ME331: Introduction to Heat Transfer
Spring 2017

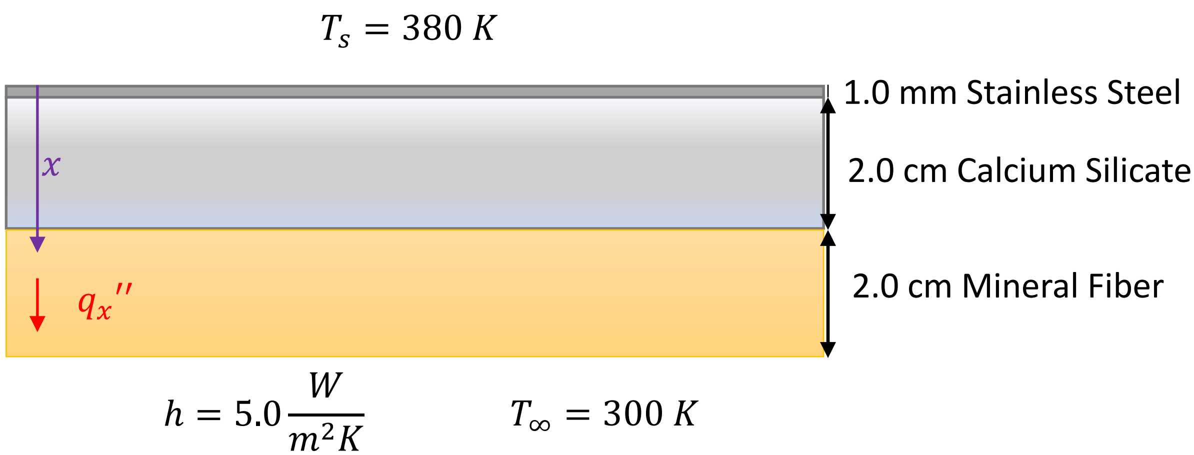
ME 331 Homework Assignment #2

Problem 1

A composite wall consists of a 1.0 mm thick AISI 304 stainless steel plate, 2.0 cm of industrial calcium silicate insulation, and 2.0 cm of industrial mineral fiber blanket insulation. The stainless steel surface is maintained at 380 K while the other side loses heat to the ambient air at 300 K, with a convection heat transfer coefficient of 5.0 W/m2-K. Estimate the heat flow per unit area.

Find: **Heat flux (heat flow per unit area), q_x'', at the wall side in contact with air.**

Schematic:



Assumptions:

We assume steady state with no thermal generation, so that $\frac{\partial T}{\partial t} = 0$ and $\dot{q} = 0$.

Uniform boundary conditions at both the steel and mineral fiber boundaries.

One dimensional conduction and constant conductivities:

$$k_{ss} = 14.9 \frac{W}{m \cdot K};k_{Ca_2SiO_4} = 0.059 \frac{W}{m \cdot K}; k_{mf} = 0.040 \frac{W}{m \cdot K}$$

Analysis:

We use a lumped resistance approach where $q = \frac{\Delta T}{R_{total}}$ and the total resistance is comprised of four resistors in series (three materials plus convection) and $\Delta T = (380 - 300)K = 80\ K$.

$$R_{total} = \left(\frac{L}{kA}\right)_{SS} + \left(\frac{L}{kA}\right)_{CS} + \left(\frac{L}{kA}\right)_{MF} + \frac{1}{hA}$$

$$A \cdot R_{total} = \frac{0.001\ m}{14.9 \frac{W}{mK}} + \frac{0.02\ m}{0.059 \frac{W}{mK}} + \frac{0.02\ m}{0.040 \frac{W}{mK}} + \frac{1}{5 \frac{W}{m^2K}} = 1.039 \frac{m^2K}{W}$$

$$q'' = \frac{\Delta T}{A \cdot R_{total}} = \frac{80\ K}{1.039\frac{m^2K}{W}} = 77.0\frac{W}{m^2}$$

$$\boxed{q'' = 77.0\frac{W}{m^2}}$$

Comments

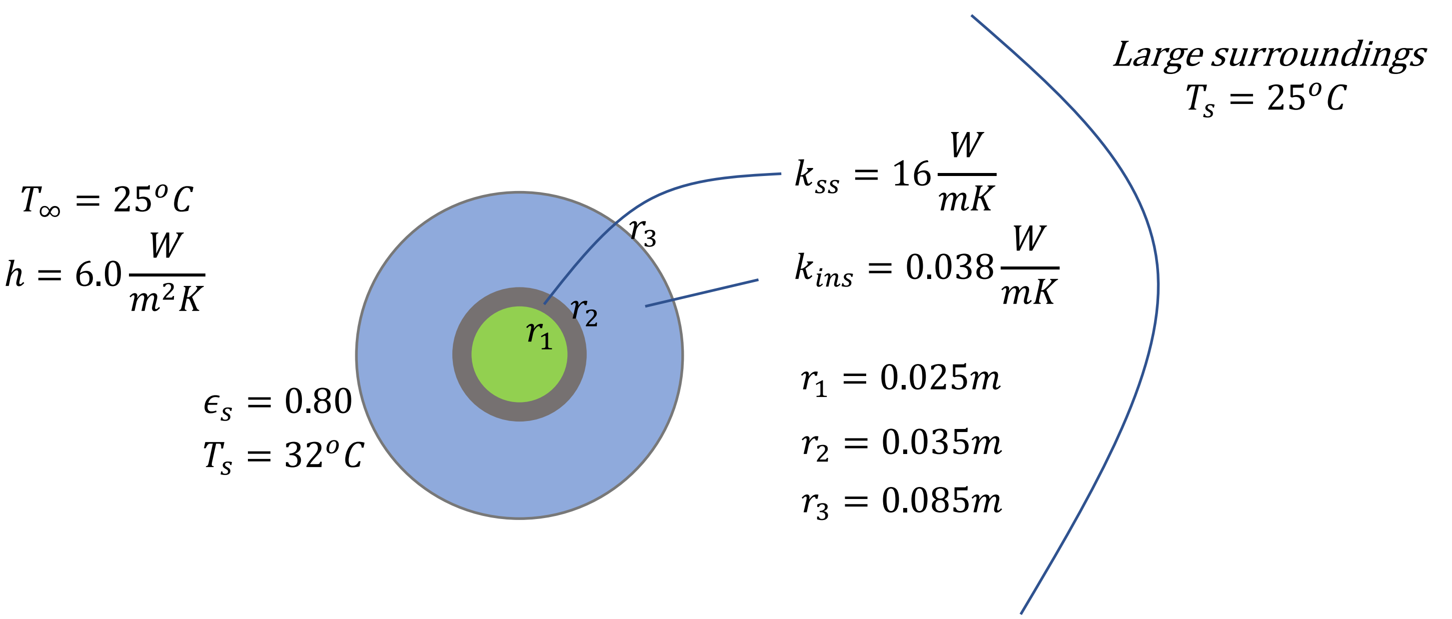
We can see that only the resistance of the stainless steel is negligible (naturally since it is the thinnest one and has the greatest conductivity).

Problem 2

A 2.0 m long cylindrical chemical reactor has an inside diameter of 5.0 cm, has a 1.0 cm thick stainless steel shell, and is insulated on the outside by a layer of medium-density (ρ = 28 kg/m3) fiberglass blanket insulation (0.038 W/m-K), 5.0 cm thick. The ambient air is at 25°C, and the surroundings can be assumed to be large and also at 25°C. The convective heat transfer coefficient between the insulation and air is 6.0 W/m^2-K, and the gray surface emissivity (and absorptivity) of the insulation is 0.80. At steady state the outer surface of the insulation is measured at 32°C. Draw the thermal circuit and determine the temperature of the inner surface of the stainless steel shell. Take k = 16 W/m-K for the stainless steel.

Find: The inner surface temperature of the stainless steel T_r

Schematic:



Assumptions:

We assume steady state, 1-D heat transfer with no thermal generation inside stainless steel or insulation, $\dot{q} = 0$. We assume a gray surface in a large enclosure, thus:

$$q_{radiation} = \epsilon\sigma A(T_s^4 - T_{surr}^4)$$

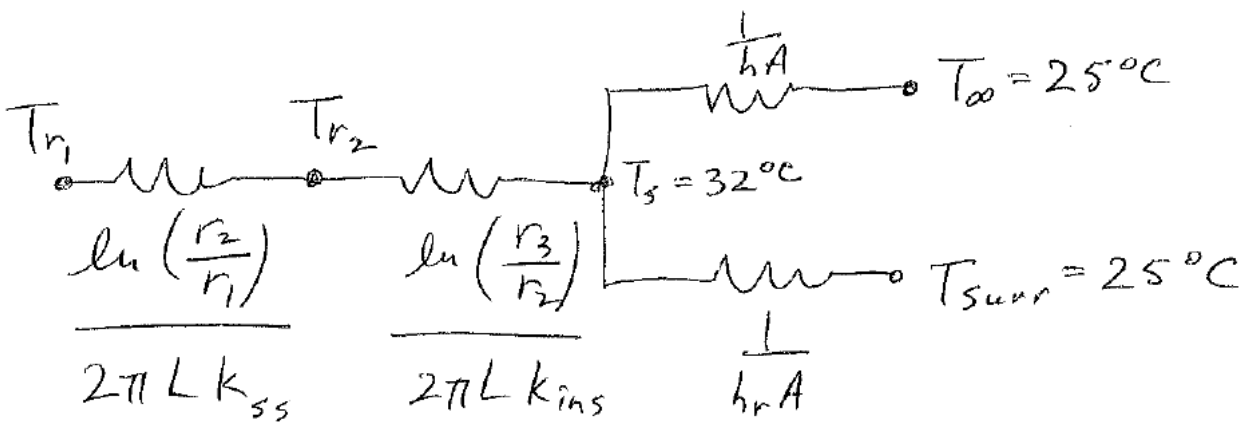
Analysis:

A differential control volume in the cylindrical radial coordinate is used to obtain

$$\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$$

The solution of this heat diffusion equation yields the resistance for a cylindrical shell.

The thermal circuit has two conduction resistances in series followed by convection and radiation resistances in parallel.



Where

$$q_{radiation} = \epsilon\sigma A(T_s^4 - T_{surr}^4) = h_r A(T_s - T_{surr})$$

$$h_r = \epsilon\sigma(T_s + T_{surr})(T_s^2 + T_{surr}^2)$$

And

$$q_{total} = q_{rad} + q_{conv} = \epsilon\sigma A(T_s^4 - T_{surr}^4) + hA(T_s - T_\infty)$$

$$q''_{total} = 0.80 \cdot 5.67 * 10^{-8}\frac{W}{m^2K^4}[305\ K^4 - 298\ K^4] + 6.0\frac{W}{m^2K^4}(32 - 25)K$$

This same q''_{total} must flow through the conduction resistances.

$$q''_{total} = \frac{T_{r_1} - 32^oC}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk_{ss}} + \frac{ln\left(\frac{r_3}{r_2}\right)}{2\pi Lk_{ins}}}$$

Solving we get

$$T_{r_1} = 184.6^oC$$

Comments:

As in problem 1, the stainless steel layer has negligible resistance as compared to the insulating layer. The outer convection and radiation resistances are comparable and neither can be neglected.

Problem 3

A 1.0 mm thick copper plate (k = 386 W/m-K) is sandwiched between two 5.0 mm thick epoxy boards (k = 0.26 W/m-K) that are 15 cm x 20 cm in size. If the thermal contact resistance on each side of the copper plate is estimated to be 8.3x10^{-3} K/W, determine the error involved in the total thermal resistance of the plate if thermal contact resistances are ignored.

Find: The heat rates with and without $R_{contact}$

Schematic:

Assumption:

1-D steady state conduction with no heat generation ($\dot{q} = 0$) and constant conductivity for each material. Hence we have:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

With no contact resistance we have three resistances in series, with contact resistance we have five resistances in series.

Analysis:

We have that

$$q = \frac{\Delta T}{R_{total}}$$

Hence for the same ΔT we will get a different q depending on whether or not contact resistance is included.

Without contact resistance:

$$R_{total} = 2\left(\frac{L}{kA}\right)_{epoxy} + \left(\frac{L}{kA}\right)_{cu}$$

$$R_{total} = 2\left(\frac{0.0005m}{0.26\frac{W}{mK}\cdot 0.15m\cdot 0.20m}\right) + \frac{0.001m}{386\frac{W}{mK}\cdot 0.15m\cdot 0.20m} = 1.28\frac{K}{W}$$

With contact resistance

$$R_{total} = 1.28\frac{K}{W} + 2\cdot 0.0083\frac{K}{W} = 1.30\frac{K}{W}$$

Thus for instance, for a $\Delta T = 10\ K$ we have that

$$q = 7.8\ W\ without\ R_c$$

$$q = 7.7\ W\ with\ R_c$$

The error is given by

$$\boxed{error = \frac{1.30 - 1.28}{1.30} = 1.3\%}$$

Hence for a given ΔT the error due to neglecting the contact resistance results in a 1.3% error in q.

Comments:

For this problem, the contact resistances are small compared to the thermal resistance of the epoxy boards. However, contact resistances should be considered in general to check whether they constitute a significant portion of the total resistance.

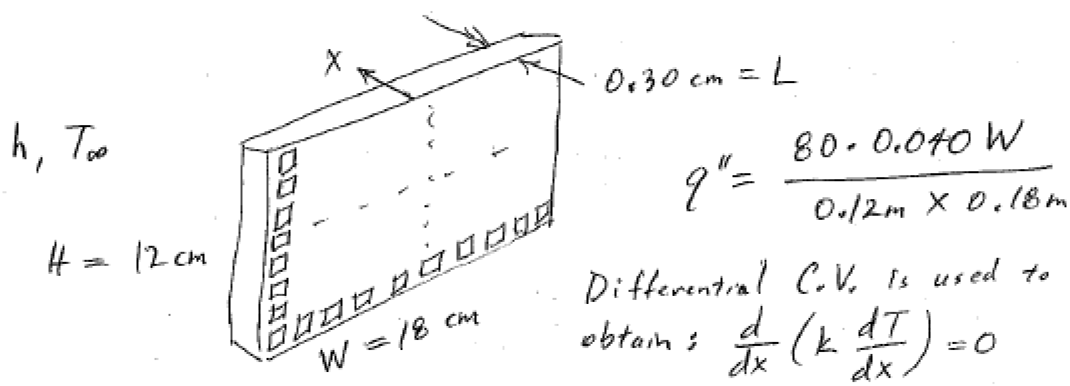
Problem 4

A 0.30 cm thick, 12 cm high, and 18 cm long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.040 W. The board is impregnated with copper filling and has an effective thermal conductivity of 20 W/m-K. All the heat generated in the chips is conducted across the circuit board and is dissipated from the backside of the board to a medium at 40°C with a heat transfer coefficient of 50 W/m2-K. (a) Determine the temperatures on the two sides of the circuit board. (b) Now a 0.20 cm thick, 12 cm high, 18 cm long aluminum plate (k = 237 W/m-K) with a total of 864 aluminum pin fins each of diameter 0.25 cm and length 2.0 cm, is attached to the back side of the circuit board with a 0.020 cm thick epoxy adhesive (k = 1.8 W/m-K). Determine the new temperatures on the two sides of the circuit board.

(a)

Find: Temperature on both sides of the board.

Schematic:



Assumptions: We assume steady state and uniform heat flux q'' at the right face and uniform convective boundary condition at the left face. One-dimensional conduction through the board with constant conductivity.

Analysis:

for constant k and under the assumptions stated above the heat diffusion equation simplifies to:

$$\frac{d^2T}{dx^2} = 0$$

Hence

$$T = c_1 x + c_2$$

The boundary conditions are given by:

$$q'' = -k\frac{dT}{dx}\bigg|_{x=L} = h[T|_{x=L} - T_\infty]$$

$$q'' = -kc_1 = h(c_1 L + c_2 - T_\infty)$$

$$c_1 = -\frac{q''}{k} \qquad c_2 = \frac{q''}{h} + T_\infty + \frac{q''L}{k}$$

$$T = q''\left(\frac{L}{k} + \frac{1}{h}\right) + T_\infty - \frac{q''}{k}x$$

Hence

$$T|_{x=0} = \frac{3.2W}{0.12m \cdot 0.18m}\left(\frac{0.003m}{20\frac{W}{mK}} + \frac{1}{50\frac{W}{m^2K}}\right) + 40^oC = 42.985^oC$$

$$T|_{x=L} = \frac{3.2W}{0.12m \cdot 0.18m}\left(\frac{1}{50\frac{W}{m^2K}}\right) + 40^oC = 42.963^oC$$

Both sides are practically at 43^oC

Hence, we can say that conduction resistance through the board is negligible. The convection resistance on backside of the board results in a 3^oC temperature drop from the board at 43^oC to the freestream at $T_\infty = 40^oC$.

(b)

Now consider the case with the pin fins.

Find: Temperature on both sides of the circuit board.

Assumption: We assume steady state, 1-D conduction through the circuit board and epoxy adhesive. Three conduction resistances in series with fin array resistance. Same uniform heat flux $q''|_{x=0}$ at the right face. At the left (back) of the array we assume same h, T_∞.

Analysis:

$$q = \frac{\Delta T}{R_{total}} = 80 \cdot 0.040\ W = 3.2\ W$$

$$R_{total} = \left(\frac{L}{kA}\right)_{board} + \left(\frac{L}{kA}\right)_{epoxy} + \left(\frac{L}{kA}\right)_{aluminum} + R_{fin\ array}$$

$$R_{fin\ array} = \frac{1}{hA_t\eta_{array}}$$

$$\eta_{array} = 1 - \frac{N \cdot A_f}{A_t}(1 - \eta_f)$$

$$N = 864\ pin\ fins$$

For a single fin $A_f = \pi DL + \frac{\pi D^2}{4} = \pi \cdot 0.0025m \cdot \left(0.02m + \frac{0.0025m}{4}\right) = 1.62 * 10^{-4}m^2$

Total finned surface area $A_t = NA_f + A_b$

$$A_b = H \cdot W - N\frac{\pi D^2}{4} = 0.12m \cdot 0.18m - 864 \cdot \pi \cdot \frac{(0.0025m)^2}{4}$$

$$A_b = 0.01736\, m^2$$

$$A_t = 0.157\, m^2$$

For a single pin fin $\eta_f = \frac{q_f}{hA_f\theta_b}$

$$\eta_f = \frac{1}{A_f}\sqrt{\frac{PkA_c}{h}}\frac{\sinh(mL) + \frac{h}{mK}\cosh(mL)}{\cosh(mL) + \frac{h}{mK}\sinh(mL)}$$

$$m = \sqrt{\frac{hP}{kA_c}}$$

For a single pin fin with convection at fin tip

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h\pi D}{\frac{k\pi D^2}{4}}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4 \cdot 50\frac{W}{m^2K}}{237\frac{W}{mK} \cdot 0.0025m}} = 18.37\, m^{-1}$$

$$\eta_f = 0.955 \qquad with\ k_{Al} = 237\frac{W}{mK}\ , L = 0.02\ m\ and\ h = 50\frac{W}{m^2K}$$

$$\eta_{array} = 1 - \frac{864 \cdot 1.62 * 10^{-4}m^2}{0.157m^2}(1 - 0.955) = 0.960$$

$$R_{total} = \frac{0.003m}{20\frac{W}{mK} \cdot A} + \frac{0.0002m}{1.8\frac{W}{mK} \cdot A} + \frac{0.002m}{237\frac{W}{mK} \cdot A} + \frac{1}{50\frac{W}{m^2K} \cdot 0.157m^2 \cdot 0.960}$$

$$A = 0.0216\, m^2$$

$$R_{total} = 0.145\frac{K}{W}$$

$$q = \frac{T|_{x=0} - T_\infty}{R_{total}}$$

$$T|_{x=0} = qR_{total} + T_\infty = 3.2W \cdot 0.145\frac{K}{W} + 40^oC = 40.46^oC$$

On the backside of the circuit board (x=0.003 m)

$$q = \frac{T|_{x=0} - T|_{x=0.003m}}{\left(\frac{L}{kA}\right)_{board}}$$

$$T|_{x=0.003m} = T|_{x=0} - q\left(\frac{L}{kA}\right)_{board} = 40.46^oC - 3.2W\frac{0.003m}{20\frac{W}{mK}\cdot 0.0216m^2}$$

$$T|_{x=0.003m} = 40.44^oC$$

Hence both sides are practically at 40.5^oC

Comments:

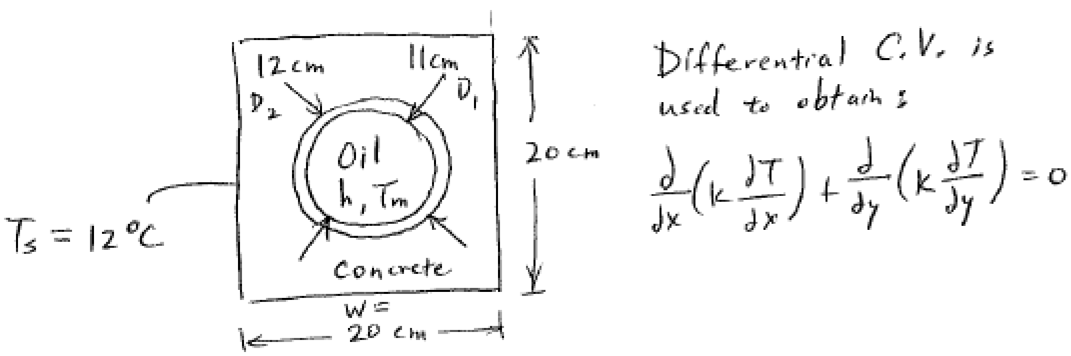
We can see that due to the high efficiency of the array η_{array} the maximum temperature at the chips is merely $\sim 0.5^oC$ higher than the ambient free stream temperature.

Problem 5

A long, 12.0 cm O.D., 11.0 cm I.D., plain carbon steel pipeline for oil is centrally enclosed in 20 cm square concrete (k = 0.80 W/m-K) along the pipe's entire length. The concrete enclosed pipe assembly is submerged in seawater with the outer surface of the concrete maintained at 12°C. The oil has a bulk mean temperature of 100°C with an inner surface heat transfer coefficient of 350 W/ m^2-K. Find the heat transfer rate per meter of pipe length.

Find: The rate of heat transfer from the pipe per unit length.

Schematic:



Assumption:

Steady state, and no thermal generation. Uniform boundary conditions at the inner surface of the steel pipe and outer surface of concrete. Two-dimensional conduction requiring use of conduction shape factor. Constant conductivity in each material. At $T = 370K, k_{steel} = 58\frac{W}{mK}$. Two conduction resistances in series with convection resistance.

Analysis:

$$q = \frac{\Delta T}{R_{total}} \qquad where\ \Delta T = (100 - 12)K = 88K$$

$$R_{total} = \frac{1}{hA} + \frac{\ln\left(\frac{D_2}{D_1}\right)}{2\pi L k_{steel}} + \frac{1}{S k_{concrete}}$$

$$S = \frac{2\pi L}{\ln\left(\frac{1.08 \cdot w}{D_2}\right)} \qquad and\ w = 20\ cm$$

$$LR_{total} = \frac{1}{h\pi D_1} + \frac{ln\left(\frac{D_2}{D_1}\right)}{2\pi k_{steel}} + \frac{ln\left(\frac{1.08 \cdot w}{D_2}\right)}{2\pi k_{concrete}}$$

$$LR_{total} = \frac{1}{350\frac{W}{m^2K}\pi \cdot 0.11m} + \frac{ln\left(\frac{0.12}{0.11}\right)}{2\pi \cdot 58\frac{W}{mK}} + \frac{ln\left(\frac{1.08 \cdot 0.20m}{0.12m}\right)}{2\pi \cdot 0.80\frac{W}{mK}}$$

$$\frac{q}{L} = \frac{88\ K}{0.125\,\frac{mK}{W}} = 700\,\frac{W}{m}$$

$$\boxed{\frac{q}{L} = 700\,\frac{W}{m}}$$

Comments:

In this case, the steel pipe wall resistance is negligible. Without the thermal resistance associated with the surrounding concrete, the heat loss per unit length of the oil pipeline would be much greater. The concrete serves both as a thermal insulator and to reduce pipe corrosion which would otherwise occur at a fast rate in sea water.