Ann Mescher — April 10, 2017

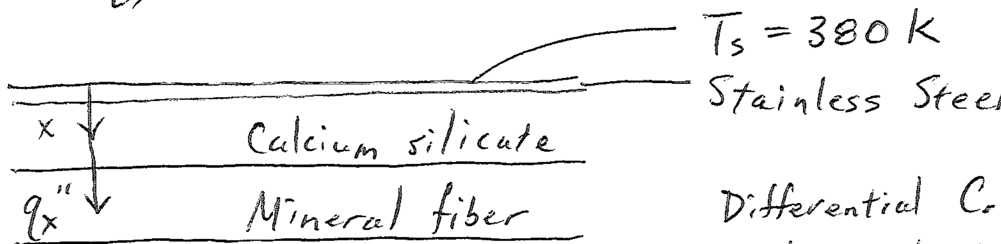
HW Assignment #2

Problem #1

Statement: Composite plate comprised of AISI 304 Stainless Steel (1.0 mm thick), calcium silicate insulation (2.0 cm thick), mineral fiber blanket (2.0 cm thick) with outer surface of stainless steel at 380 K. Opposing side with $h = 5.0 \frac{W}{m^2K}$, $T_\infty = 300K$.

Find: Heat flux q_x''.

Schematic:



$h = 5.0 \frac{W}{m^2K}$ $\quad T_\infty = 300K$

Differential C.V. is used to obtain:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

Assumptions: Steady state, and no thermal generation, so $\frac{dT}{dt} = 0$ and $\dot{q} = 0$. Uniform boundary conditions at outer surfaces of stainless steel and mineral fiber. One dimensional conduction and each material is assumed to have constant conductivity: $k_{ss} = 14.9 \frac{W}{mK}$;

at $T = 365K$, $k_{\text{Calcium Silicate}} = 0.059 \frac{W}{mK}$;

at $T = 310K$, $\rho = 24 \frac{kg}{m^3}$, $K_{\text{mineral fiber}} = 0.040 \frac{W}{mK}$.

Analysis: $q = \frac{\Delta T}{R_{total}}$ in which the total resistance is comprised of four resistors in series, and

$$\Delta T = (380-300)K = 80\,K$$

Problem # 1 continued

$$R_{total} = \left(\frac{L}{kA}\right)_{ss} + \left(\frac{L}{kA}\right)_{cs} + \left(\frac{L}{kA}\right)_{MF} + \frac{1}{hA}$$

$$A\,R_{total} = \frac{0.001\text{ m}}{14.9\ \frac{\text{W}}{\text{mK}}} + \frac{0.02\text{ m}}{0.059\ \frac{\text{W}}{\text{mK}}} + \frac{0.02\text{ m}}{0.040\ \frac{\text{W}}{\text{mK}}} + \frac{1}{5.0\ \frac{\text{W}}{\text{m}^2\text{K}}}$$

$$q'' = \frac{\Delta T}{A\,R_{total}} = \frac{80\text{ K}}{1.039\ \frac{\text{m}^2\text{K}}{\text{W}}} = \boxed{77\ \frac{\text{W}}{\text{m}^2}}$$ Answer

Comments: Only the thermal resistance of the stainless steel is negligible.

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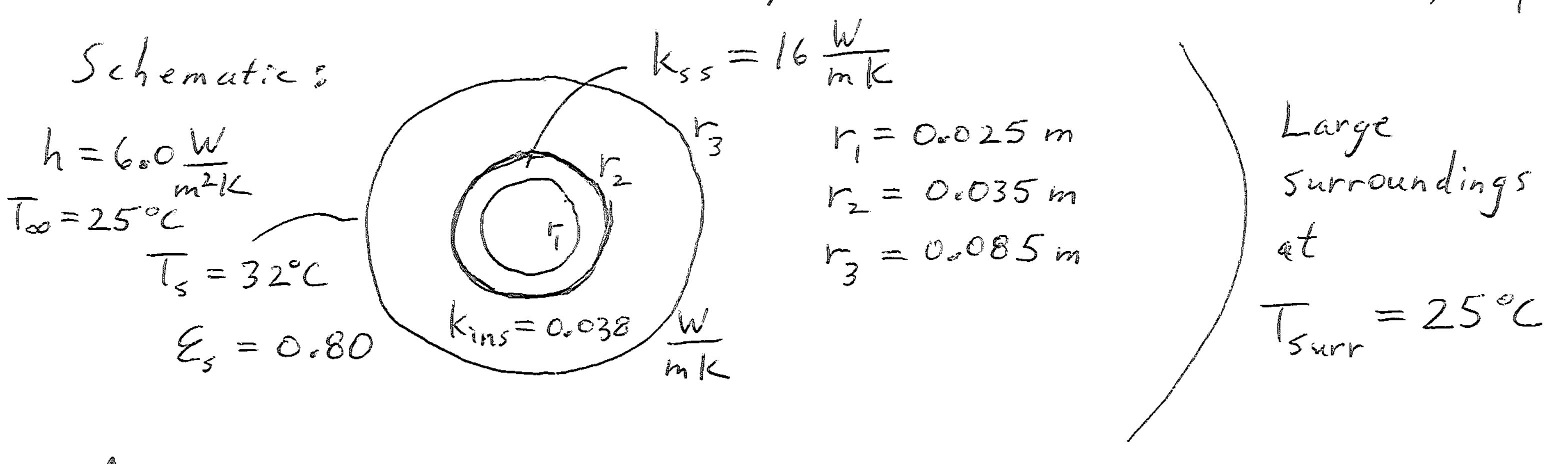
HW Assignment #2

Problem #2

Statement: A 2.0 m long chemical reactor (I.D. 5.0 cm) has a 1.0 cm thick stainless steel shell with an outer fiberglass blanket insulation 5.0 cm thick $\left(k_{ins} = 0.038 \frac{W}{mK}\right)$.
The insulation surface is at $T_s = 32°C$ and has gray emissivity $\varepsilon_s = 0.80$. The convection coefficient is $6.0 \frac{W}{m^2K}$ while the ambient air and surroundings are at 25°C.

Find: The inner surface temperature of the stainless steel, T_{r_1}.

Schematic:

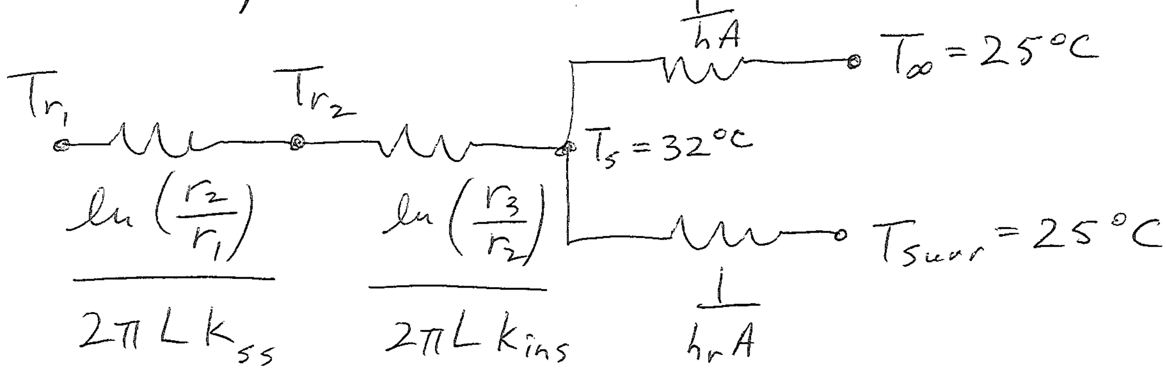


Assumptions: Steady-state, 1-D heat transfer with no thermal generation inside stainless steel or insulation, $\dot{q} = 0$.
For gray surface in large enclosure, $q_{radiation} = \varepsilon \sigma A (T_s^4 - T_{surr}^4)$

Analysis: A differential C.V. in the cylindrical radial coordinate is used to obtain $\frac{d}{dr}\left(kr \frac{dT}{dr}\right) = 0$. Solving this heat diffusion equation yields the resistance for a cylindrical shell.

Problem #2 continued

The thermal circuit has two conduction resistances in series followed by convection and radiation resistances in parallel.



where $q_{radiation} = \varepsilon\sigma A\left(T_s^4 - T_{surr}^4\right) = h_r A\left(T_s - T_{surr}\right)$

and $h_r = \varepsilon\sigma\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right)$

From the outer surface temperature, $T_s = 32°C$,

$$q_{total} = q_{rad} + q_{conv} = \varepsilon\sigma A\left(T_s^4 - T_{surr}^4\right) + hA\left(T_s - T_\infty\right)$$

$$q''_{total} = 0.80 \cdot 5.67\times10^{-8}\,\frac{W}{m^2K^4}\left[305\,K^4 - 298\,K^4\right] + 6.0\,\frac{W}{m^2K^4}\,(32-25)\,K$$

This same q_{total} flows through the conduction resistances.

$$q_{total} = \frac{T_{r_1} - 32°C}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_{ss}} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi L K_{ins}}}$$

$$\boxed{T_{r_1} = 185\ °C}$$ Answer

Comments: Here again, the stainless steel shell has negligible resistance compared to $R_{insulation}$. The outer convection and radiation resistances are comparable and neither can be neglected.

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HW Assignment #2

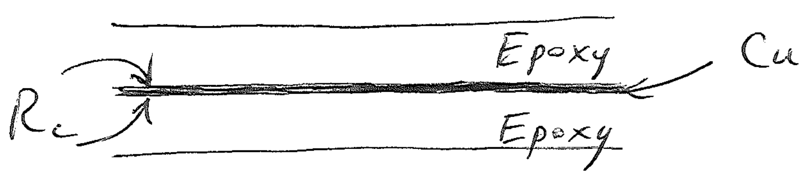
April 10, 2017

Problem #3

Statement: Copper plate of conductivity $k_c = 386 \frac{W}{mK}$ (1.0mm thick) is sandwiched between two 5-mm thick epoxy boards ($k_{epoxy} = 0.26 \frac{W}{mK}$) with contact resistance on each side of the copper plate, $R_c = 8.3 \times 10^{-3} \frac{K}{W}$. Plate and boards are each 15 cm X 20 cm.

Find: The heat rates for the cases with and without $R_{contact}$.

Schematic:



Differential C.V. is used to obtain $\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$

Assumptions: 1-D, steady-state conduction, $\dot{q} = 0$. Constant conductivity for each material. Three resistances in series for $R_c = 0$. Five resistances in series for case with contact resistances.

Analysis: For the same ΔT, there will be different q, depending on whether or not contact resistance is included.

$$q = \frac{\Delta T}{R_{total}}$$

Without contact resistance, $R_{total} = 2\left(\frac{L}{kA}\right)_{epoxy} + \left(\frac{L}{kA}\right)_{cu}$

$$R_{total} = 2\left(\frac{0.005\,m}{0.26\,\frac{W}{mK}\cdot 0.15\,m\cdot 0.20\,m}\right) + \frac{0.001\,m}{386\,\frac{W}{mK}\cdot 0.15\,m\cdot 0.20\,m}$$

$$= 1.28\ \frac{K}{W}$$

With contact resistance, $R_{total} = 1.28\,\frac{K}{W} + 2\cdot 0.0083\,\frac{K}{W}$

$$R_{total} = 1.30\ \frac{K}{W}$$

For a ΔT of 10 K for example:

$q = 7.8$ W without R_c

$q = 7.7$ W with R_c

Error in R_{total} is $\dfrac{1.30-1.28}{1.30} = \boxed{1.3\%}$ Answer

For a fixed ΔT, the error in neglecting contact resistance results in a 1.3% error in q.

Comments: In this problem, the contact resistances are small compared to the thermal resistance of the epoxy boards. Generally, contact resistances should be considered to check whether they are a significant portion of the overall resistance.

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HW Assignment #2

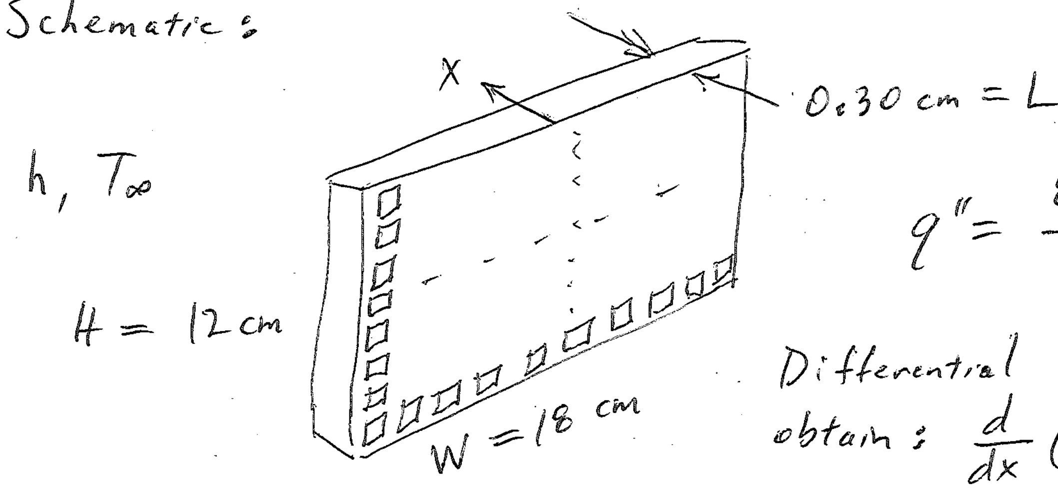
April 10, 2017

Problem # 4 a)

Statement: A circuit board (0.30 cm X 12 cm X 18 cm) of conductivity $k = 20 \frac{W}{mK}$ has 80 equally spaced chips on one side, each dissipating 0.040 W. All generated heat is conducted across the board and is dissipated to a medium with $h = 50 \frac{W}{m^2 K}$ and $T_\infty = 40°C$.

Find: Temperature on both sides of board.

Schematic:



$$q'' = \frac{80 \cdot 0.040\ W}{0.12 m \times 0.18 m}$$

Differential C.V. is used to obtain: $\frac{d}{dx}\left(k \frac{dT}{dx}\right) = 0$

Assumptions: Steady-state. Uniform heat flux q'' at right face and uniform convective boundary condition at left face. One-dimensional conduction through the board with constant conductivity.

Analysis: For constant k, the heat diffusion equation becomes $\frac{d^2T}{dx^2} = 0$ with general solution

$$T = C_1 x + C_2$$

Problem #4a) continued

Boundary conditions: $q'' = -k \frac{dT}{dx}\Big|_{x=0}$

$$-k \frac{dT}{dx}\Big|_{x=L} = h\left[T\big|_{x=L} - T_\infty\right]$$

$$q'' = -k C_1 = h(C_1 L + C_2 - T_\infty)$$

$$C_1 = -\frac{q''}{k} \qquad C_2 = \frac{q''}{h} + T_\infty + \frac{q'' L}{k}$$

$$T = q''\left(\frac{L}{k} + \frac{1}{h}\right) + T_\infty - \frac{q''}{k}x$$

$$T\big|_{x=0} = \frac{3.2\ W}{0.12\,m \times 0.18\,m}\left(\frac{0.003\,m}{20\ \frac{W}{mK}} + \frac{1}{50\ \frac{W}{m^2K}}\right) + 40°C = 42.985\ °C$$

$$T\big|_{x=L} = \frac{3.2\ W}{0.12\,m \times 0.18\,m} \cdot \frac{1}{50\ \frac{W}{m^2K}} + 40°C = 42.963\ °C$$

Both sides of the board are practically at $\boxed{43\ °C}$

Conduction resistance through the board is negligible.

Convection resistance on backside of board results in a 3°C temperature drop from the board at 43°C to the freestream $T_\infty = 40°C$.

Ann Mescher April 10, 2017

HW Assignment #2

Problem #4 b)

Statement: Previous configuration as in Problem #4a) with an aluminum ($k = 237 \frac{W}{mK}$) plate (0.20 cm × 12 cm × 18 cm) and fin array (864 aluminum pin fins, $D = 0.25$ cm, $L = 2.0$ cm) bonded onto the backside of circuit board with 0.020 cm thick epoxy ($k = 1.8 \frac{W}{mK}$).

Find: Temperature on both sides of circuit board.

Assumptions: Steady-state. 1-D conduction through circuit board and epoxy adhesive. Three conduction resistances in series with fin array resistance. Same boundary condition of uniform $q''\big|_{x=0}$ at right (front) side. Same h, T_∞ at left (back) side for fin array.

Analysis:
$$q = \frac{\Delta T}{R_{total}} \quad \text{where} \quad q = 80 \cdot 0.040\,W = 3.2\,W$$

and
$$R_{total} = \left(\frac{L}{kA}\right)_{board} + \left(\frac{L}{kA}\right)_{epoxy} + \left(\frac{L}{kA}\right)_{aluminum} + R_{\substack{fin\\array}}$$

where
$$R_{\substack{fin\\array}} = \frac{1}{h A_t \eta_{array}}$$
and where

$$\eta_{array} = 1 - \frac{N A_f}{A_t}(1 - \eta_f) \quad , \quad N = 864 \text{ pin fins}$$

Problem #4 b) continued

Surface area of single fin, $A_f = \pi D L + \frac{\pi D^2}{4}$

$$A_f = \pi \cdot 0.0025\,m \left(0.02\,m + \frac{0.0025\,m}{4}\right) = 1.62 \times 10^{-4}\ m^2$$

Total finned surface area, $A_t = N A_f + A_b$, where

prime area $A_b = H \cdot W - N\frac{\pi D^2}{4} = 0.12\,m \cdot 0.18\,m - \frac{864 \cdot \pi \cdot (0.0025\,m)^2}{4}$

$$A_b = 0.01736\ m^2$$

$$A_t = 0.157\ m^2$$

Efficiency of a single pin fin, $\eta_f = \frac{q_f}{h A_f \theta_b}$

$$\eta_f = \frac{1}{A_f}\sqrt{\frac{PkA_c}{h}}\ \frac{\sinh(mL) + \frac{h}{mk}\cosh(mL)}{\cosh(mL) + \frac{h}{mk}\sinh(mL)} \quad \text{where } m = \sqrt{\frac{hP}{kA_c}}$$

for a single pin fin with convection at fin tip $(x=L)$.

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4 \cdot 50\,\frac{W}{m^2K}}{237\,\frac{W}{mK} \cdot 0.0025\,m}} = 18.37\ m^{-1}$$

$\eta_f = 0.955$ (with $k_{Al} = 237\,\frac{W}{mK}$, $L = 0.02\,m$, $h = 50\,\frac{W}{m^2K}$)

$$\eta_{array} = 1 - \frac{864 \cdot 1.62 \times 10^{-4}\,m^2}{0.157\ m^2}(1 - 0.955) = 0.960$$

$$R_{total} = \frac{0.003\,m}{20\,\frac{W}{mK} \cdot A} + \frac{0.0002\,m}{1.8\,\frac{W}{mK} \cdot A} + \frac{0.002\,m}{237\,\frac{W}{mK} \cdot A} + \frac{1}{50\,\frac{W}{m^2K} \cdot 0.157\,m^2 \cdot 0.960}$$

where $A = 0.0216\ m^2$

$$R_{total} = 0.145\ \frac{K}{W}$$

$$q = \frac{T|_{x=0} - T_\infty}{R_{total}}$$

Problem #4 b) continued

$$T\Big|_{x=0} = q\,R_{total} + T_\infty = 3.2\,W \cdot 0.145\,\frac{K}{W} + 40^\circ C = 40.46\,^\circ C$$

On the backside of the circuit board ($x = 0.003\,m$),

$$q = \frac{T\Big|_{x=0} - T\Big|_{x=0.003m}}{\left(\frac{L}{kA}\right)_{board}}$$

$$T\Big|_{x=0.003m} = T\Big|_{x=0} - q\left(\frac{L}{kA}\right)_{board} = 40.46^\circ C - 3.2\,W\,\frac{0.003\,m}{20\,\frac{W}{mK}\cdot 0.0216\,m^2}$$

$$T\Big|_{x=0.003m} = 40.44\,^\circ C$$

Both sides of the board are practically at $\boxed{40.5\,^\circ C}$

Comment:

Due to the high efficiency η_{array} of the fin array design, the maximum temperature at the chips is only ~0.5 °C higher than the ambient freestream $T_\infty = 40\,^\circ C$.

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HW Assignment #2

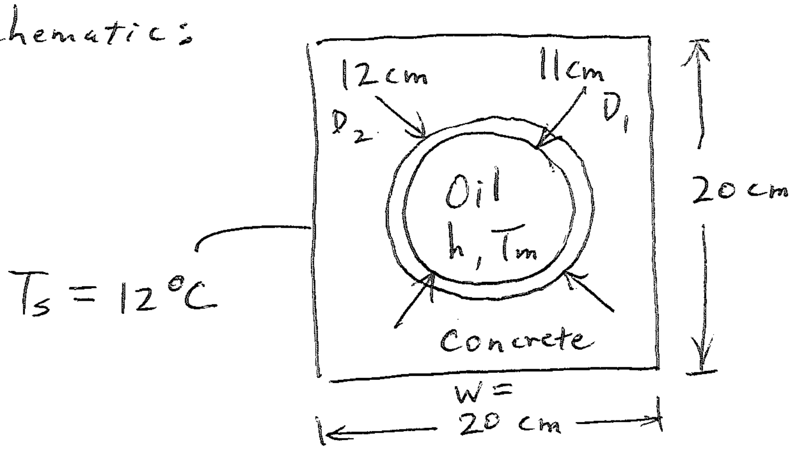
April 10, 2017

Problem #5

Statement: Oil at $T_m = 100°C$ and with $h = 350 \frac{W}{m^2K}$ flows in a plain carbon steel pipe (12 cm O.D., 11 cm I.D.) which is enclosed in 20 cm square concrete ($k = 0.80 \frac{W}{mK}$). The outer surface of the concrete is at 12 °C.

Find: The rate of heat transfer from the pipe per unit length.

Schematic:



Differential C.V. is used to obtain:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = 0$$

Assumptions: Steady-state, $\frac{\partial T}{\partial t} = 0$, and no thermal generation, $\dot{q} = 0$. Uniform boundary conditions at inner surface of steel pipe and outer surface of concrete. Two-dimensional conduction requiring use of conduction shape factor. Constant conductivity in each material. At $T = 370$ K, $k_{steel} = 58 \frac{W}{mK}$. Two conduction resistances in series with convection resistance.

Analysis: $q = \frac{\Delta T}{R_{total}}$ where $\Delta T = (100 - 12)\,K = 88\ K$

Problem # 5 continued

$$R_{total} = \frac{1}{hA} + \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi L k_{steel}} + \frac{1}{S k_{concrete}}$$

where $S = \dfrac{2\pi L}{\ln\left(\dfrac{1.08\, w}{D_2}\right)}$ and $w = 20\ cm$

$$L R_{total} = \frac{1}{h \pi D_1} + \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi k_{steel}} + \frac{\ln\left(\frac{1.08\, w}{D_2}\right)}{2\pi k_{concrete}}$$

$$= \frac{1}{350 \frac{W}{m^2 K} \pi \cdot 0.11\, m} + \frac{\ln\left(\frac{0.12}{0.11}\right)}{2\pi \cdot 58 \frac{W}{m K}} + \frac{\ln\left(\frac{1.08 \cdot 0.20 m}{0.12 m}\right)}{2\pi \cdot 0.80 \frac{W}{m K}}$$

$$\frac{q}{L} = \frac{88\ K}{0.125 \frac{m K}{W}} = \boxed{700 \frac{W}{m}}$$ Answer

Comments: In this case, the steel pipe wall resistance is negligible. Without the thermal resistance associated with the surrounding concrete, the heat loss per unit length of the oil pipeline would be much greater. The concrete serves both as a thermal insulator and to reduce pipe corrosion which would otherwise occur at a fast rate in sea water.