ME331: Introduction to Heat Transfer
Spring 2017

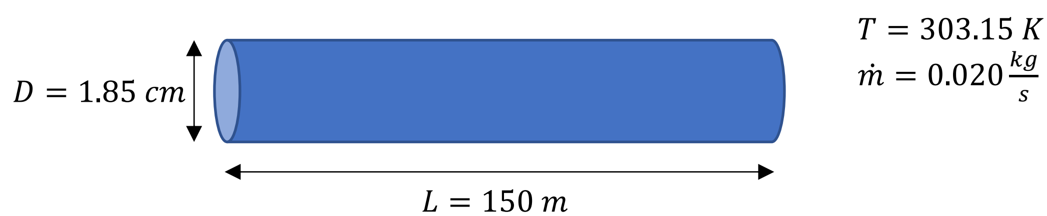
ME 331 Homework Assignment #6

Problem 1

Statement: Water at 30°C flows through a long 1.85 cm diameter tube at a mass flow rate of 0.020 kg/s.

Find: The mean velocity (u_m), maximum velocity (u_{MAX}), and the pressure difference (Δp) between two sections $\Delta x = 150$ m apart in the tube.

Schematic:



Assumptions:

1. Incompressible fully developed flow.
2. Constant properties for water at 30°C = 303.15K. Interpolating from Table A.6,

$\rho = 995.8\ kg/m^3$, $\quad \mu = 8.01 \times 10^{-4}\ N \cdot s/m^2$

Analysis:

From the mass flow rate, density, and cross sectional area, the mean velocity is found using eq. 8.5:

$$u_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\frac{\pi}{4}D^2\rho} = \frac{0.020\frac{kg}{s}}{\frac{\pi}{4}(0.0185\ m)^2 \cdot 995.8\frac{kg}{m^3}} = 0.074721\frac{m}{s}$$

$$u_m = 7.5 \times 10^{-2}\frac{m}{s}$$

ANSWER

Determine whether the flow is laminar or turbulent.

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{995.8\frac{kg}{m^3} \cdot 0.074721\frac{m}{s} \cdot 0.0185\ m}{8.01 \times 10^{-4}\ N \cdot s/m^2} = 1{,}718.8$$

The Reynolds number is less than the critical number of 2300; thus the flow is laminar. The velocity profile is given by equation 8.15:

$$u(r) = 2\,u_m\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

The maximum velocity at r = 0 is:

$$u_{MAX} = u(r=0) = 2\,u_m = \; 0.1494\,m/s$$

$$\boxed{u_{MAX} = 0.15\,\frac{m}{s}}$$

ANSWER

The pressure gradient is related to the mean velocity by equation 8.14:

$$u_m = -\frac{1}{8\mu}\left(\frac{dp}{dx}\right)r_o^2$$

For fully developed laminar flow, the pressure gradient is constant:

$$\frac{dp}{dx} = -\frac{8\mu u_m}{r_o^2} = -\frac{8\cdot 8.01\times 10^{-4}\,N\cdot\frac{s}{m^2}\cdot\frac{0.074721m}{s}}{(0.00925\quad m)^2} = -5.5948\,Pa/m$$

The pressure drop over 150 m length of pipe is:

$$|\Delta p| = \left|\frac{dp}{dx}\right|L = 5.5948\,\frac{Pa}{m}\cdot 150\,m = 839.2194\,Pa$$

$$\boxed{|\Delta p| = 840\,Pa}$$

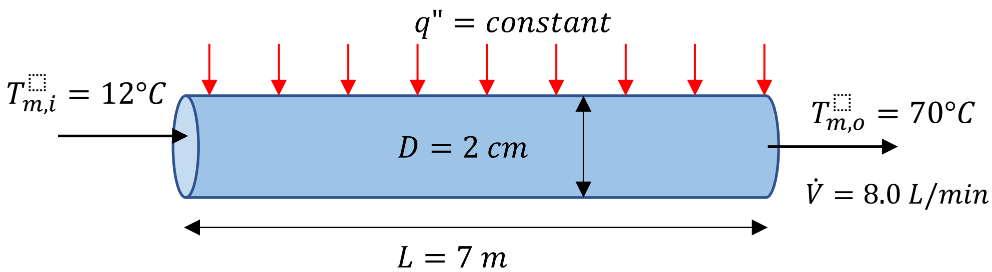
ANSWER

Problem 2

Statement: You want to heat water from 12°C to 70°C as it flows through a 2.0 cm internal diameter, 7.0 m long tube. The tube is equipped with an electric resistance heater, which provides uniform heating at the outer surface of the tube. The heater is well insulated, so that in steady operation all the thermal energy generated in the heater is transferred to the water in the tube. You want the system to provide hot water at a rate of 8.0 L/min.

Find: The required power rating of the resistance heater, and the inner surface temperature at the outlet ($T_{s,o}$).

Schematic:



Assumptions:

1. Steady flow conditions.
2. Uniform surface heat flux.
3. Inner surface of the tube is smooth.
4. Properties evaluated at the mean fluid temperature of (70+12)/2 = 41°C:

$$\rho = 992.1\ kg/m^3$$
$$k = 0.631\ W/mK$$
$$\nu = \frac{\mu}{\rho} = 0.658 \times 10^{-6}\ m^2/s$$
$$c = 4179\ J/kgK$$
$$Pr = 4.32$$

Analysis:

The mass flow rate through the tube and the required power rating of the resistance heater are:

$$\dot{m} = \rho\dot{V} = (992.1\ kg/m^3)(0.008\ m^3/min) = 7.937\ kg/min\ = 0.132\ kg/s$$

$$q = \dot{m}\ c\ \ (T_{m,o} - T_{m,i}) = \left(0.132\frac{kg}{s}\right)\left(4179\frac{J}{kgK}\right)(70-12)K = 32.06\ kW$$

$q = 32\ kW$ **ANSWER**

To determine the inner surface temperature at the tube outlet we first determine whether the flow is laminar or turbulent.

$$u_m = \frac{\dot{V}}{A_c} = \frac{\left(\frac{8}{60}\times10^{-3}\right)\frac{m^3}{s}}{\frac{\pi(0.02\,m)^2}{4}} = 0.4244\,\frac{m}{s}$$

$$Re = \frac{u_m D}{\nu} = \frac{\left(0.4244\frac{m}{s}\right)(0.02\,m)}{0.658\times10^{-6} m^2/s} = 12{,}890$$

This value is greater than the critical Reynolds number of 2300 for pipe flow. Therefore, the flow is turbulent and the entry length is estimated at 10 pipe diameters:

$$x_{fd,t} \approx 10D \quad = 10(0.02\,m) = 0.2\,m$$

Since the entry length is much shorter than the total tube length, the thermal and hydrodynamic profiles of the turbulent flow are assumed to be fully developed at L = 7 m. The local Nusselt number is determined from:

$$Nu = \frac{hD}{k} = 0.023 Re^{0.8} Pr^{0.3} = 0.023(12890)^{0.8}(4.32)^{0.4} = 80.2$$

The local heat transfer coefficient is:

$$h = \frac{k}{D} Nu = \frac{0.631\,\frac{W}{mK}}{0.02\,m}(80.2) = 2530\,W/m^2K$$

Using q $= hA(T_{s,o} - T_{m,o})$, the inner surface temperature of the tube at the outlet is:

$$T_{s,o} = \frac{q}{hA} + T_{m,o} = \frac{32{,}060\,W}{2530\frac{W}{m^2K}\cdot\pi(0.02\,m)(7\,m)} + 70°C = 98.8°C$$

$$T_{s,o} = 99°C$$

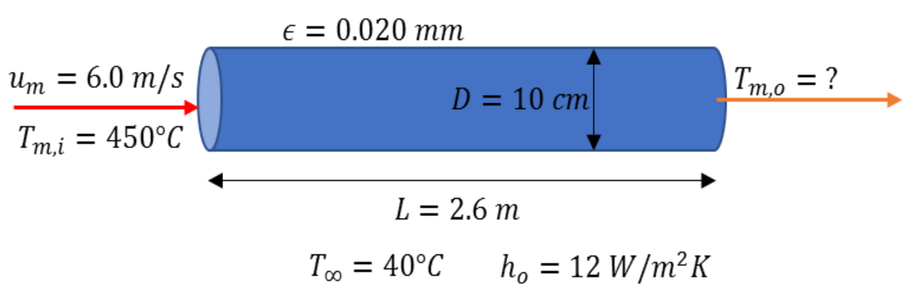
ANSWER

Problem 3

Statement: A typical automotive exhaust pipe carries exhaust gas at a mean velocity of 6.0 m/s. If the gas leaves the manifold at a temperature of 450°C and the muffler is located 2.6 m away, estimate the temperature of the gas as it enters the muffler. Assume an ambient air temperature of 40°C and a heat transfer coefficient of 12 W/m^{2}K for the outside surface of the exhaust pipe. The exhaust pipe is thin walled and has a diameter of 10 cm. The exhaust pipe inner roughness Is ε = 0.020 mm.

Find: The approximate temperature of the exhaust gas as it enters the muffler ($T_{m,o}$).

Schematic:



Assumptions:

1. Incompressible, fully developed flow.

2. Constant properties of air at 723 K are assumed for the exhaust gas.

$\rho = 0.4821\ kg/m^3$ $\quad c_p = 1.081 \times 10^{-3} J/kgK$ $\quad Pr = 0.698$

$\mu = 3.461 \times 10^{-5} N \cdot s/m^2$ $\quad \nu = 7.193 \times 10^{-5} m^2/s$ $\quad k = 5.36 \times 10^{-2}\ W/mK$

Analysis:

$$Re_D = \frac{u_m D}{\nu} = \frac{6.0 \frac{m}{s} \times 0.1\ m}{7.193 \times 10^{-5} m^2/s} = 8.341 \times 10^3 > 2300$$

The flow is turbulent, but possibly transitional (not fully turbulent) since $2300 < Re_D < 10{,}000$.

$$\frac{\epsilon}{D} = \frac{0.020 \times 10^{-3} m}{0.1 m} = 0.0002$$

From Figure 8.3, the friction factor is:

$$f = 0.033$$

For fully developed turbulent flow in a smooth tube, the local Nusselt number and local heat transfer coefficient are:

$$Nu_D = \frac{\left(\frac{f}{8}\right)(Re_D - 1000)Pr}{1 + 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}(Pr^{2/3} - 1)} = 25.586$$

$$h_i = Nu_D \cdot \frac{k}{D} = 13.714\frac{W}{m^2 \cdot K}$$

However, since $Re_D < 10{,}000$ and since $\frac{L}{D} = 26 < 60$, the calculated h_i could be either an overestimate or an underestimate of $\bar{h}_i$. Lacking any better estimate, use $\overline{Nu}_D \approx Nu_D$, $\bar{h}_i \approx h_i$.

$$R_{tot} = R_{conv,i} + R_{conv,o} = \frac{1}{h_i PL} + \frac{1}{h_o PL} = \frac{1}{\pi \times 0.1m \times 2.6m}\left(\frac{1}{13.714} + \frac{1}{12}\right)m^2 \cdot \frac{K}{W} = 0.19129\frac{K}{W}$$

Then, from equation 8.45b,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{1}{\dot{m}c_p R_{tot}}\right)$$

$$T_{m,o} = T_\infty - (T_\infty - T_{m,i})\ exp\left(-\frac{1}{\dot{m}c_p R_{tot}}\right) = 371.4°C$$

$T_{m,o} = 370°C$ **ANSWER**

Comment:

$$q = \frac{\Delta T_{lm}}{R_{tot}} = -1900\ W$$

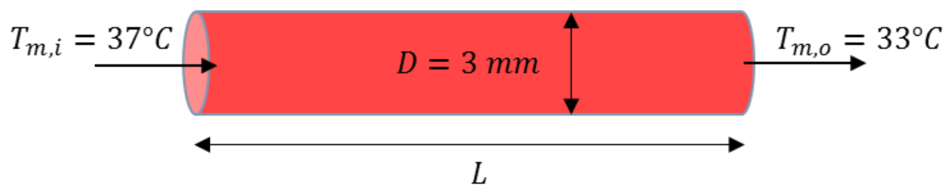
The negative sign indicates that heat flows from gas in the exhaust pipe to the ambient air environment.

Problem 4

Statement: A blood purification system removes blood from a patient through a 3.0 mm diameter tube at a mass flow rate of 0.090 kg/min. The blood exits the patient at 37°C and cools to 33°C just prior to entering a filter and heat exchanger that raises the blood temperature back to 37°C. Approximate the properties of blood with those of water.

Find: Estimate the rate of heat transfer from the blood and the heat transfer coefficient, assuming fully developed blood flow in the tube.

Schematic:



Assumptions:

1. Incompressible, fully developed flow.
2. Constant properties of water at 35°C = 308.15 K are assumed for blood.

$$\rho = 993.8\ kg/m^3$$
$$k = 0.625\ W/mK$$
$$\mu = 7.22 \times 10^{-4}\ N\ s\ /\ m^2$$
$$c = 4178\ J/kgK$$

3. Constant surface heat flux or constant surface temperature may be assumed.

Analysis:

$$q = \dot{m}\ c\ \ (T_{m,o} - T_{m,i}) = \frac{0.090}{60}\frac{kg}{s} \times\ 4178 \frac{J}{kgK} \times (33 - 37)K = -25.07\ W$$

$q = -25W$ **ANSWER**

The negative sign indicates that the blood is transfering heat to its surrounding environment.

The Reynolds number is:

$$Re_D = \frac{u_m D}{\nu} = \frac{\dot{m}D}{\rho A_c \nu} = 881.7 < Re_{cr}$$

For fully developed laminar flow, constant surface heat flux, $Nu_D = \frac{hD}{k} = 4.36$

$$h = 4.36 \cdot \frac{k}{D} = 908.3 \frac{W}{m^2 \cdot K}$$

$h = 910 \ \frac{W}{m^2 \cdot K}$ **ANSWER**

For fully developed laminar flow, constant surface temperature, $Nu_D = \frac{hD}{k} = 3.66$

$$h = 3.66 \cdot \frac{k}{D} = 762.5 \frac{W}{m^2 \cdot K}$$

$h = 760 \ \frac{W}{m^2 \cdot K}$ **ANSWER**

Comments:

This process of blood cooling is unlikely to be either constant heat flux or surface temperature. It is also unlikely for it to be fully developed. Therefore, these answers are first order estimates. If more information is given about the outer convection condition, this problem could be solved more accurately using a similar method as in problem 3.

Problem 5

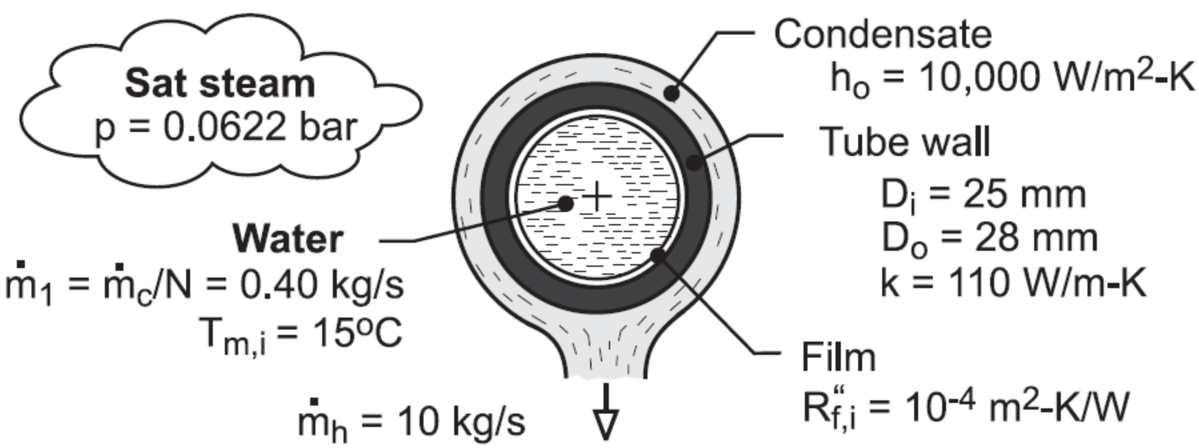
Statement: The condenser of a steam power plant contains N = 1000 brass tubes (k_t = 110 W/m-K), each of inner and outer diameter, D_i = 25 mm and D_o = 28 mm. Steam condensation on the outer surfaces of the tubes is characterized by a convection coefficient of h_o = 10,000 W/m^2-K.

Find: **(a) If cooling water from a nearby lake is pumped through the condenser tubes at a mass flow rate of $\dot{m}_c$ = 400 kg/s, what is the overall heat transfer coefficient U_o based on the tube outer surface area? Water properties may be approximated as constant: μ = 9.60 x 10^{-4} N-s/m^2, k = 0.60 W/m-K, and Pr = 6.6.**

(b) If after extended operation, fouling results in an additional resistance of $R''_{f,I}$ = 10^{-4} m^2-K/W, at the inner surface of the tubes, what is the value of U_o?

(c) If water is extracted from the lake at 15°C, and 10 kg/s of steam at 0.0622 bar are to be condensed, what is the temperature of the cooling (lake) water leaving the condenser? The specific heat of the water is 4180 J/kg-K.

Schematic:



Assumptions:

1. Water is incompressible, and viscous dissipation is negligible.
2. Fully developed flow in tubes.
3. Negligible fouling on outer tube surfaces, D_o.
4. Water properties inside tubes are given as follows:

   c = 4180 J/kg·K, μ = 9.6 × 10^{-4} N·s/m^2, k = 0.60 W/m·K, Pr = 6.6.
5. From Table A-6, properties outside tubes are for saturated vapor / liquid water at p = 0.0622 bar: Tsat = 310 K, h_{fg} = 2.414 × 10^6 J/kg.

Analysis:

(a) Without fouling resistance, Eq. 11.5 yields:

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k_t} + \frac{1}{h_o}$$

$Re_{D_i} = \frac{4\dot{m}_1}{\pi D_i \mu} = 21{,}220$; thus the flow in the tubes is turbulent. From eq 8.60:

$$h_i = \left(\frac{k}{D_i}\right) 0.023 Re_{D_i}^{4/5} Pr^{0.4} = 3398\, W/m^2K$$

$$U_o = \left[\frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{D_o\, ln\left(\frac{D_o}{D_i}\right)}{2k_t} + \frac{1}{h_o}\right]^{-1} = \left[\frac{1}{3398\, W/m^2K}\left(\frac{28}{25}\right) + \frac{0.028m\, ln\left(\frac{28}{25}\right)}{2\times 110\, W/mK} + \frac{1}{10{,}000\, W/m^2K}\right]^{-1} = 2252\, \frac{W}{m^2K}$$

$U_o = 2250 \frac{W}{m^2K}$ **ANSWER**

(b) With fouling, Eq. 11.5 yields:

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{D_o \ln\left(\frac{D_o}{D_i}\right)}{2k_t} + \frac{1}{h_o} + \left(\frac{D_o}{D_i}\right) R''_{f,i}$$

$$U_o = \left[4.44 \times 10^{-4} m^2K/W + \left(\frac{D_o}{D_i}\right) R''_{f,i}\right]^{-1} = 1798\, \frac{W}{m^2K}$$

$U_o = 1800\, \frac{W}{m^2K}$ **ANSWER**

(c) The rate of heat transfer to water inside the tubes equals the rate at which thermal energy is extracted from steam outside of the tubes.

$$\dot{m}_h h_{fg} = \dot{m}_c\, c\, \left(T_{m,o} - T_{m,i}\right)$$

$$T_{m,o} = T_{m,i} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c} = 15°C + \frac{10\frac{kg}{s}\times 2.414\times 10^6 \frac{J}{kg}}{400\frac{kg}{s}\times 4180\frac{J}{kgK}} = 29.4°C$$

$T_{m,o} = 30°C$ **ANSWER**

Comments: (1) The largest contribution to the thermal resistance is from convection at the interior surface of the tubes. To increase U_o, h_i could be increased by increasing $\dot{m}_c$. (2) Note that the outlet temperature of the cooling water $T_{m,o} = 29.4°C$ = 302.5 K remains below T_{sat} = 310 K, as required.

Problem 6

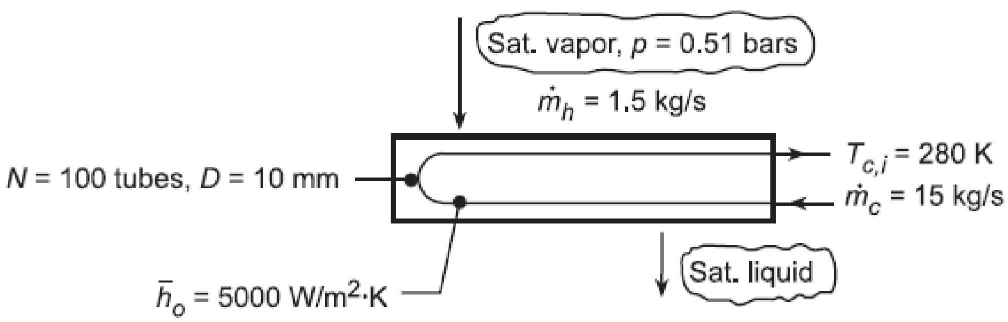
Statement: In a Rankine power system, 1.5 kg/s of steam leaves the turbine as saturated vapor at 0.51 bar. The steam is condensed to saturated liquid by passing it over the tubes of a shell-and-tube heat exchanger, while cooling liquid water, having an inlet temperature of $T_{c,I}$ = 280 K, is passed through the tubes. The condensing heat exchanger contains 100 thin-walled tubes, each 10 mm diameter, and the total cooling water flow rate through the tubes is 15 kg/s. The average convection coefficient associated with condensation on the outer surface of the tubes may be approximated as h_o = 5000 W/m²-K. Appropriate property values for the cooling liquid water are c = 4178 J/kg-K, μ = 700 x 10^{-6} kg/s-m, k = 0.628 W/m-K, and Pr = 4.6.

Find: (a) What is the liquid water outlet temperature from the tubes?

(b) What is the required tube length (per tube)?

(c) After extended use, deposits accumulating on the inner and outer tube surfaces result in a cumulative fouling factor of 0.0003 m²-K/W. For the prescribed inlet conditions and the computed tube length, what mass fraction of the vapor is condensed?

Schematic:



Assumptions:

1. Negligible heat loss to the surroundings.
2. Negligible wall conduction resistance and fouling (initially). With fouling, R_f'' = 0.0003 m²·K/W.
3. Cooling liquid water properties treated as constant:

   c = 4178 J/kg·K, μ = 700 × 10^{-6} kg/s·m, k = 0.628 W/m·K, Pr = 4.6.

4. From Table A.6, for saturated steam at p = 0.51 bar, T_{sat} = 355 K, h_{fg} = 2.304 × 10^{6} J/kg.

Analysis:

(a) From an overall energy balance, $q_h = \dot{m}_h h_{fg} = q_c = \dot{m}_c c_c (T_{c,o} - T_{c,i})$

$$T_{c,o} = T_{c,i} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_c} = 280\ K + \frac{1.5\frac{kg}{s} \times 2.304 \times 10^6 \frac{J}{kg}}{15\frac{kg}{s} \times 4178\frac{J}{kg}} = 335.1\ K$$

$T_{c,o} = 335\ K$ **ANSWER**

(b) With $C_r = C_{min}/C_{max} \rightarrow 0$, $NTU = -\ln(1-\epsilon)$,

$$\text{where} \quad \epsilon = \frac{q}{q_{MAX}} = \frac{\dot{m}_c c_c(T_{c,o} - T_{c,i})}{\dot{m}_c c_c(T_{h,i} - T_{c,i})} = \frac{335.1 - 280}{355 - 280} = 0.7347$$

$$NTU = -\ln(1 - 0.7347) = 1.327 = UA/C_{min}$$

With negligible tube wall thickness, the overall heat transfer coefficient U is: $\frac{1}{U} = \frac{1}{\bar{h}_i} + \frac{1}{\bar{h}_o}$.

The Reynolds number for internal flow inside a singe tube is:

$$Re_D = \frac{4\dot{m}_c}{\pi D \mu} = \frac{4 \times 15\frac{kg}{s}/100}{\pi(0.01) \times 700 \times 10^{-6} kg/s \cdot m} = 27{,}284$$

Using the Dittus-Boelter correlation (equation 8.60) for fully developed turbulent pipe flow,

$$Nu_D = 0.023 Re_D^{4/5} Pr^n = 0.023(27{,}284)^{4/5}(4.6)^{0.4} = 149.8$$

Assuming the tube length is sufficiently long such that the internal flow is fully developed for the majority of the tube length (L > 60 D):

$$\bar{h}_i \approx h_i = \left(\frac{k}{D}\right) Nu_D = 9{,}408 \frac{W}{m^2 K}$$

With $U = \left[\left(\frac{1}{9408}\right) + \left(\frac{1}{5000}\right)\right]^{-1} \frac{W}{m^2 K} = 3{,}265 \frac{W}{m^2 K}$, the heat transfer area is:

$$A = \dot{m}_c c_c \left(\frac{NTU}{U}\right) = 15\frac{kg}{s}\left(4178\frac{J}{kgK}\right)\left(\frac{1.327}{3{,}265\frac{W}{m^2 K}}\right) = 25.5 m^2$$

The tube length is $L = \frac{A}{N\pi D} = \frac{25.5m^2}{100\pi(0.01m)} = 8.11\ m$, satisfying the condition that L > 60 *0.01 m = 0.6 m.

$L = 8.1\ m$ **ANSWER**

(c) With fouling, the overall heat transfer coefficient is $1/U_{wf} = 1/U_{wof} + R_f''$.

Hence,

$$\frac{1}{U_{wf}} = (3.063 \times 10^{-4} + 3 \times 10^{-4}) m^2 \cdot K/W$$

$$U_{wf} = 1649\, W/m^2 \cdot K$$

$$NTU = U_{wf}A/C_{min} = (1649\, W/m^2 \cdot K \times 25.5m^2)/(15\, kg/s \times 4178\, J/kg \cdot K) = 0.670$$

From equation 11.35a,

$$\epsilon = 1 - \exp(-NTU) = 1 - \exp(-0.670) = 0.488$$

$$q = \epsilon q_{max} = 0.488 \times 15\frac{kg}{s} \times 4178\frac{J}{kg\cdot K}(355 - 280)K = 2.296 \times 10^6 W$$

Without fouling, the heat rate was:

$$q = \dot{m}_h h_{fg} = 1.5\frac{kg}{s} \times 2.304 \times 10^6 \frac{J}{kg} = 3.456 \times 10^6 W$$

Hence,

$$\frac{\dot{m}_{h,wf}}{\dot{m}_{h,wof}} = \frac{2.296 \times 10^6}{3.456 \times 10^6} = 0.664$$

The reduced condensation rate with fouling is then:

$$\dot{m}_{h,wf} = 0.664 \times 1.5\frac{kg}{s} = 0.996\frac{kg}{s}$$

$\dot{m}_{h,wf} = 1.0\frac{kg}{s}$ **ANSWER**