HW Assignment # 3

Problem # 1

Statement: Steady-state soldering iron tip. Cylindrical with 3 mm diameter and 10 mm long. $T_\infty = T_{surr} = 20°C$, $h = 20\ W/m^2\text{-}K$

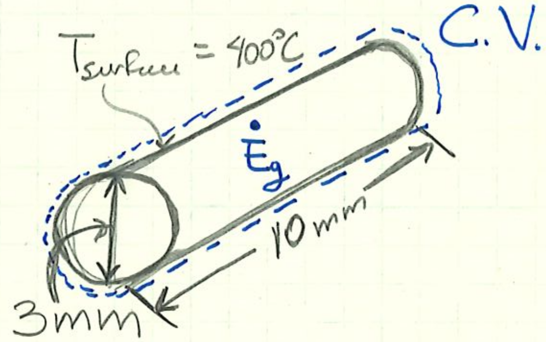
$\varepsilon_{new} << 1$, $\varepsilon_{aged} = 0.8$, $T_{surface} = 400°C$

Find: Required power for two cases:

a) New tip, $\varepsilon \simeq 0$

b) Aged tip, $\varepsilon = 0.8$

Schematic:



$T_{surr} = T_\infty = 20°C$

Assumptions:
- Constant properties, steady state.
- Heat loss from one end and peripheral surfaces.
- Uniform surface temperature and heat transfer coefficient.
- Aged tip is treated as a small gray surface in a large enclosure.
- For new tip, radiation is negligible.

Analysis

- Starting with Energy Balance

$$\cancel{E_{in}} - E_{out} + E_g = \Delta \cancel{E_{ST}}$$

0 steady state

$$E_g = E_{out}$$

$$E_g = \text{Power Supplied} = \dot{q}_c$$

$$E_{out} = q_{conv} + q_{radiation}$$

- First calculate E_{out}:

$$q_{conv} = h A_s (T_s - T_\infty)$$

$$A_s = 2\pi r L + \pi r^2$$
$$= 2\pi(1.5\cdot10^{-3}\,m)(0.01m) + \pi(1.5\cdot10^{-3}m)^2$$
$$= 1.013\cdot10^{-4}\,m^2$$

$$q_{conv} = 20\,\frac{W}{m^2K}(1.013\cdot10^{-4}m^2)(673-293)K$$
$$= 0.769\,W$$

- Second calculate radiation for $\varepsilon = 0.8$

$$q_{radiation} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4)$$

$$= 0.8(5.67\cdot10^{-8}\,\frac{W}{m^2K^4})(1.013\cdot10^{-4}m^2)(673^4-293^4)K^4$$

$$= 0.9088\,W$$

Problem # 1 continued:

- For new tip:

$$q_{\substack{new\\Tip}} = q_{conv}$$

$$\boxed{q_{\substack{new\\tip}} = 0.77\ W}$$

$$\dot{q}_{\substack{new\\tip}} = \frac{q_{\substack{new\\tip}}}{Volume} = \frac{0.77\ W}{\pi (1.5 \cdot 10^{-3} m)^2 (0.01 m)}$$

$$\boxed{\dot{q}_{\substack{new\\tip}} = 1.09 \cdot 10^{7}\ \frac{W}{m^3}}$$

- For aged tip:

$$q_{\substack{aged\\Tip}} = q_{conv} + q_{radiation} = 0.769 W + 0.9088 W$$

$$\boxed{\begin{aligned} q_{\substack{aged\\tip}} &= 1.68 W \\ \dot{q}_{\substack{aged\\tip}} &= 2.37 \cdot 10^{7}\ \frac{W}{m^3} \end{aligned}}$$

Comments:

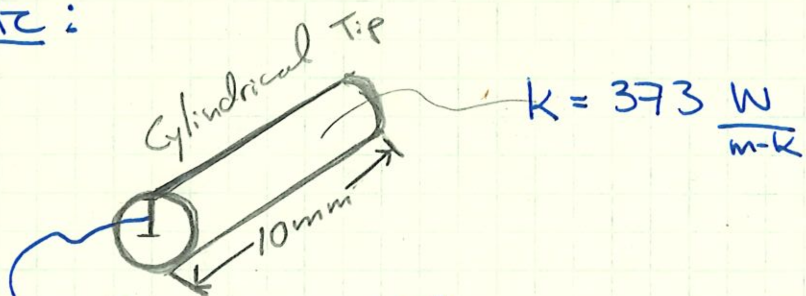
For the new polished tip radiation was neglected. The old tip has more than twice the power requirement for the new Tip.

Problem #2

Statement: Same steady-state tips (both new and aged) as in problem #1.

Find: Steady-state temperature distributions for new and aged tips.

Schematic:



$R = \frac{3mm}{2} = 1.5 \cdot 10^{-3} m$

Assumptions:
- Constant properties
- 1-D in radial direction.
- Steady-state
- Soldering tip constructed from copper with $k = 373 \frac{W}{m\text{-}K}$ at an averge temp of 700 K.

Analysis:

The Heat Diffusion Equation in cylindrical coordinates is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)}^{0} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}^{0} + \dot{q} = \cancel{\rho c\frac{\partial T}{\partial t}}^{0}$$

Since we have assumed 1-D and steady state.

So we have:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q} = 0$$

- Boundary conditions:

$$T|_{r=R} = 673\text{ K} \quad \text{(outer surface)}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{(center line adiabatic due to symmetry)}$$

- First solve for new tip condition, $q_{rad} = 0$

Separate variables and integrate

Since k = constant

$$\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \dot{q} = 0$$

$$k\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\dot{q}r$$

$$kr\frac{\partial T}{\partial r} = -\dot{q}\left(\frac{r^2}{2}\right) + C_1$$

$$\Rightarrow \quad \frac{dT}{dr} = \frac{-\dot{q}r}{2k} + \frac{C_1}{kr}$$

Integrating again:

$$T(r) = -\frac{\dot{q}r^2}{4k} + C_2 + \frac{C_1}{k}\ln r$$

- Now apply B.C. at $r=0$: $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = \left.\left(-\frac{\dot{q}r}{2k} + \frac{C_1}{kr}\right)\right|_{r=0} = \left.\frac{C_1}{kr}\right|_{r=0} = 0 \Rightarrow \underline{C_1 = 0}$$

Problem #2 continued

- Now apply B.C. at $r=R$

$$T\Big|_{r=R} = 673\,K$$

$$673K = -\frac{\dot{q}R^2}{4k} + C_2$$

$$\Rightarrow \quad C_2 = 673K + \frac{\dot{q}R^2}{4k}$$

$$\Rightarrow \quad T(r) = -\frac{\dot{q}r^2}{4k} + \frac{\dot{q}R^2}{4k} + 673K$$

$$\boxed{T(r) = \frac{\dot{q}}{4k}(R^2 - r^2) + 673\,K}$$

- Substituting $\dot{q}$ for the new tip (convection only)

$$T(r) = \frac{1.09\cdot 10^7 \frac{W}{m^3}}{4k}(R^2 - r^2) + 673\,k$$

Where we used $\dot{q}_{\substack{new\\Tip}}$ from problem #1

- Substituting $\dot{q}$ for the old tip ($\dot{q}_{\substack{aged\\tip}}$ from problem #1)

$$T(r) = \frac{2.37\cdot 10^7 \frac{W}{m^3}}{4k}(R^2 - r^2) + 673\,K$$

- The maximum temperature occurs at $r=0$.

Thus, for the new tip:

$$T_{\substack{max\\new}} = \frac{\dot{q}}{4k}(R^2-\cancel{r^2}) + 673\,K = \frac{\dot{q}R^2}{4k} + 673\,K$$

$$= \frac{1.09\cdot 10^7 \frac{W}{m^3}}{4\cdot 373 \frac{W}{mK}}(1.5\cdot 10^{-3}m)^2 + 673\,K$$

$$= 0.0164\,K + 673\,K \simeq 673\,K = 400°C$$

$$\boxed{T_{\substack{MAX\\new}} = 400°C}$$ Answer

For the old tip:

$$T_{\substack{MAX\\old}} = \frac{2.37\cdot 10^7 \frac{W}{m^3}}{4\cdot 373 \frac{W}{mK}}(1.5\cdot 10^{-3}m)^2 + 673\,K$$

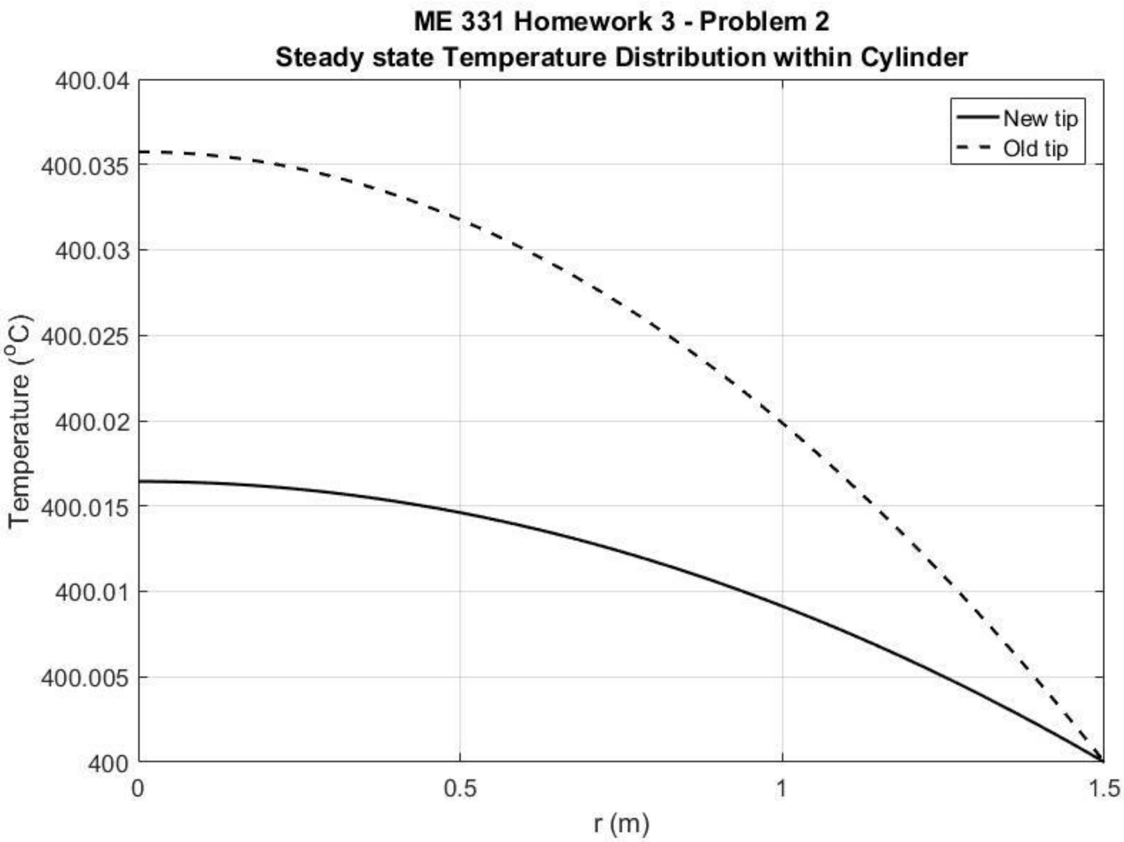
$$= 0.0357\,K + 673\,K \simeq 673\,K = 400°C$$

$$\boxed{T_{\substack{MAX\\old}} = 400°C}$$ Answer

See attached plots and MATLAB code.

Comments: Due to negligible conduction resistance, there are extremely small thermal gradients within the soldering iron tips, whether for the new or aged cases.

Problem #2 continued



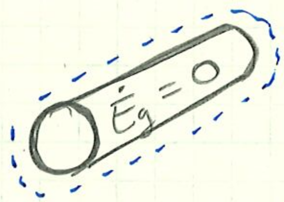

```
%% Plot temperature profiles for new and old tips

r = 0:1e-5:1.5e-3;
R = 1.5e-3; % m
k = 373; %W/m-K

% New tip - convection onlt
qnew = 1.09e7; %W/m^3
Tnew = (qnew/(4*k))*(R^2-r.^2)+673;

% Old tip - convection plus radiation
qold = 2.37e7;  %W/m^3
Told = (qold/(4*k))*(R^2-r.^2)+673;

figure(1); clf
plot(1000*r,Tnew-273,'k','Linewidth',1.6); grid on; hold on
plot(1000*r,Told-273,'k--','Linewidth',1.6);
xlabel('r (m)'); ylabel('Temperature (^oC)')
legend('New tip','Old tip')

title({'ME 331 Homework 3 - Problem 2';'Steady state Temperature Distribution
within Cylinder'})
set(gca,'Fontsize',13)
```

Problem 3

Statement: Soldering iron tips (both new and aged) operating at steady state, then turned off.

Find: Temperature within tip as a function of time for both cases.

Schematic:



C.V. Lumped!

Assumptions: $T_{initial} = 673\ K$

- Negligible temperature changes within tips, Apply lumped capacitance model.

- Constant properties at $T = 400k$

$\rho = 8933\ kg/m^3$; $k = 393\ \frac{W}{mK}$; $c = 397\ \frac{J}{kg\,K}$

$h = 20\ W/(m^2K)$

Analysis

- Check Biot # = $Bi = \frac{h L_c}{k}$

Where $L_c = \frac{\text{Volume}}{\text{Surface Area}} = \frac{\pi r^2 L}{\pi r^2 + 2\pi r L} = 6.9767 \cdot 10^{-4} m$

$$Bi = \frac{20\ \frac{W}{m^2K} \cdot 6.9767 \cdot 10^{-4} m}{393\ \frac{W}{mK}} = 3.5 \cdot 10^{-5} << 0.1$$

So lumped works!

Problem 3 continued:

- Now apply conservation of Energy to lumped C.V.

$$\dot{E}_{stored} = -\dot{E}_{out}$$

$$\rho V c \frac{dT}{dt} = -hA_s(T - T_\infty)$$ for new tip

$$\Rightarrow \int_0^t \frac{-hA_s}{\rho V c}\,dt = \int_{T_i}^{T} \frac{1}{T - T_\infty}\,dT$$

$$-\frac{hA_s}{\rho V c} t = \ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right)$$

$$\Rightarrow \boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA_s}{\rho V c} t\right)}$$ For new tip.

- For the aged tip we have to account for radiation as well.

We get

$$\frac{dT}{dt} = \frac{-A_s}{\rho V c}\left[h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{surr}^4)\right]$$

Where $T_{surr} = T_\infty = 20°C = 293\,K$

$\varepsilon = 0.8$ and we can solve numerically

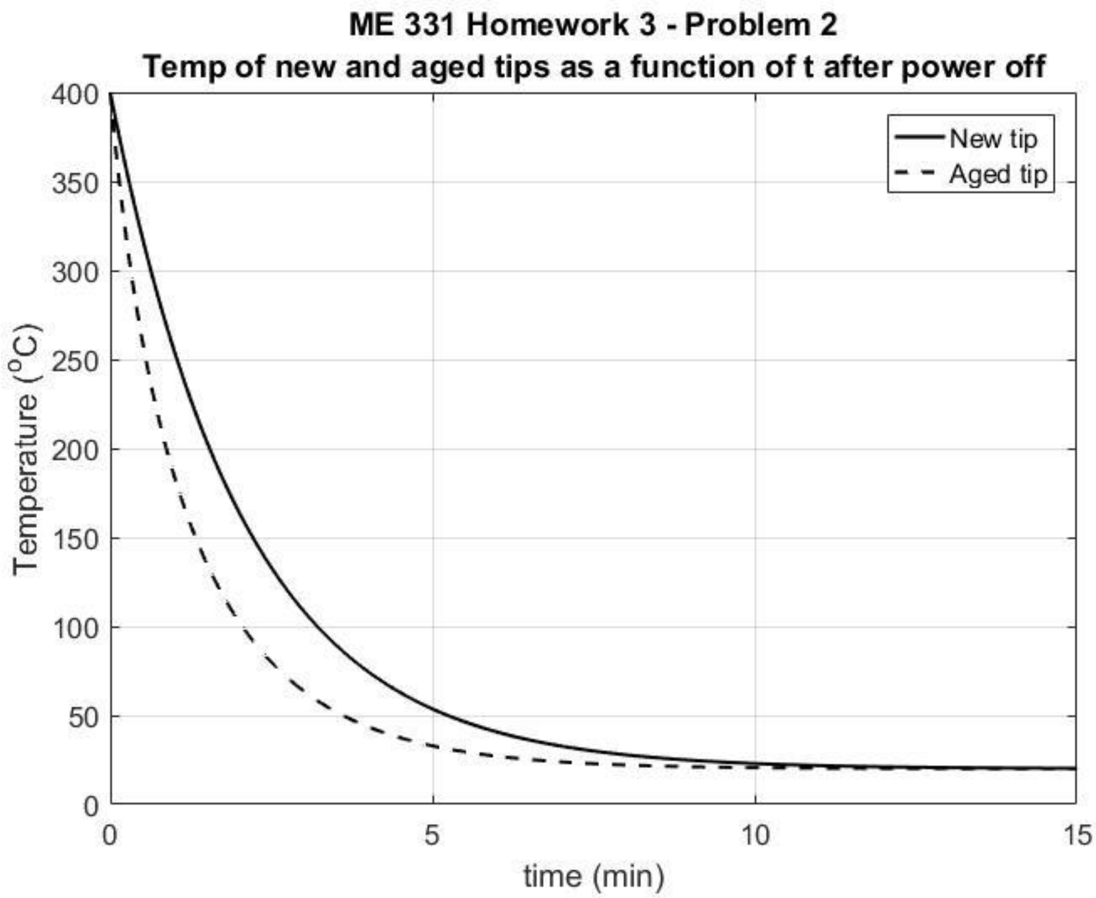
For a sufficiently small time step we can write

$$\frac{dT}{dt} \approx \frac{\Delta T}{\Delta t}$$

For temperatures at times $t_1, t_2, t_3, \ldots, t_{n-1}, t_n, t_{n+1}, \ldots$

$$T_{n+1} = T_n + \Delta t\left\{-\frac{A_s}{\rho V c}\left[h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{surr}^4)\right]\right\}$$

Problem #3 continued



Comments:

The aged tip cools more rapidly due to radiation.

(see MATLAB code below)

Problem #3 continued

```
%% ME331 : Homework 3
%% P3

r = 1.5e-3;    L = 0.01; % m
k = 393;       c = 397;
V = pi*(r^2)*L; A = pi*r^2 + 2*pi*r*L;
Lc = V/A

h = 20;

Bi = h*Lc/k

rho = 8933; sigma = 5.67e-8; epsilon = 0.8;
Tinf = 293; To = 673;
%% Plot new tip Temps
t = 0:2:1000;
T = Tinf+(To-Tinf)*exp(-(h*A/(rho*V*c))*t);

figure(1); clf
plot(t/60,T-273,'k','Linewidth',1.6); grid on; hold on


%% Plot old tip temps
const = -A/(rho*V*c);
fun = @(T)(const*( h*(T-Tinf) +epsilon*sigma*(T^4-Tinf^4) ) );

deltat = 1e-2; %small time step 1/100 s
t = 0:deltat:1000;
T = zeros(size(t));
T(1) = 673; %initial condition
for k = 2:length(t)
    T(k) = T(k-1) + deltat*fun(T(k-1));
end


plot(t/60,T-273,'k--','Linewidth',1.6);
xlabel('time (min)'); ylabel('Temperature (^oC)')
legend('New tip','Aged tip')

title({'ME 331 Homework 3 - Problem 2'; ...
    'Temp of new and aged tips as a function of t after power off'})
set(gca,'Fontsize',13)

xlim([0 15])
```

Problem # 4

Statement: Water main is positioned below ground to avoid freezing. Initial soil temperature is 20°C, and ground surface temperature is −25°C for 100 days.

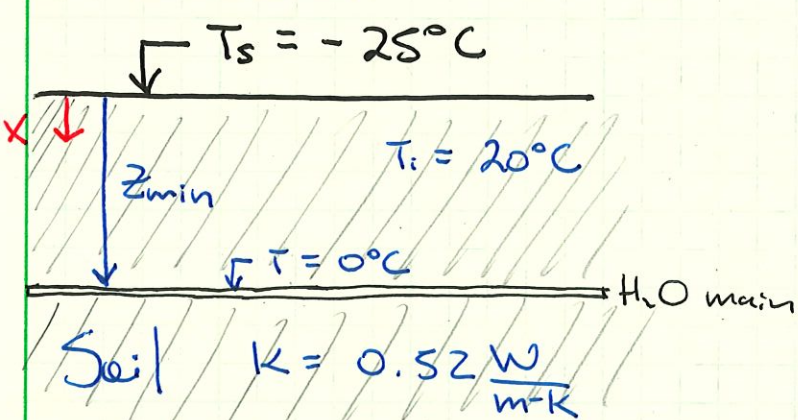
- Ground - soil $T_i = 20°C$
- Avoid freezing H_2O main, $T > 0°C$
- T_s of soil = −25°C for 100 days.

Find: − Minimum depth (z_{min}) for H_2O main to avoid freezing.

$\Rightarrow T(z_{min}, 100 \text{ days}) = 0°C$

Assumptions:
- Soil is a semi-infinite medium
- Constant surface temp.
- Constant properties

Schematic:



Values from Table A.3
values at 300 K

$k_{soil} = 0.52 \frac{W}{mK}$

$\rho_{soil} = 2050 \frac{kg}{m^3}$

$C_{soil} = 1840 \frac{J}{kg\,K}$

$$\alpha = \frac{k}{\rho c} = \frac{0.52 \frac{W}{mK}}{2050 \frac{kg}{m^3} \cdot 1840 \frac{J}{kg\,K}} = 1.3786 \cdot 10^{-7} \frac{m^2}{s}$$

Analysis:

- Semi infinite medium with constant T_s

Since we have semi infinite medium with constant T_s, equation (5.60) from (Incropera, 2011) applies:

$$\frac{T - T_s}{T_i - T_s} = \operatorname{erf}\eta \quad \text{with} \quad \eta = \frac{x}{(4\alpha t)^{1/2}}$$

$$\Rightarrow \frac{T(z_{min}, 100\,\text{days}) - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{z_{min}}{2\sqrt{\alpha t}}\right)$$

$$\operatorname{erf}\left(\frac{z_{min}}{2\sqrt{\alpha t}}\right) = \frac{0 - (-25)\,K}{20 - (-25)\,K} = \frac{25}{45} = \frac{5}{9} = 0.5556$$

let $w = \dfrac{z_{min}}{2\sqrt{\alpha t}} \Rightarrow \operatorname{erf}(w) = \dfrac{5}{9}$

Interpolating in table B.2

$$W = 0.5407 = \frac{z_{min}}{2\sqrt{\alpha t}} \Rightarrow z_{min} = 2 \cdot 0.5407\sqrt{\alpha t}$$

$$z_{min} = 2 \cdot 0.5407 \cdot \sqrt{1.3786 \cdot 10^{-7}\,\frac{m^2}{s} \cdot 100 \cdot 3600 \cdot 24\,s}$$
$$= 1.1803\,m$$

Rounding up to ensure no freezing possibility with $-25°C$ surface temperature for as long as 100 days.

$$\boxed{z_{min} = 1.2\,m}$$

Problem #5

Statement: Fire door (planar solid hardwood) insulated on interior surface. 4.0 cm thick and with uniform initial temperature $T_i = 25°C$. Test surface of door opposing the insulated surface by exposing to ambient temp of 925°C with $h_{eff} = 80 \frac{W}{m^2K}$

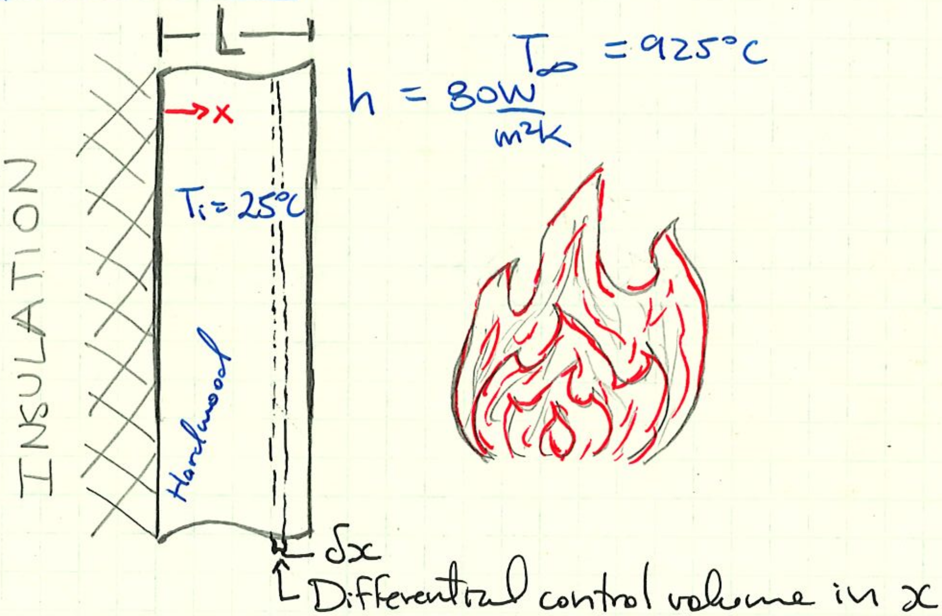
Find: Temperature distribution as a function of time. When is single term approximation inaccurate? Show improvement by adding terms of the series.

- Plot temperature distribution as a function of time
- Range of times when single term approximation is inaccurate
- Using single-term approximation, Find $T(x,t)$ and plot for different times t.
- Plot single term approximation for $F_o < 0.2$, compare to many term approx.

Assumptions:

- Perfect insulation @ $x=0$ → Fire door is half of planar wall, $L = 4.0$ cm
- Constant properties
- 1-D conduction in x-direction

Schematic:



Hardwood at 300K - Table A.3

$\rho = 720 \frac{kg}{m^3}$ $\qquad c = 1255 \frac{J}{kg\,K}$

$k = 0.16 \frac{W}{m\text{-}K}$ $\qquad \alpha = \frac{k}{\rho c} = \dfrac{0.16 \frac{W}{mK}}{720 \frac{kg}{m^3} \cdot 1255 \frac{J}{kgK}}$

$$\alpha = 1.77 \cdot 10^{-7} \frac{m^2}{s}$$

Analysis:

Since we assume perfect insulation on one side we can assume the door behaves as a half planar wall (see for instance Example 5.5 in textbook)

Hence we have:

$$\frac{T - T_\infty}{T_i - T_\infty} = \theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 F_o)\cos(\zeta_n x^*)$$

(5.34) $\qquad$ (5.42a)

with $x^* = \frac{x}{L}$, $F_o = \frac{\alpha t}{L^2}$

$$C_n = \frac{4 \sin \zeta_n}{2\zeta_n + \sin(2\zeta_n)} \quad (5.42b)$$

$$\zeta_n \tan \zeta_n = Bi \quad (5.42c)$$

$$Bi = \frac{hL}{k}$$

We can check Bi

$$Bi = \frac{hL}{k} = \frac{80 \frac{W}{m^2K} \cdot 0.04m}{0.16 \, W/m\text{-}K} = 20 > 0.1$$

Clearly we cannot use lumped capacitance.

- Single term approximation is valid when $F_o > 0.2$

$$F_o > 0.2, \quad F_o = \frac{\alpha t}{L^2} \Rightarrow t > \frac{0.2 L^2}{\alpha}$$

$$t > \frac{0.2 \cdot (0.04m)^2}{1.77 \cdot 10^{-7} m^2/s} = 1807.2s$$

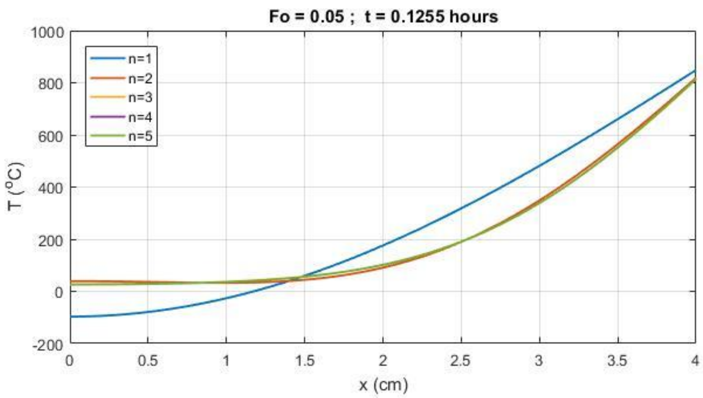
Thus

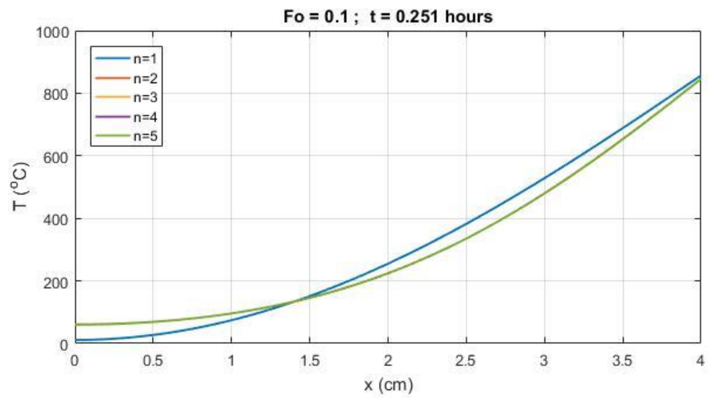
Single term approx is valid for $t > 1800s$
is inaccurate for $0 < t < 1800s$

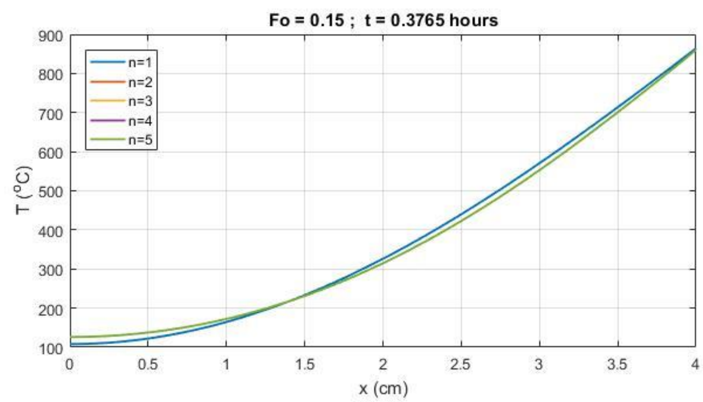
For temp. profile we have:

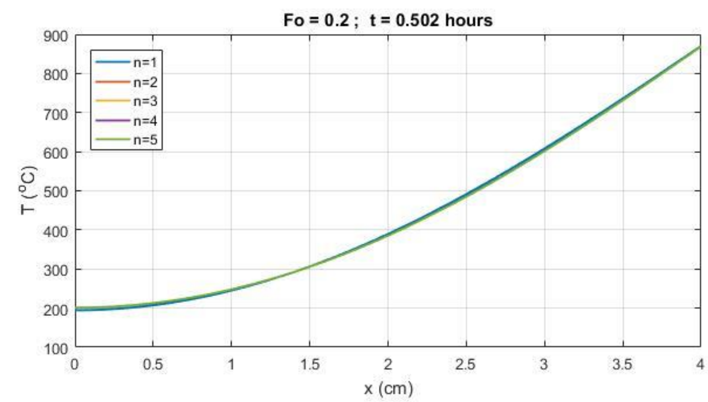
$$T = T_\infty + (T_i - T_\infty) \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 F_o) \cos(\zeta_n x^*)$$

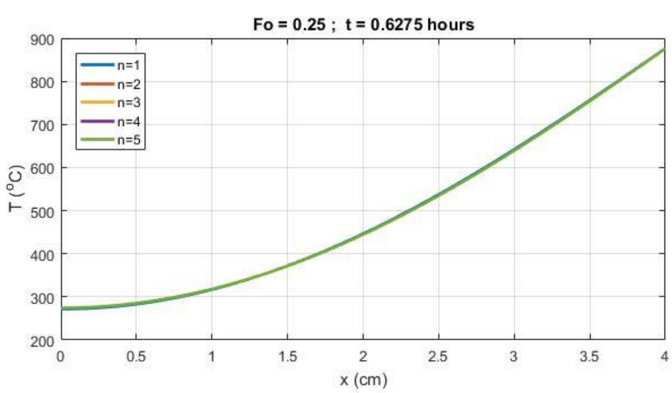
See plots and code below.

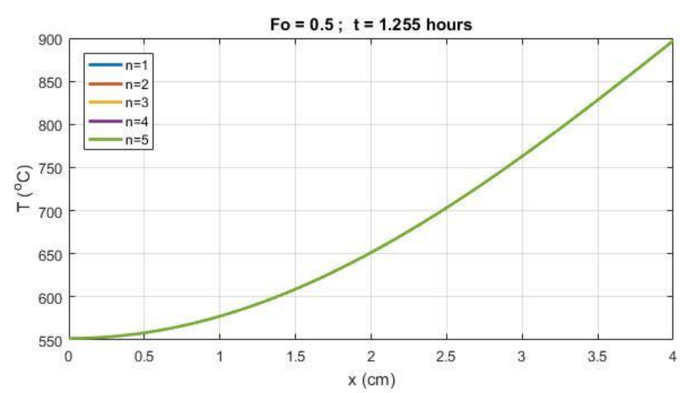

Fo = 0.05 ; t = 0.1255 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)

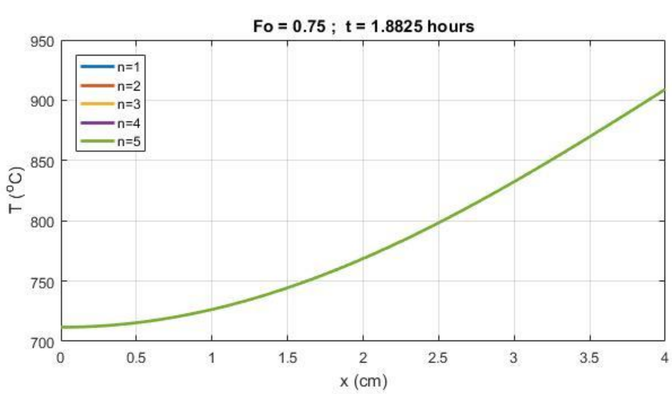

Fo = 0.1 ; t = 0.251 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)

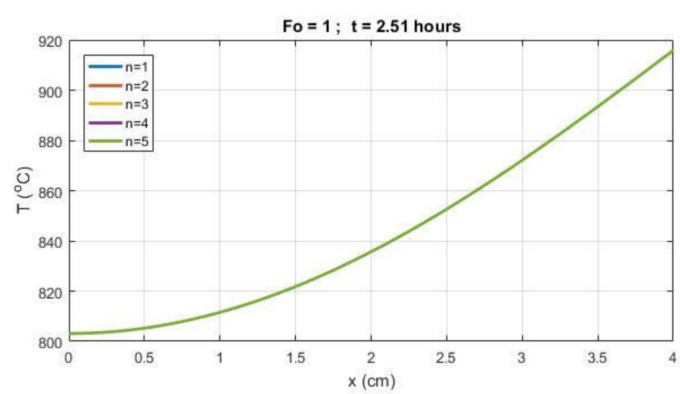

Fo = 0.15 ; t = 0.3765 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)

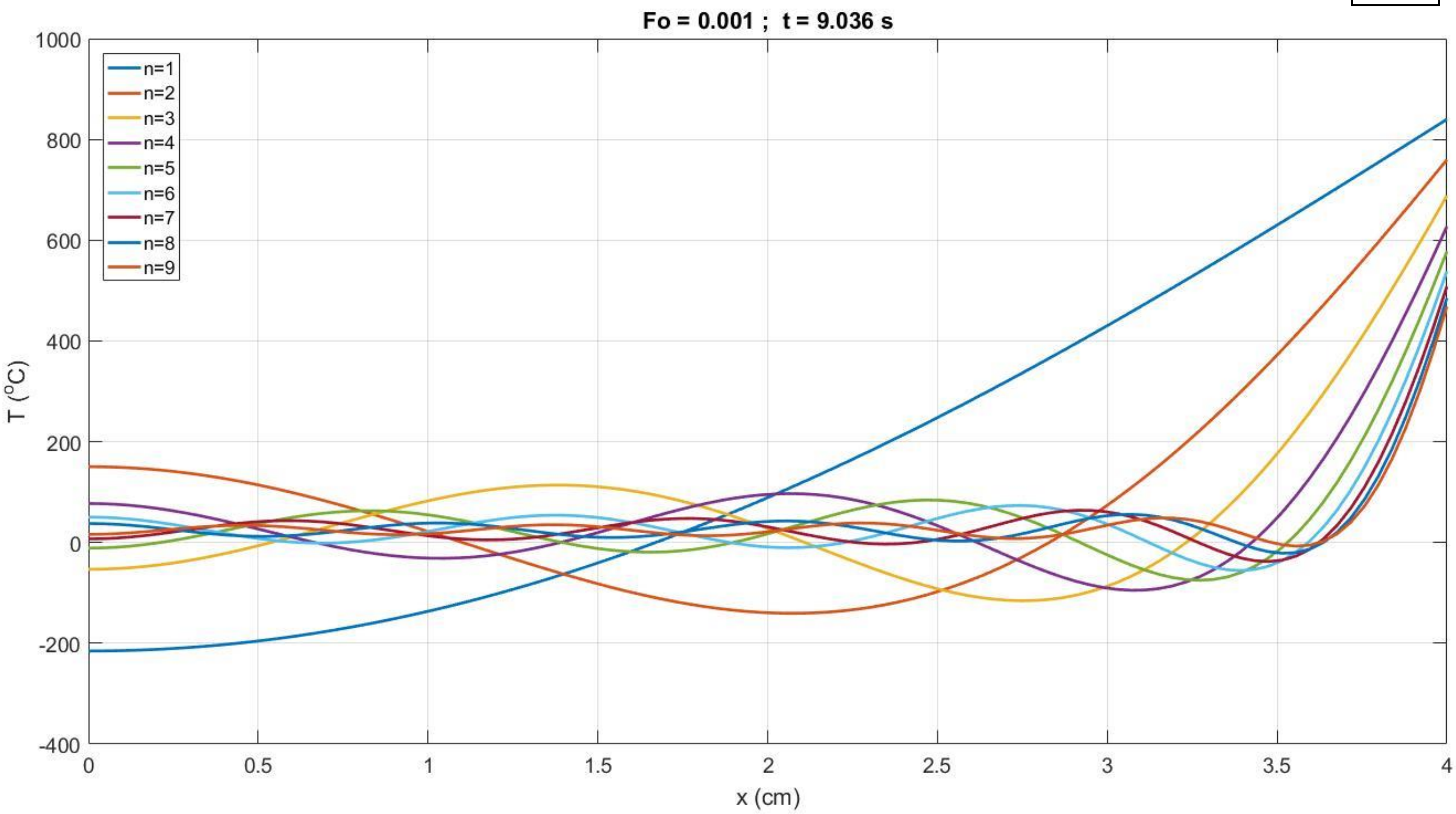

Fo = 0.2 ; t = 0.502 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)

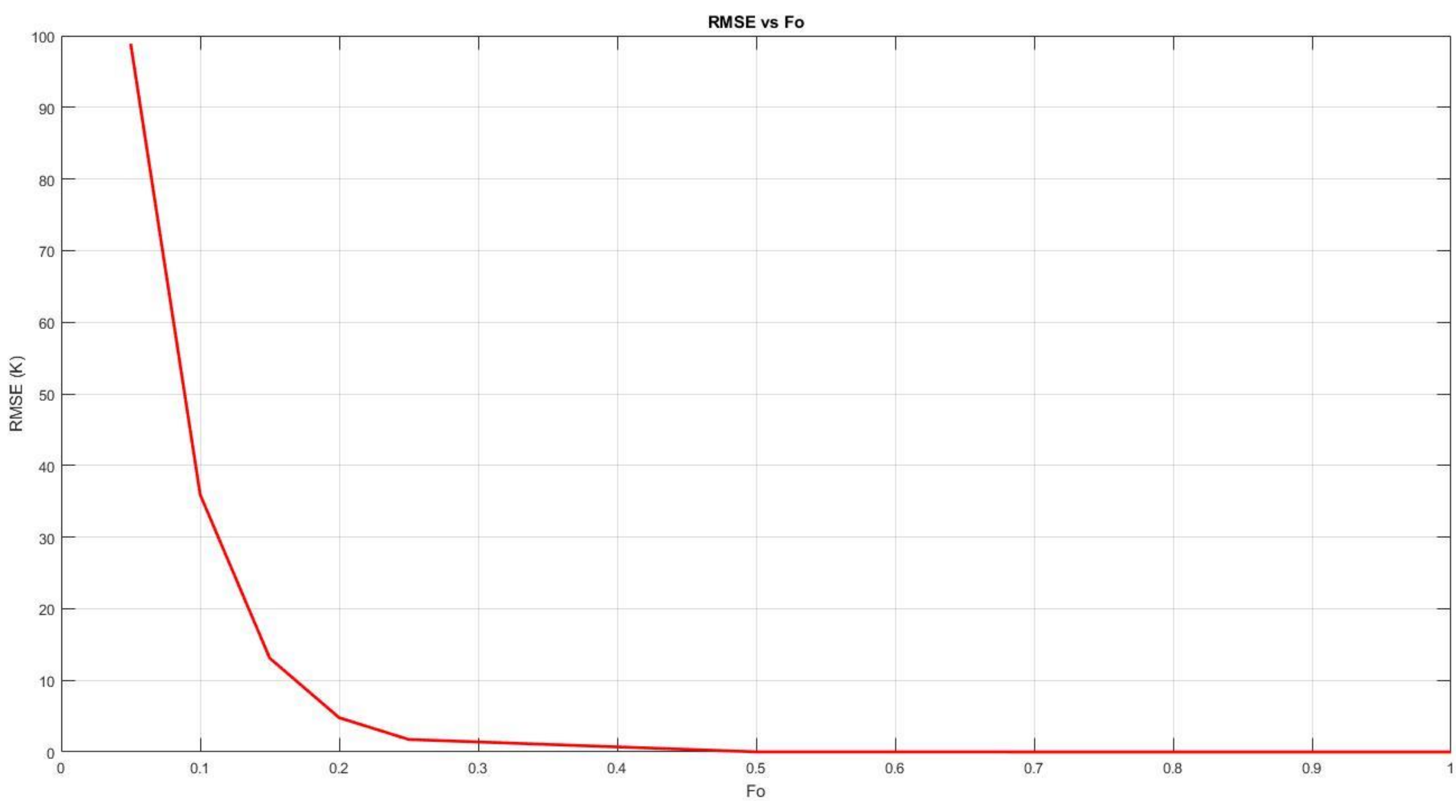

Fo = 0.25 ; t = 0.6275 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)


Fo = 0.5 ; t = 1.255 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)


Fo = 0.75 ; t = 1.8825 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)


Fo = 1 ; t = 2.51 hours
n=1
n=2
n=3
n=4
n=5
T (°C)
x (cm)





RMSE computed from the 5 the error between the 1st term and five terms approximations.

Comments:

We see that for large enough values of the Fo number the series converges pretty quickly (as seen in the RMSE plor above). However the series does not converge quickly early in the process (at small t, small Fo). For instance, as appreciated in the plot for Fo = 0.001 it is evident that we need many more terms in the series to obtain an accurate solution.

MATLAB code:

```
clear; clc

%% ME331 : Homework 3

%% Problem 4
% properties
rho = 2050; k = 0.52; c = 1840; alpha = k/(rho*c);
L = 4e-2;
% solution of eqn 5.60
fun = @(w)(erf(w)-5/9);
w = fzero(fun,0.54) ;
Zmin = 2*w*sqrt(alpha*100*3600*24); % Zmin in m

%% Problem 5
% Properties
rho = 720; k = 0.16; c = 1255; alpha = k/(rho*c)
h = 80; % W/(m^2-K)

% for valid one term approx
tone = 0.2*(L^2)/alpha % t in s


%%
% find the zeta_n and Cn
Bi = h*L/k

fun = @(x)(x.*tan(x)-Bi);
pnn = 1;
aold = -2;
zeta = [];
for knn = 0.3:0.5:30
    a = fzero(fun,knn);

    if abs(fun(a))<1e-9
        if abs(a-aold)>1e-6
            zeta(pnn) = a;
            pnn = pnn+1;
            aold = a;
        end
    end
end
```

```
zeta = unique(zeta);
zeta = zeta'; % i like column vectors
Cn = 4*sin(zeta)./(2*zeta+sin(2*zeta));

[zeta Cn]

Tinf = 925+273.15;
To = 25+273.15;
x = linspace(0,0.04);

%% PLot first inaccurate
krm = 1;
figure(1); clf
for FoM = 1:4
    Fo = 0.05*FoM; t = (L^2)*Fo/alpha;
    for nterms = 1:5
        T  = Temp_sol(x,t,zeta,Cn,To,Tinf,L,alpha,nterms);
        eval(['subplot 22' num2str(FoM)])
        plot(x*100,T-273.15,'Linewidth',1.4); hold on
    end
    xlabel('x (cm)'); ylabel('T (^oC)'); grid on
    title(['Fo = ' num2str(Fo) ' ;  t = ' num2str(t/3600) ' hours'])
    legend('n=1','n=2','n=3','n=4','n=5','Location','Northwest')
    % get RMSE

    Fofo(krm) = Fo;
    RMSE(krm) = sqrt(mean((T -
Temp_sol(x,t,zeta,Cn,To,Tinf,L,alpha,1)).^2));  % Root Mean Squared Error
    krm = krm+1;
end




%% PLot accurate
figure(2); clf
for FoM = 1:4
    Fo = 0.25*FoM; t = (L^2)*Fo/alpha;
    for nterms = 1:5
        T  = Temp_sol(x,t,zeta,Cn,To,Tinf,L,alpha,nterms);
        eval(['subplot 22' num2str(FoM)])
        plot(x*100,T-273.15,'Linewidth',2); hold on
    end
    xlabel('x (cm)'); ylabel('T (^oC)'); grid on
    title(['Fo = ' num2str(Fo) ' ;  t = ' num2str(t/3600) ' hours'])
    legend('n=1','n=2','n=3','n=4','n=5','Location','Northwest')
    Fofo(krm) = Fo;
    RMSE(krm) = sqrt(mean((T -
Temp_sol(x,t,zeta,Cn,To,Tinf,L,alpha,1)).^2));  % Root Mean Squared Error
    krm = krm+1;
end

[Fofo' RMSE']

%% for very small Fo
```

```
figure(3); clf
Fo = 0.001; t = (L^2)*Fo/alpha;
for nterms = 1:9
    T  = Temp_sol(x,t,zeta,Cn,To,Tinf,L,alpha,nterms);
    plot(x*100,T-273.15,'Linewidth',2); hold on
end
xlabel('x (cm)'); ylabel('T (^oC)'); grid on
    title(['Fo = ' num2str(Fo) ' ;  t = ' num2str(t) ' s'])
    legend('n=1','n=2','n=3','n=4','n=5','n=6','n=7','n=8','n=9',...
    'Location','Northwest')
set(gca,'Fontsize',14)

figure(4); clf
plot(Fofo,RMSE,'r','Linewidth',2); xlabel('Fo'); ylabel('RMSE (K)')
grid on
title('RMSE vs Fo')

function [ T ] = Temp_sol(x,t,zeta,Cn,To,Tinf,L,alpha,nterms)
%UNTITLED2 Summary of this function goes here
%   Detailed explanation goes here
m = length(x) ;
theta = zeros(size(x));
zeta  = zeta(1:nterms);
Cn  = Cn(1:nterms);
Fo = alpha*t/(L^2);
for km = 1:m
    theta(km) = sum(Cn.*exp(-(zeta.^2)*Fo).*cos(zeta.*x(km)./L));
end

T = Tinf+(To-Tinf)*theta;


end
```