Home Experiment 2 Report

Your Name

ME 331: Introduction to Heat Transfer

04/21/2017

1. Abstract

Thermo-receptors in human fingers can give an indication of an object's temperature relative to the skin temperature at which such receptors are typically maintained. The experiment described in this report takes advantage of thermo-reception to estimate the time at which the temperature of a long thin metallic wire (fashioned from a conventional wire clothes hanger) reaches a temperature near 35 °C. A conventional oven was used to raise the temperature of the wire to 177°C and then the wire was allowed to cool in ambient room-temperature air until it reached human skin temperature. Observations of the time at which the wire reached human skin temperature were comparable to the values estimated using a lumped capacitance model. Given the sizeable uncertainties regarding the object's composition and properties, we note that such a rough approximation obtained without the use of any sophisticated equipment may be useful in practical engineering situations where a quick assessment may be required.

2. Introduction / Theory

Sensing of temperature differences relative to skin temperature is enabled by the presence of hot and cold receptors found immediately under the skin surface. Human thermoreception relies on at least three types of sensory receptors: cold receptors, warmth receptors and pain receptors (Hall, 2016). Figure 1 shows the response of the different types of receptors as a function of temperature as well as an indication of the subjective experience associated with given temperature ranges.

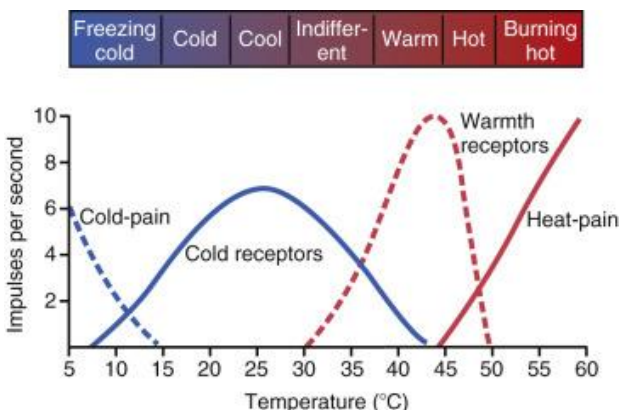

Figure 1: Discharge frequencies at different skin temperatures of a cold-pain fiber, a cold fiber, a warmth fiber, and a heat-pain fiber. Notice how the subjective experience is labelled as indifferent between about 28°C and 37°C (taken from Hall, 2016).

Furthermore, studies indicate that there is a neutral zone (from 31-36°C) where a subject experiences neither warmth nor cold. Eliciting a sensation of warmth also depends on the starting skin temperature. If the initial skin temperature is 31°C then a half-degree increase will cause a warming sensation; if the initial temperature is 36°C, then even a fifth of a degree increase will result in a subjective sensation of warmth (Schmidt, 1986).

In this report, we compare the perceived temperatures with estimates derived from heat transfer theory. We can utilize the lumped capacitance method as long as the following condition is satisfied:

$$Bi = \frac{hL_c}{k} < 0.1 \tag{1}$$

Where $L_c = V/A$ is a characteristic length. For a long cylinder the characteristic length is half the radius. In this case, the lumped capacitance method is valid. Assuming negligible radiation for a well-polished metal surface (for which emissivity approaches zero) the time evolution of the temperature of the wire can be expressed as:

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right)t\right] \tag{2}$$

where h is the convection heat transfer coefficient, A_s is the surface area, V is the volume, k is the thermal conductivity, c is the specific heat, ρ is the density, T_i is the initial temperature of the wire, T_∞ is the air temperature at a large distance away from the body and $\theta(x) \equiv T(x) - T_\infty$.

Empirical correlations can be used to derive an estimate of the natural convection heat transfer coefficient. For instance, with the Prandtl and Rayleigh numbers we can use (see sec 9.6.3 of Bergman & Incropera, 2011):

$$\overline{Nu}_D = \frac{\bar{h}D}{k_{air}} = \left\{0.60 + \frac{0.387 Ra_D^{\frac{1}{6}}}{\left[1 + (0.559/Pr)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right\}^2 \tag{3}$$

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu_{air}\alpha_{air}} \tag{4}$$

$$\alpha = \frac{k}{\rho c} \tag{5}$$

Using this correlation, the natural convection heat transfer coefficient is estimated at h = 15 W/m^{2}K.

3. Procedure

A long wire (shown in Figure 2) was fashioned from a wire hanger of unknown metallic composition. The approximately straight wire was polished with sandpaper to eliminate any possible rust, residue or coating.

Figure 2: Long wire, L=0.90 m made from sanded metal wire hanger.

We cut the wire used in Home Lab 1 to obtain a section of wire 40 cm long. This section of wire was heated to a uniform temperature of approximately 177^{o}C in a conventional electric kitchen oven (no calibration in the oven's temperature setting was performed). The wire was extracted from the oven using oven mitts and placed on the supports as shown in Figure 3. The time was observed at which the temperature of the wire's surface reached approximately 35 ^{o}C using finger tip thermo-reception as described previously for Home Lab 1.

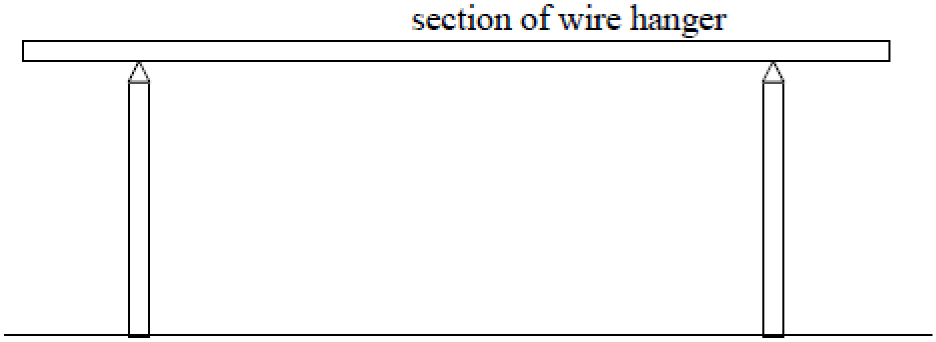

Figure 3: Schematic for Home Lab 2.

4. Results

The results and properties of the wire as well as other relevant experimental parameters are listed in Table 1 for Home Lab 2. The uncertainty ranges were estimated considering the devices limitations as well as the operator's limitations (e.g. limited temperature sensitivity in the fingers).

Table 1: Results and parameters for Home Lab 2.

Wire's Length, L	0.40 +/- 0.005m
Wire's Diameter	0.090 in = 1.1 +/- 0.05 mm
Oven temperature/wire's initial temperature T_i	177+/-0.3ºC
Room Temperature, T_∞	19.4+/-0.3ºC
Time at which wire's temperature equaled skin temperature, t_{ts}	240 +/- 30 s

5. Analysis

We calculated the wire's temperature as a function of time after it had been taken out of the oven. We ascertain that the lumped capacitance approach is valid given that the estimated Biot number was $Bi \approx \frac{15\frac{W}{m^2K}2\cdot10^{-3}m}{20\frac{W}{m\cdot K}} = 0.0015$. Thus, from equation 2 we can estimate the convection heat transfer coefficient and using the empirical estimate of said coefficient we can generate the time series to estimate the time at which the cylinder's temperature was equal to skin temperature.

Using this approach, we estimated that for a wire made of steel, the convection coefficient would be approximately equal to 22 W/m^2K and assuming the wire was made of aluminum the coefficient would be approximately equal to 14 W/m^2K. These values can be compared with the estimated value of 17 W/m^2K from equation 3.

Finally, we calculated the times at which the wire would be at skin temperature to be 5.2 minutes assuming steel and 3.3 minutes assuming aluminum. These estimates are about 30% and 20% different from the observed value of 4 minutes.

6. Discussion

We were able to achieve reasonable agreement between our observations and calculated values with a 30% difference between the estimated cooling time and the observed cooling time. It is important to emphasize the numerous sources of error, approximations and assumptions that were adopted for this analysis. A major source of uncertainty arises from our lack of knowledge regarding the composition of the wire hanger. We considered two possible alternatives for the wire material and used different nominal values for its thermal properties, but the actual composition may be significantly different from what we conjectured. Such assumption is of consequence since properties like thermal conductivity can vary by an order of magnitude from one material to another.

In the present analysis, we neglected radiation. Radiation plays a major role in heat transfer, especially for coated hangers for which the emissivity can approach that of a blackbody. In Figure 4 below we include a comparison of radiative and convective heat fluxes for a blackbody, where $q''_{conv} = h(T - T_\infty)$ and $q''_{rad} = \sigma(T^4 - T_\infty^4)$. We note that radiative heat transfer is not negligible compared to convective heat transfer for blackbody radiation.

Moreover, there was significant uncertainty in our sensing “device”. As mentioned in the introduction, subjective sensations of warmth and cold may depend on the initial skin temperature, which varies within and across individuals. In spite of the numerous limitations of this study, we have demonstrated that first order approximations to heat transfer processes can be deduced from sensations obtained with our bare hands.

7. Conclusion

Thermo-reception of the human skin was used to determine the time at which a metal wire’s surface temperature was comparable to 35°C. The observations were in reasonable agreement with the calculations from empirical correlations. The experiments and analysis presented here illustrate how our senses can be used to assess temperature of objects close to the human body temperature and how such assessments can be used to quickly validate a certain mathematical approach to a practical engineering situation.

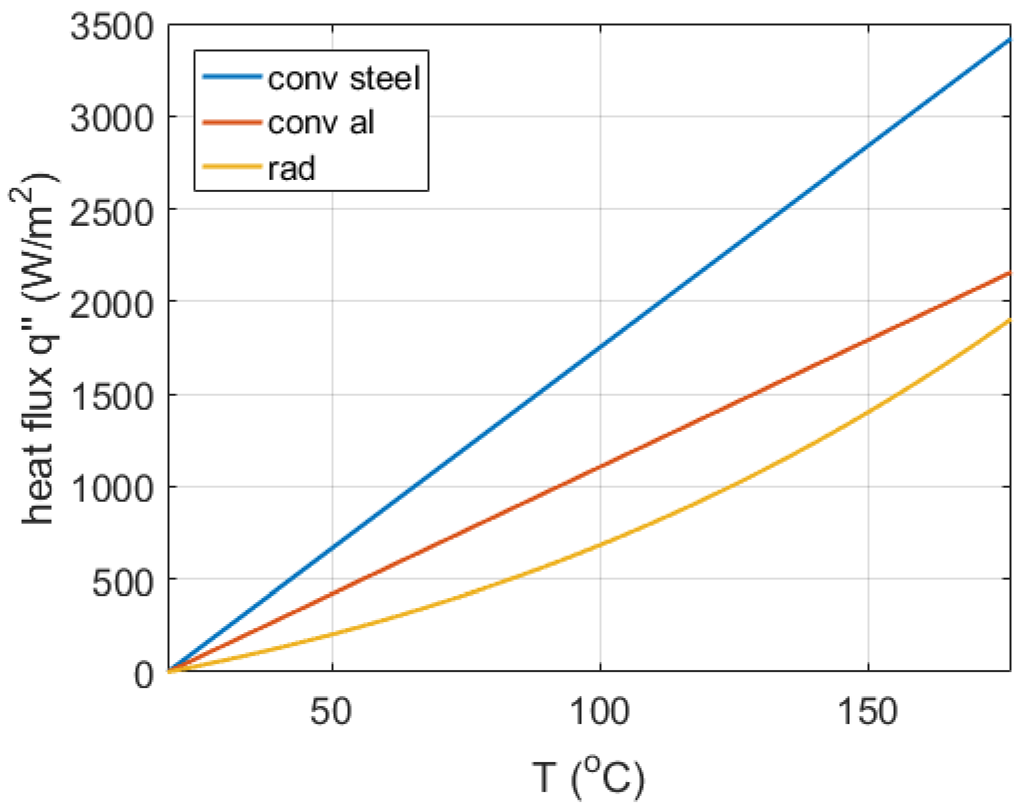

Figure 4: Estimated convective and radiation heat fluxes for the range of temperatures spanned in this experiment. The convective heat flux was estimated using the different estimated convection coefficients for steel and aluminum.

References

Bergman, T., & Incropera, Frank P. (2011). Fundamentals of heat and mass transfer. (7th ed. / Theodore L. Bergman [and others]. ed.). Hoboken, NJ: Wiley.

Hall, J. (2016). Guyton and Hall textbook of medical physiology (13th ed., Guyton Physiology). Philadelphia, PA: Elsevier.

Schmidt, R., & Altner, Helmut. (1986). Fundamentals of sensory physiology (3rd, rev. and expanded ed.). Berlin ; New York: Springer-Verlag.

APPENDIX: MATLAB CODE

```
close all; clear; clc
%%  Lab Experiment 2
%% Define properties

R = (0.09*2.54e-2)/2;    % radius of the wire in m
L = 0.4;                 % Length in m
Tinf = 19.4+273.15;      % room temperature in K

Tb  = 34+273.15;         % body (skin) temperature in K
To  = 176.667+273.15;    % temp at t=0

%% to find h
% use empirical correlations

g = 9.81;
Tf = ((To+Tinf)/2+Tinf)/2; % film temp at point of Tbody
% for air at 300K from table A.4
alpha = 22.5e-6; kair = 26.3e-3; Pr = 0.707; nu = 15.89e-6;
beta = 3.43e-3; % volumetric thermal expansion coefficient (1/K)
Ra = (g*beta*(Tf-Tinf)*(2*R)^3)/(nu*alpha)
NuD = ( 0.60 + (0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27))^2

hthe = NuD*kair/(2*R)  % theory h


%% Part 1
%% assume thermal conductivity steel
k= 15; rho = 7700; c = 500; %stainless steel
Bi = hthe*(2*R)/k
t = 4*60; To = 177+273.15; %350F
V = pi*(R^2)*L;
A = 2*pi*(R^2)+2*pi*R*L ;
hs = -rho*V*c/(A*t)*log( (Tb-Tinf)/(To-Tinf) )

% assume thermal conductivity of aluminum
k = 200; rho = 2700; c = 900; % aluminum
Bi = hthe*(2*R)/k
t = 4*60; To = 177+273.15; %350F
V = pi*(R^2)*L;
A = 2*pi*(R^2)+2*pi*R*L ;
ha = -rho*V*c/(A*t)*log( (Tb-Tinf)/(To-Tinf) )

es = abs(hs-hthe)/hthe
ea = abs(ha-hthe)/hthe


%% Part 2
%% assume thermal conductivity steel
k= 15; rho = 7700; c = 500; %stainless steel
h = hthe;
% T = @(t)(Tinf+(To-Tinf)*exp(-(hA/(rho*V*c))*t));
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```
tTb = -rho*V*c/(h*A)*log( (Tb-Tinf)/(To-Tinf) ); tTb/60
es = abs(tTb-t)/t

% assume thermal conductivity of aluminum
k = 200; rho = 2700; c = 900; % aluminum
% T = @(t)(Tinf+(To-Tinf)*exp(-(hA/(rho*V*c))*t));
tTb = -rho*V*c/(h*A)*log( (Tb-Tinf)/(To-Tinf) ) ; tTb/60

ea = abs(tTb-t)/t


 clear; close all; clc
%%
sig = 5.67e-8; %stefan boltzmann constant

R = (0.09*2.54e-2)/2;    % radius of the wire in m
L = 0.4;                 % Length in m
Tinf = 19.4+273.15;      % room temperature in K

Tb  = 34+273.15;         % body (skin) temperature in K
To  = 176.667+273.15;    % temp at t=0

%% to find h
% use empirical correlations

hsteel = 21.7484;
hal = 13.7269;

T = linspace(Tinf,To);

qco_s = hsteel*(T-Tinf);
qco_a = hal*(T-Tinf);
qrad = sig*(T.^4-Tinf^4);
T = T-273.15;
plot(T,qco_s,T,qco_a,T,qrad,'Linewidth',1.6); grid on
legend('conv steel','conv al','rad','Location','Northwest')
set(gca,'Fontsize',14)
xlabel('T (^oC)'); ylabel('heat flux q'''' (W/m^2)')
xlim([T(1) T(end)])
```