivity of $\varepsilon = 0.90$ standing in a large room enclosure at $23°C$ with room air at the same temperature of $23°C$ with a natural convection coefficient of $h = 5.0 \frac{W}{m^2 K}$.

Find: Convective and radiative heat losses from person.

Schematic:





Assumptions: For gray surface, $\varepsilon_{person} = 0.90$, in a large enclosure, $q_{radiation} = \varepsilon \sigma A (T_s^4 - T_{surr}^4)$

Analysis: $q_{radiation} = 0.90 \cdot 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \cdot 1.7\ m^2 (305.15\ K^4 - 296.15\ K^4)$

$q_{convection} = hA(T_s - T_\infty) = 5.0 \frac{W}{m^2 K} \cdot 1.7\ m^2 (32 - 23)\ K$

Answer: $q_{total} = q_{radiation} + q_{convection} = 84.9\ W + 76.5\ W$

$\boxed{q_{total} = 160\ W}$ High heat loss for high T_s!

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Problem # 4

Statement: Heat diffusion equation is given in simplest form for a medium as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t},$$

where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity.

Find: a) transient or steady?

b) 1-D, 2-D, or 3-D?

c) thermal generation, $\dot{q} = ?$

d) constant or variable conductivity k?

Assumptions: The medium is a solid or a non-convecting fluid.

Answer: a) Transient because of the term $\frac{\partial T}{\partial t}$.

b) 2-D because of the terms $\frac{\partial^2 T}{\partial x^2}$ and $\frac{\partial^2 T}{\partial y^2}$.

c) No thermal generation; $\dot{q}$ must be 0.

d) Constant k because for variable k,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = \rho c_p \frac{\partial T}{\partial t}.$$

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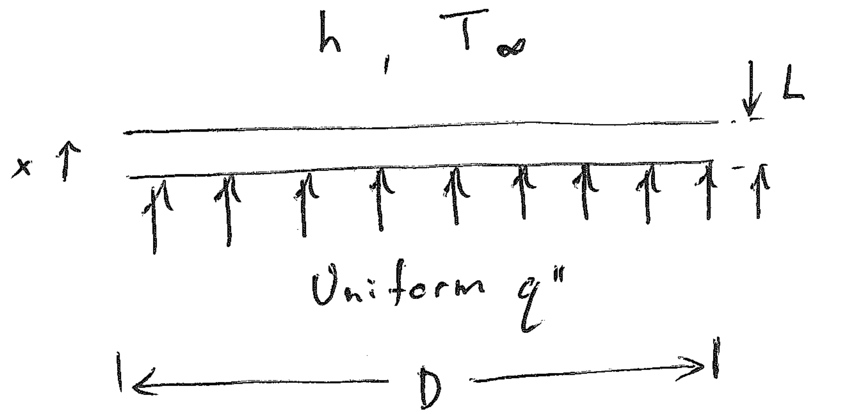
Problem #5

Statement: Stainless steel pan bottom conducts heat one-dimensionally to boil water @ standard P_{atm}.
At steady-state, $T_{water} = 100°C$, $h = 3400 \frac{W}{m^2K}$.
Pan geometry is known: $D = 20\,cm$ and $L = 0.30\,cm$.
Heat flux to bottom of pan is $q'' = \frac{0.85 \times 1000\,W}{A}$

Find: Steady-state temperature distribution through thickness of the pan bottom; differential equation and boundary/initial condition(s) for 1-D transient conduction in pan bottom.

Schematic:



Differential CV is used to obtain:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho c \frac{\partial T}{\partial t}$$

Assumptions:
- Standard atmospheric pressure with water boiling point at 100°C.
- Constant stainless steel thermal conductivity $k_{ss} = 15\ W/mK$.

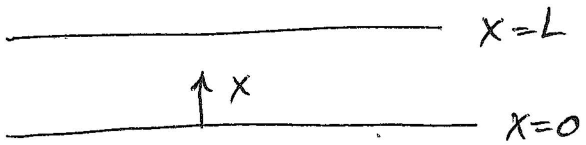
Problem # 5 continued.

Assumptions (continued): • Shallow pan such that side walls have negligible effect on 1-D heat transfer through pan bottom.

• No thermal energy generation, $\dot{q} = 0$

Analysis: For the more general 1-D transient case, both h and T_∞ are variable. The heat diffusion equation reduces to: $k_{ss} \dfrac{d^2 T}{dx^2} = \rho c \dfrac{dT}{dt}$.

Boundary conditions are imposed at $x = 0, L$.



$$-k_{ss}\left(\frac{dT}{dx}\bigg|_{x=L}\right) = h\left[\left(T\big|_{x=L} - T_\infty\right)\right]$$

$$q'' = -k_{ss}\left(\frac{dT}{dx}\bigg|_{x=0}\right)$$

To solve the transient problem, an initial condition is required and hence a further assumption. For example, the water and pan can be assumed to be at an initial uniform temperature of $T = 20°C$.

$$T(x, t=0) = 20°C \quad \text{and} \quad T_\infty(t=0) = 20°C.$$

Also at $t=0$, $h(t=0) = 0$, or $\left.\dfrac{dT}{dx}\right|_{\substack{x=L \\ t=0}} = 0$.

To solve transient conduction problem, convection analysis is also needed to obtain h and T_∞ as functions of time, $h(t)$ and $T_\infty(t)$.

Steady-state Analysis : $k_{ss} \frac{d^2T}{dx^2} = \cancel{\rho c \frac{dT}{dt}}^{\,0}$ No initial condition.

$T_\infty = 100\,°C$; $h = 3400 \frac{W}{m^2 K}$; $q'' = \frac{850\ W}{\pi (0.10 m)^2}$

Boundary conditions as before with fixed T_∞ and h.

$q'' = -k_{ss} \left.\frac{dT}{dx}\right|_{x=0}$; $-k_{ss} \left.\frac{dT}{dx}\right|_{x=L} = h\left[\left(T\right|_{x=L} - T_\infty\right]$

From the reduced heat diffusion equation,

$\frac{d^2T}{dx^2} = 0$ and therefore $\frac{dT}{dx} = \text{constant} = -\frac{q''}{k_{ss}}$

$T(x) = C_1 x + C_2 = -\frac{q''}{k_{ss}} x + C_2$ Linear!

Using the boundary condition at $x = L$, solve for $T(x=L)$.

$q'' = h\left[T(x=L) - T_\infty\right]$; $T(x=L) = \frac{q''}{h} + T_\infty$

$$T(x=L) = \frac{850\ W}{\pi (0.10 m)^2 \cdot 3400 \frac{W}{m^2 K}} + 100\,°C = 107.96\,°C \simeq 108\,°C$$

Obtain C_2 by substituting $T(x=L)$ at $x=L$ into the general solution:

$$T(x=L) = \frac{q''}{h} + T_\infty = -\frac{q''}{k_{ss}} L + C_2$$

$$C_2 = \frac{q''}{h} + \frac{q''}{k_{ss}} L + T_\infty = T(x=L) + \frac{850\ W \cdot 0.003\ m}{\pi (0.10 m)^2 \cdot 15 \frac{W}{m K}}$$

$$C_2 = 113.37\,°C = T(x=0)$$

Answer:

$$\boxed{T = -1804 \frac{°C}{m} x + 113.4\,°C}$$

Linear temperature decrease through pan thickness!