ME331: Introduction to Heat Transfer
Spring 2017

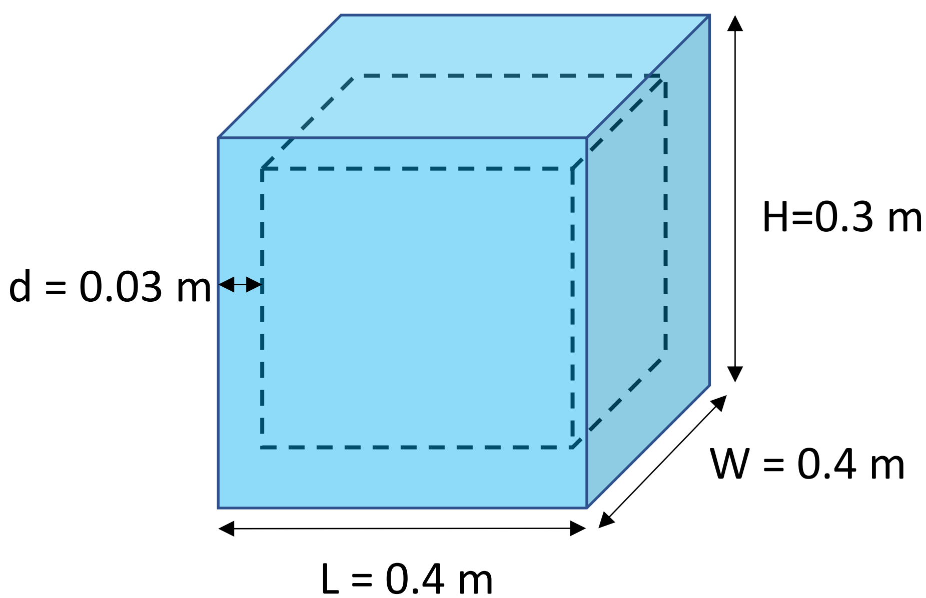
ME 331 Homework Assignment #1

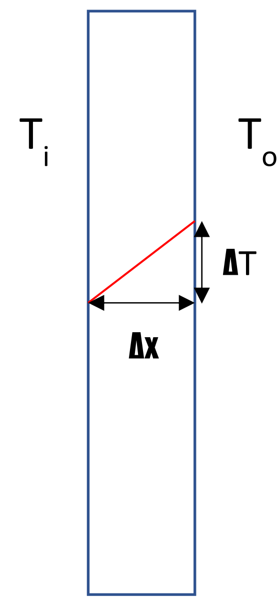
Problem 1

An ice chest whose outer dimensions are 30 cm x 40 cm x 40 cm is made of 3.0 cm thick Styrofoam (k = 0.033 W/m-K) Initially, the chest is filled with 40 kg of ice at 0.0°C, and the inner surface temperature of the ice chest can be taken to be 0.0°C at all times. The heat of fusion of ice at 0°C is 333.7 kJ/kg, and the surrounding ambient air is at 30°C. Disregarding any heat transfer from the 40 cm x 40 cm base of the ice chest, determine how long it will take for the ice in the chest to melt completely if the outer surfaces of the ice chest are at 8°C.

Find: How long (Δt) it takes to melt all the ice.

Schematic:





Assumptions: We assume one dimensional steady state heat transfer with constant and uniform material properties.

Analysis:

We start with conservation of energy (eqn. 1.12b)

$$\Delta E_{st} = E_{in} - E_{out} + E_g$$

If we focus on the control volume inside the chest (in dashed lines above) we know that:

$$\Delta E_{st} = 40\ kg \cdot 333.7 \frac{kJ}{kg} = 13348\ kJ = 1.3348 \cdot 10^7\ J$$

We have 1-D steady state conduction through the top and four sides[1] so we can write:

$$\frac{q}{A} = q'' = k\frac{dT}{dx} = k\frac{\Delta T}{\Delta x} = 0.033\frac{W}{m \cdot K} \cdot \frac{(8-0)K}{0.03m} = 8.8\frac{W}{m^2}$$

$$q = A \cdot q'' = [2(L-d \cdot H-d+W-d \cdot H-d)+L-d \cdot W-d] \cdot q''$$

$$q = [2(0.37 \cdot 0.27 + 0.37 \cdot 0.27) + 0.37 \cdot 0.37]m^2 \cdot 8.8\frac{W}{m^2} = 4.7212\, W$$

Under the steady assumption then we can say:

$$\Delta E_{st} = E_{in} - E_{out} + E_g = E_{in} - E_{out} = q \cdot \Delta t$$

Since we have no energy being generated

$$\Delta t = \frac{\Delta E_{st}}{q} = \frac{1.3348 \cdot 10^7\, J}{4.7212\frac{J}{s}} = 2.83 \cdot 10^6 s \approx 32.7228\, days$$

Answer

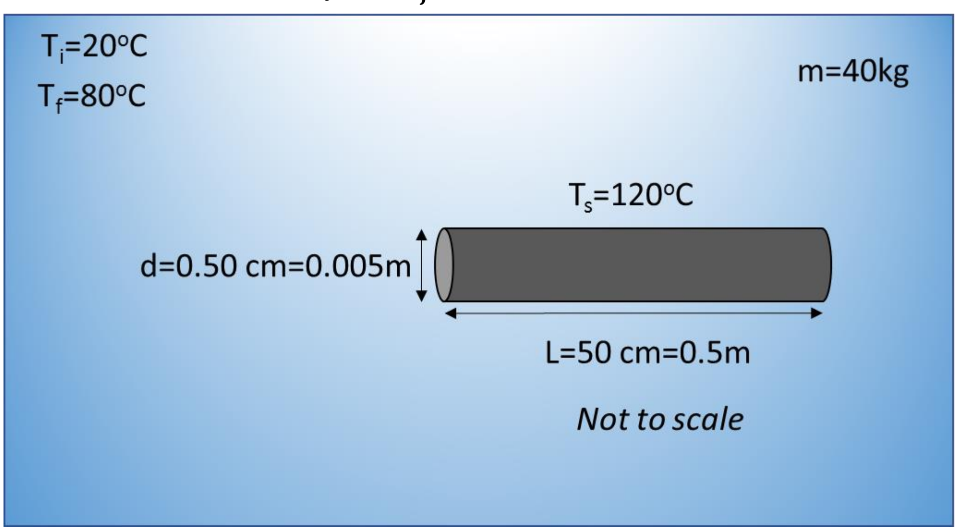
Respecting significant figures (2 digits in problem statement):

$$\boxed{\Delta t = 33\, days}$$

Problem 2

A 50 cm long, 800 W electric resistance heating element with diameter 0.50 cm and surface temperature 120°C is immersed in 40 kg of water initially at 20°C. Determine how long it will take for this heater to raise the water temperature to 80°C. Also, determine the convection heat transfer coefficients at the beginning and at the end of the heating process.

Find: How long (Δt) it takes to raise the temperature of the water from 20^oC to 80^oC, as well as the initial and final transfer coefficients (h_i and h_f).



[1] I am cheating a bit here in order to make the problem one dimensional. I am not accounting for the difference in area between the inner and outer surfaces of the chest.

Assumptions: **The tank is insulated, so that there is negligible heat loss from the water to the surroundings. Water circulates so that the water temperature is approximately uniform. The specific heat of the liquid water from 20^oC to 80^oC is treated as constant. There is negligible radiation transfer.**

Analysis:

To solve this problem we assume that the temperature of the water through the domain is uniform (a pretty violent assumption) and that the water is in an insulated container.

The change in the stored energy of the water is given by the change in its temperature, namely:

$$\Delta E = mC\Delta T = 40kg \cdot 4180 \frac{J}{kg \cdot K} \cdot (80 - 20)K = 1.0032 \cdot 10^7 J$$

We know the power consumed by the resistance, and since the resistance's temperature is assumed to be constant we can also assume that all of the power is going to the water.

$$P = \frac{\Delta E}{\Delta t}$$

$$\Delta t = \frac{\Delta E}{P} = \frac{1.0032 \cdot 10^7 J}{800 \frac{J}{s}} = 12540s \approx 3.4833\ hr$$

Answer

$$\boxed{\Delta t = 3.5\ hr}$$

By definition the convection coefficient is given by:

$$q'' = h(T_s - T_\infty)$$

$$h = \frac{q''}{(T_s - T_\infty)} = \frac{q}{A(T_s - T_\infty)}$$

And here we have that $T_\infty = T_{H_2O}$ since we assume uniform water temperature. Also $q = P = 800W$ and $A = 2\pi r^2 + 2\pi rL = 2\pi r(r + L) = 2\pi \cdot 2.5 \cdot 10^{-3}m(2.5 \cdot 10^{-3} + 0.5)m = 7.89 \cdot 10^{-3}m^2$

$$h_i = \frac{800W}{7.89 \cdot 10^{-3}m^2(120 - 20)K} = 1.0135 \cdot 10^3 \frac{W}{m^2 \cdot K}$$

$$h_f = \frac{800W}{7.89 \cdot 10^{-3}m^2(120 - 80)K} = 2.5338 \cdot 10^3 \frac{W}{m^2 \cdot K}$$

Answer

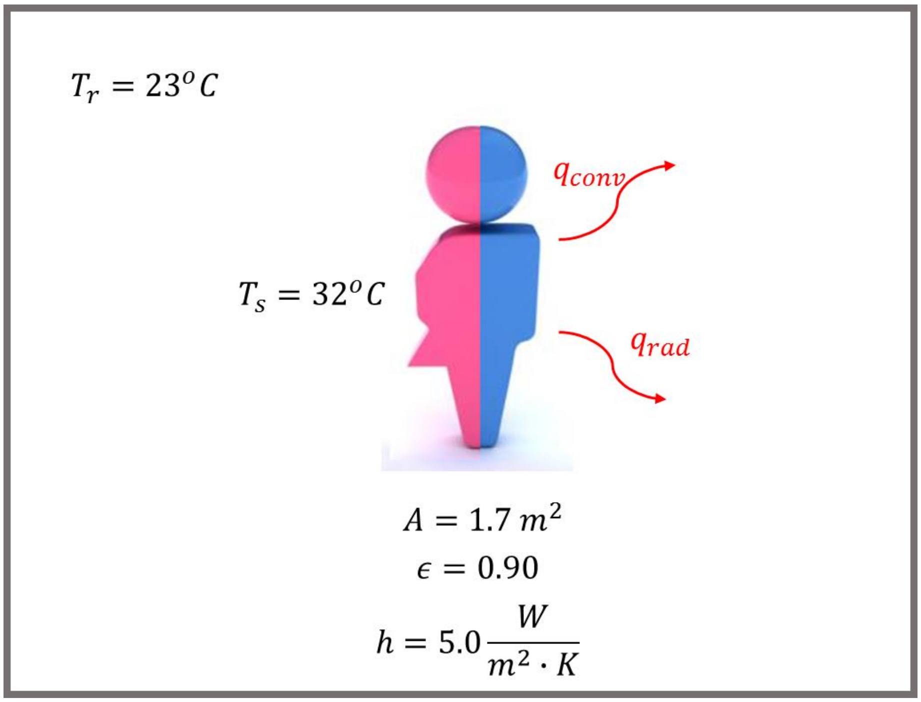
Again, respecting significant digits:

$$\boxed{\begin{aligned} h_i &= 1.0 \cdot 10^3 \frac{W}{m^2 \cdot K} \\ h_f &= 2.5 \cdot 10^3 \frac{W}{m^2 \cdot K} \end{aligned}}$$

Problem 3

Consider a person standing in a room at 23°C. Determine the total rate of heat transfer from this person if the exposed surface area and the skin temperature of the person are 1.7 m2 and 32°C, respectively, and the convection heat transfer coefficient is 5.0 W/m2-K. Take the emissivity of the skin and clothes to be 0.90, and assume the temperatures of the inner surfaces of the room to be the same as the air temperature.

Find: Radiative and convective losses from person.



Assumptions: **For gray surface $\epsilon_{person} = 0.90$, in a large enclosure, $q_{rad} = \epsilon\sigma A(T_s^4 - T_r^4)$**

Analysis:

We assume steady state so neither the room's nor the person's temperature changes over time. Under this assumption the total rate of heat transfer is the summer of the convective and radiative transfers (notice no conduction since we have no temperature gradients through a conductor).

$$q_{tot} = q_{conv} + q_{rad}$$

$$q_{tot} = hA(T_s - T_r) + \epsilon\sigma A(T_s^4 - T_r^4)$$

$$q_{tot} = 5.0 \frac{W}{m^2 \cdot K} \cdot 1.7\, m^2 (32 - 23)K + 0.9 \cdot 5.67 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}(305.15^4 - 296.15^4)K^4$$

$$q_{tot} = 76.5\, W + 84.89W = 161.39\, W$$

So we see that under ordinary circumstances for humans the radiative and convective heat transfers for humans are similar.

Answer

$$\boxed{q_{tot} = 160\ W}$$

Again, just 2 significant digits.

Problem 4

Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

We assume the medium is a solid or a non-convecting fluid.

(a) Is the heat transfer steady or transient?

The heat transfer is **transient** as indicated by the presence of the $\frac{1}{\alpha}\frac{\partial T}{\partial t}$ term. This means that in general the time derivative is non-zero (i.e. temperature varies with time).

(b) Is the heat transfer one-, two-, or three-dimensional?

The heat transfer is **two dimensional** since there are x and y derivatives, but no z derivatives.

(c) Is there heat generation in the medium?

There is **NO heat generation** in the medium as the equation is homogeneous (no source $\dot{q}$ terms), i.e. it depends only on T and derivatives of T. This means that energy is merely being transferred from one place to another.

(d) Is the thermal conductivity of the medium constant or variable?

The **thermal conductivity of the medium is constant**, as is evident by the lack of its derivatives.

$$\alpha = \frac{k}{\rho c_p}$$

Recall that in general we have (eqn 2.19 in Bergman, 2011):

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

For two dimensional case this reduces to

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Without sources we get

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = \rho c_p \frac{\partial T}{\partial t}$$

Finally, for constant conductivity

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\rho c_p\frac{\partial T}{\partial t}$$

And

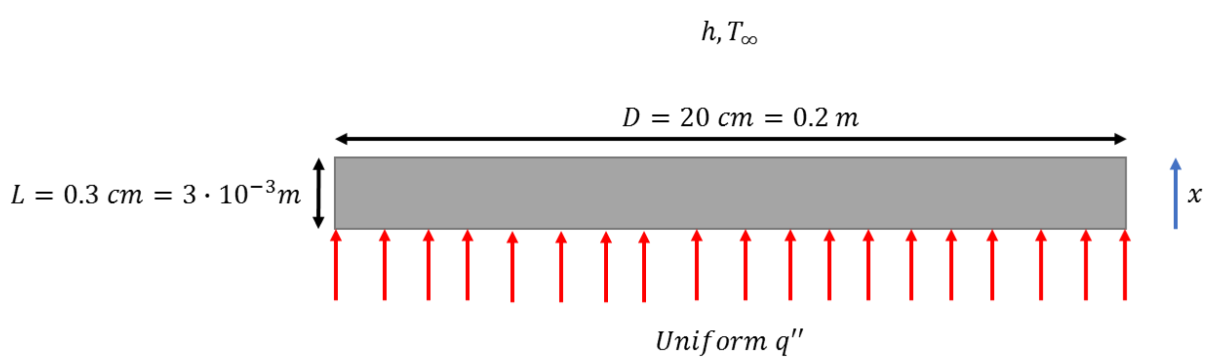
$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{\rho c_p}{k}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The expression on top.

Problem 5

Consider a steel pan used to boil water on top of an electric range. The bottom section of the pan is L = 0.30 cm thick and has a diameter of D = 20 cm. The electric heating unit on the range top consumes 1000 W of power during cooking, and 85 percent of the heat generated in the heating element is transferred uniformly to the pan. Heat transfer from the top surface of the pan bottom to the water is by convection with a heat transfer coefficient of h. Assuming constant thermal conductivity and one-dimensional heat transfer through the pan bottom, express the mathematical formulation (differential equation and boundary conditions) of this heat conduction process during steady state. Solve for the steady state temperature distribution through the thickness of the pan bottom for h = 3400 W/m2K. Now express the mathematical formulation for the transient heat conduction process (differential equation and boundary conditions – do not solve) with variable heat transfer coefficient h and variable water temperature T_∞.

Find: **Steady state temperature distribution through thickness of the pan bottom; differential equation and boundary conditions for 1-D transient conduction in pan bottom.**



Assumptions

Standard atmospheric pressure with water boiling at 100^oC

Constant stainless steel thermal conductivity $k = 15\frac{W}{m\cdot K}$

We neglect the effect of the side walls (so the problem becomes 1D)

No thermal energy generation (where would it come from?)

Analysis

From problem 4 we know we have:

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t}$$

Since the problem is 1D, conductivity is constant and there are no sources.

For the boundary conditions at bottom (x=0) and top (x=L) we have:

At the bottom, constant heat flux from heating unit

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q''$$

At the top, convection to water,

$$-k\frac{\partial T}{\partial x}\bigg|_{x=L} = h(T|_{x=L} - T_\infty)$$

For the steady state case, we know that $h \; and \; T_\infty$ are constant so for the steady state the problem reduces to:

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q''$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=L} = h(T|_{x=L} - T_\infty)$$

The general solution to the ODE above is

$$T = ax + b$$

For some constants a and b.

We have that $\frac{\partial T}{\partial x} = a = -\frac{q''}{k}$ from the first boundary condition

So

$$-k\frac{\partial T}{\partial x}\bigg|_{x=L} = -ka = q'' = h(T|_{x=L} - T_\infty) = h(aL + b - T_\infty)$$

$$b = \frac{q''}{h} + T_\infty - aL = q''\left(\frac{1}{h} + \frac{L}{k}\right) + T_\infty$$

So the solution is:

$$T = T_\infty + q''\left(\frac{1}{h} + \frac{L - x}{k}\right)$$

From this equation we see that the temperature decreases through the pan thickness.

Since we are told that the water is boiling, we have that $T_\infty = 100^oC$

Also we have $q = 0.85 \cdot 1000\ W$ (85% heat transfer) so that $q'' = \frac{850W}{A} = \frac{850W}{\pi \cdot (0.1m)^2} = 2.7 \cdot 10^4\,\frac{W}{m^2}$

We can assume (arbitrarily) that $k = 15\,\frac{W}{m \cdot K}$ (the value for steel varies).

So substituting values we get:

$$T = 373\ K + 2.7 \cdot 10^4\,\frac{W}{m^2}\left(\frac{1}{3400}\frac{m^2K}{W} + \frac{3 \cdot 10^{-3}m}{15\,\frac{W}{m \cdot K}} - \frac{1}{15\,\frac{W}{m \cdot K}}\,x\right)$$

$$T = 373\ K + 13.37K - 1804\,\frac{K}{m} \cdot x = 386.37K - 1804\,\frac{K}{m} \cdot x$$

$$T = 386.37K - 1804\,\frac{K}{m} \cdot x = 113.4^oC - 1804\,\frac{K}{m} \cdot x$$

From this we can get

$$T(x = L) = 113.4^oC - 1804\,\frac{K}{m} \cdot 3 \cdot 10^{-3}m = 108^oC$$

We have obtained the general transient formulation for this problem. We just recall:

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t}$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q''$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=L} = h(t)\big(T|_{x=L} - T_\infty(t)\big)$$

We have added the time dependencies for clarity. Since for this problem we have an additional derivative (time) we need an initial condition for the problem to have a unique solution. This is a much harder problem since it would require convection analysis to find $h(t) and\ T_\infty(t)$.