Cylindrical Conduction Heat Transfer Experiment

ME 331 Introduction to Heat Transfer

April 6th, 2017

Abstract

Fourier's law of thermal conduction was used to quantify the thermal conductivity of a cylindrical pipe insulating material. A bench top system was used to apply a known heat transfer rate to the inner cylindrical surface of the insulation while the temperatures at both the inner and outer surfaces of the insulation were recorded at five positions along the test specimen's length. On average, experimental values of thermal conductivity were within 10% of the manufacturer's published values [2]. Future work requires improvements in the experiment to reduce end losses and insulation density variations along the specimen's length.

Introduction

Fourier's law of thermal conductivity is a phenomenological equation which states that the energy transfer at the molecular level occurs in the direction of decreasing temperature, normal to isotherms. Furthermore, from Fourier's law, the heat transfer rate is directly proportional to: 1) the temperature gradient, 2) the area normal to heat flow, and 3) the thermal conductivity transport property, *k*. In cylindrical coordinates, the heat rate in the radial direction is:

$$q_r = -kA\frac{dT}{dr} \tag{1}$$

where q_r is the heat transfer rate in Watts (W), k is the thermal conductivity in W/m-K, A (m^2) is the area normal to the direction of heat transfer, and dT/dr is the temperature gradient in K/m. For the case of known surface temperatures, the steady-state, 1-D heat diffusion equation can be integrated twice and combined with Equation 1 to obtain:

$$q_r = \frac{2\pi Lk(T_{s,1} - T_{s,2})}{\ln\left(\frac{r_2}{r_1}\right)} \tag{2}$$

where $T_{s,1}$ is the temperature at the inner radius r_1, and $T_{s,2}$ is the temperature at the outer radius r_2. Solving Equation 2 for the thermal conductivity:

$$k = \frac{\ln\left(\frac{r_2}{r_1}\right) q_r}{2\pi L\left(T_{s,1} - T_{s,2}\right)} \tag{3}$$

where the heat transfer rate, q_r, is calculated from Ohm's law:

$$q_r = IV \tag{4}$$

and where I and V are the supplied current and voltage, respectively.

The bench top system consists of a long cylindrical cartridge heater surrounded by cylindrical insulating material, insulated ends, and 11 thermocouples. The cartridge heater provides a measured heat transfer rate at the inner surface of the insulation. Heat losses through the two ends of the cartridge heater are reduced with insulating end blocks, in order to limit heat flux primarily to the radial direction. Ten thermocouples measure temperatures at the inner and outer surfaces of the insulation at five positions along the length of the cylindrical system. These (five) thermocouple pairs provide inner and outer surface temperatures of the insulation as well as temperature gradients along the cylinder's longitudinal axis. TCo1 is located on the outer surface of the insulation, directly above TC1 (at the insulation's inner surface), TCo2 is located on the outer surface, directly above TC2, etc. Electrically supplied power enters nearest TCo5 and TC5. Thermocouples at the insulation's outer surface are held with tape against the insulation to minimize contact resistance errors. The eleventh thermocouple is used to monitor the ambient air temperature.

Procedure

The following steps were performed for data collection:

1. The cartridge heater was powered and temperatures were monitored until steady state was reached. Steady state was assumed once temperature changes were less than one degree Celsius over a time span of 30 minutes.

2. Once steady state was reached, the temperatures, voltage, and current were recorded using the data acquisition system. Concurrently, the voltage and current displayed by two multi-meters were recorded by hand. 15 data sets were recorded at 30-second intervals. The Appendix of this report lists recorded data.

Results

The thermal conductivity of the insulating material was calculated using Equations 3, and 4. The lab data was processed using Microsoft Excel. The manufacturer's published value for the thermal conductivity of the insulating material is listed as 0.0366 W/m-K [2]. The calculated temperature differences, thermal conductivities, and percentage differences between the calculated thermal conductivity and the manufacturer's published value are listed in Table 1 for each thermocouple pair location as well as for the entire insulation material. See the Appendix for detailed calculations.

Table 1. Calculated temperature differences, thermal conductivities, and comparison to manufacturer's published value.

	TC1 -TCo1	TC2–TCo2	TC3–TCo3	TC4–TCo4	TC5–TCo5	Average
Steady State ΔT (K)	38.5	35.9	32.3	29.8	27.2	32.7
Thermal Conductivity k (W/m-K)	0.034	0.037	0.041	0.044	0.048	0.040
Difference from Manufacturer's k Value (%)	- 6.8	0.3	11	19	28	9.4

Discussion

1. *In this method of calculating k, what assumptions, if any, are made about the heat flux? Does it vary along the length?*

   It is assumed that heat flux is constant along the length of the cylinder. However, as shown in Table 1 and in the Appendix, the heat flux is not constant as indicated by the varying temperature differences across the insulation thickness. An additional contributor to the varying temperature differences could be variable insulation thicknesses at the measurement locations. The outer surface thermocouples are held tightly against the insulation with tape wrapped around the cylinder at that location to secure the thermocouple and minimize contact resistance. The pressure associated with this method of fixation leads to thickness variations in the insulating material being measured.

2. *Compare the measured values of thermal conductivity for the insulation with the values published by the manufacturer. Do the measured values seem reasonable?*

   The insulation manufacturer, Armacell (Chapel Hill, NC), publishes a thermal conductivity of 0.0366 W/m-K for their Armaflex pipe insulation [2]. From this experiment, the calculated average thermal conductivity is 0.040 W/m-K, which is within 10% of the published value. Agreement within 10% seems reasonable based on the standard accuracy of ±1°C for Type K thermocouples utilized in the experiment. Furthermore, the range of calculated thermal conductivities brackets the manufacturer's value of 0.0366 W/m-K.

3. *Account for differences between the measured values of thermal conductivity and the published value. What potential sources of error are there? Estimate heat losses and discuss how these might contribute to error.*

   In addition to the uncertainly associated with the thermocouple measurements, there are several additional sources of error in the experiment.

   a. The non-uniform heat flux along the length of the cartridge heater is likely to be the largest contributor to error in the experiment.

   b. Use of Equation 3 assumes there is no contact resistance between the outer surface of the cartridge heater and the inner surface of the insulation. To quantify the error associated with contact resistance, a total thermal resistance which includes the conduction resistance and the contact resistance is used:

$$q_r = \frac{\Delta T}{R_{Conduction} + R_{Contact}} \tag{5}$$

   From this equation, it is possible to quantify the contact resistance and the corresponding air gap that would cause a 3% variation, for example, in the measured thermal conductivity. Modifying the temperature difference between the inner and outer radii to reflect a 3% difference in k results in $R''_{contact}$ = 6.11x10^{-3} m^2-K/W. Using air as the interstitial fluid, this contact resistance is equivalent to an air gap of 1.60x10^{-4} m or 160 microns which is possible for regions along the length of the cartridge heater.

   c. Thickness variations of the insulation as a result of the outer thermocouple fixation method could result in actual variations in thermal conductivity due to partial compression of the foam material. Thickness variations also result in erroneous values for the outer radius r_2 used in Equation 3.

   d. Use of Equation 4 assumes all of the heat transfer is in the radial direction with no heat losses at the two ends of the cartridge heater. The experiment was designed so that the ends of the cartridge heater are insulated to reduce end losses; however, end losses do occur and can be estimated. The end-insulating material is Temperlite, which has a thermal conductivity of 0.06 W/m-K. Assuming that one end of the cartridge heater is maintained at TC1, the opposite end maintained at TC5, and an approximate thickness of 2x10^{-2} m, Fourier's law may again be used to quantify the one-dimensional heat loss. Assuming these conditions, the heat loss through the ends could be up to 0.0180 W of the power supplied by the cartridge heater, which could cause the resulting thermal conductivity to be in error as much as 1.0%.

4. *Construct a graph of temperature vs. longitudinal distance for the outside and inside surface thermocouples. Does the temperature vary in the longitudinal direction? If so, why would the rod temperature vary?*

As indicated by Figure 1, the temperature varies along the length of the heater. The varying temperature along the length of the heater could result from these two factors: (1) the general principle of operation of an electric heater and (2) tip effects. In principle, the heat is generated by running current through a resistive heating element, which has an associated voltage drop across the length of the resistive element. This voltage drop, along with how the resistive elements are positioned and wired within the cartridge heater, is a contributing factor to the varying temperature. Second, the cartridge heater used in this experimental system has a capped tip on the end nearest TC1 and a ceramic stop at the end nearest TC5 to allow the wiring to enter the heater.

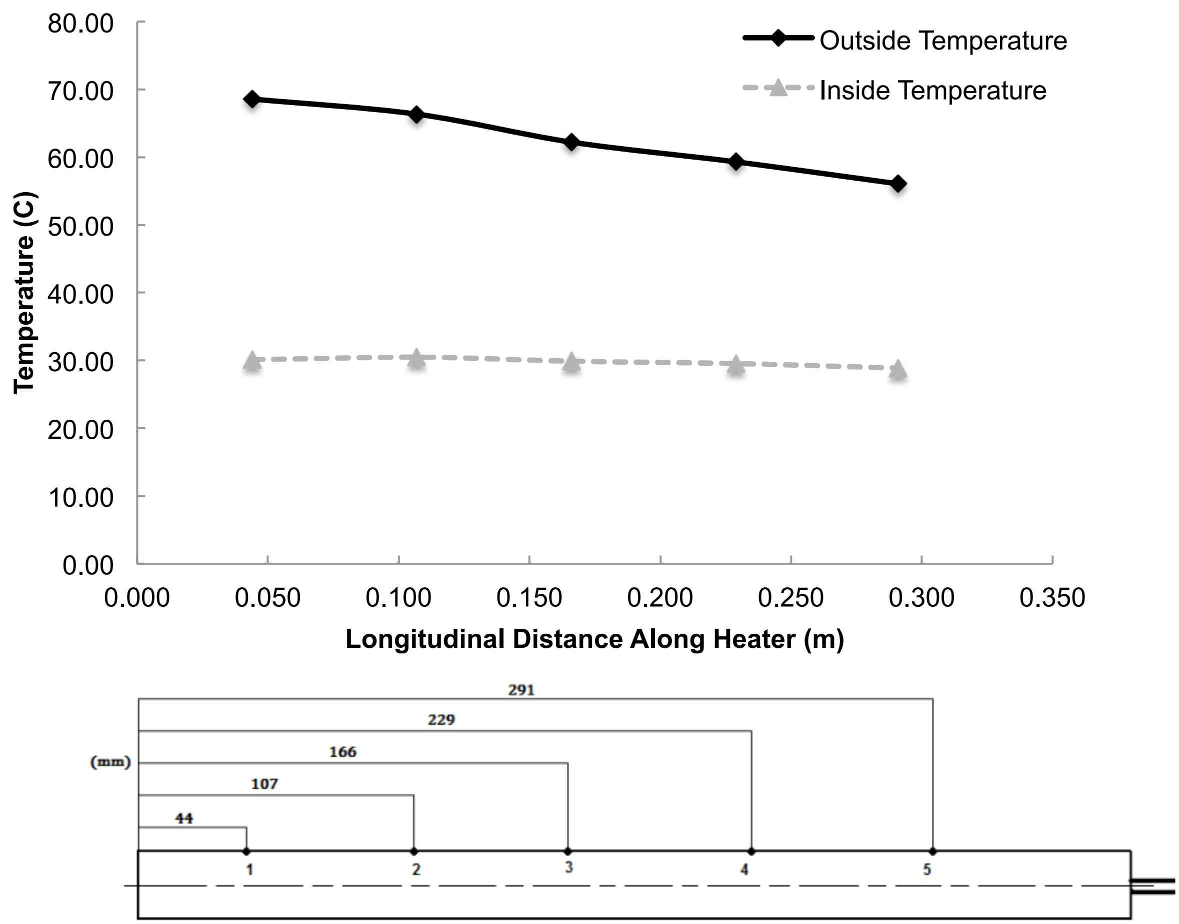

Figure 1. Surface temperature vs. longitudinal distance along the length of the system.

Since these heaters are typically submersed in a flow stream with the capped tip exposed and the sections furthest from the wire entry point experiencing the largest flow velocities, they are designed so the stream does not cause the temperature to drop substantially at the end. This requires providing extra power to the tip. In this experimental system, the tip was insulated with Temperlite material, which forced the additional power to conduct largely through the radial direction and manifest in a larger heat flux and higher temperature near the tip of the heater.

5. *Briefly discuss one or two alternative methods for measuring thermal conductivity. How does this system for measuring k compare to other methods?*

The American Society for Testing and Materials (ASTM) provides detailed guidelines for testing the thermophysical properties of materials. ASTM-C-177 and ASTM-C-518 are two active guidelines for the quantification of thermal conductivity. ASTM-C-177 positions the specimen between hot and cold plates, and using thermocouples, the temperature is measured at all the possible heat transfer paths, as shown in the left hand schematic of Figure 2. Rather than using thermocouples, ASTM-C-518 uses heat flux transducers to directly measure the heat flow through the specimen, as shown in the right hand schematic of Figure 2. Both ASTM-C-177 and ASTM-C-518 place the test specimen near a hot plate to drive heat transfer similar to the cartridge heater utilized in this experimental set up. One difference to note between ASTM standards and the utilized experimental setup is the fact that the ASTM guidelines only discuss a Cartesian geometry whereas a cylindrical geometry was tested in this experiment.

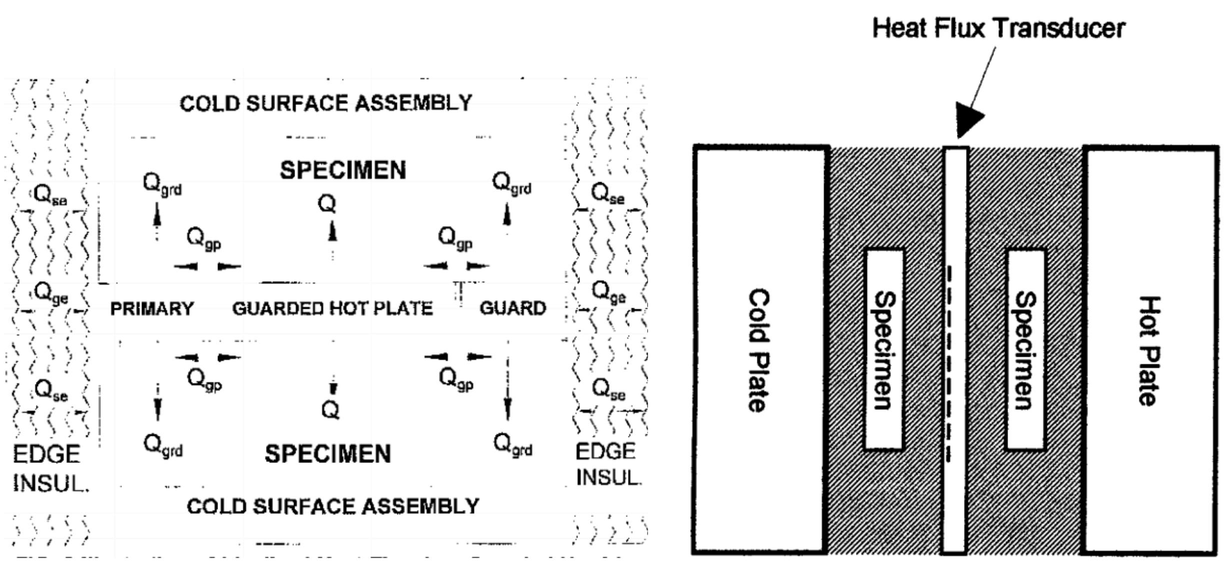

Figure 2. Idealized heat flow for ASTM-C-177 test apparatus (left) [4]. ASTM-C-518 test apparatus (right) [3].

Conclusion

Fourier's law of thermal conduction was utilized to quantify the thermal conductivity of a cylindrical pipe insulating material. Using an applied heat transfer rate at the inner surface of the insulation, along with measured temperatures at the inner and outer surfaces of the insulation, thermal conductivity was quantified. The experimental value of the average thermal conductivity was 0.040 W/m-K, which is within 10% of the manufacturer's published values [2]. The error between the measured and published thermal conductivity can be attributed to the following sources; non-uniform heat flux along the heater, neglecting thermal contact resistance, insulation thickness variations due to thermocouple fixation method, and end losses.

References

1. F.P. Incropera et. al., Introduction to Heat Transfer, 6th ed. Hoboken, NJ: Wiley, 2011.

2. http://www.armacell.us/fileadmin/user_upload/Insulation_Submittals-English/APArmaflex_Tube.Sub.EN.US.2016.pdf

3. "Standard Test Method for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus" (ASTM-C-518).

4. "Standard Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus" (ASTM-C-177).

Appendix

Geometric Data

Heater Length (l):	0.36	m
Inner Radius (ri):	0.005	m
Insulation Thickeness:	0.01905	m
Outer Radius (ro):	0.024	m

Experimental Data

Time	Date	TC 1 ∞C	TC 2 ∞C	TC 3 ∞C	TC 4 ∞C	TC 5 ∞C	TC o1 ∞C	TC o2 ∞C	TC o3 ∞C	TC o4 ∞C	TC 05 ∞C	Power Watts	Ambient ∞C	MM Volt.	MM Curr.	MM Power
8:05:25	3/29/17	68.59	66.33	62.20	59.30	56.01	29.62	30.23	29.63	29.46	29.02	1.90	24.07	20.38	0.098	1.99
8:05:55	3/29/17	68.56	66.35	62.18	59.27	55.99	29.71	30.30	29.68	29.46	29.01	1.90	24.19	20.40	0.098	1.99
8:06:25	3/29/17	68.56	66.31	62.17	59.28	55.99	29.97	30.47	29.85	29.55	28.85	1.90	24.03	20.39	0.098	1.99
8:06:55	3/29/17	68.58	66.35	62.22	59.33	56.04	30.07	30.58	29.86	29.61	28.83	1.90	24.09	20.40	0.098	1.99
8:07:25	3/29/17	68.55	66.31	62.20	59.31	56.04	30.25	30.55	29.94	29.46	28.62	1.90	24.30	20.39	0.098	1.99
8:07:55	3/29/17	68.56	66.34	62.22	59.30	56.10	30.34	30.51	29.91	29.62	28.96	1.90	23.83	20.38	0.098	1.99
8:08:25	3/29/17	68.58	66.34	62.20	59.28	56.07	30.05	30.39	29.97	29.74	29.10	1.90	24.09	20.37	0.097	1.99
8:08:55	3/29/17	68.59	66.36	62.23	59.30	56.06	29.90	30.30	29.89	29.67	29.06	1.90	24.04	20.38	0.098	1.99
8:09:25	3/29/17	68.58	66.38	62.27	59.32	56.09	30.10	30.49	29.86	29.48	28.90	1.90	24.12	20.40	0.098	1.99
8:09:55	3/29/17	68.61	66.40	62.27	59.33	56.10	30.27	30.47	29.93	29.53	28.67	1.89	24.28	20.38	0.098	1.99
8:10:25	3/29/17	68.56	66.34	62.22	59.30	56.11	30.14	30.42	29.97	29.51	28.75	1.90	24.23	20.38	0.098	1.99
8:10:55	3/29/17	68.56	66.36	62.23	59.31	56.11	30.13	30.46	29.88	29.35	28.60	1.89	23.98	20.37	0.097	1.98
8:11:25	3/29/17	68.55	66.33	62.23	59.32	56.08	30.12	30.53	29.80	29.29	28.73	1.89	24.03	20.35	0.097	1.98
8:11:55	3/29/17	68.55	66.35	62.24	59.29	56.10	30.19	30.73	30.06	29.66	28.91	1.89	23.92	20.37	0.097	1.98
8:12:25	3/29/17	68.57	66.34	62.22	59.31	56.12	30.35	30.76	30.11	29.73	29.10	1.89	23.90	20.37	0.097	1.98
	Average:	68.57	66.35	62.22	59.30	56.07	30.08	30.48	29.89	29.54	28.87	1.90	24.07	20.38	0.10	1.99
	Max Delta:	0.06	0.09	0.11	0.06	0.14	0.74	0.53	0.48	0.45	0.50	0.01	0.48	0.04	0.00	0.01

Calculated Values

Published Conductvity:	0.0366	W/m-K

a.) Overall Average Temperatures

Average Inside Temp:	62.50	C
Average Outside Temp:	29.77	C
Average Temp Delta:	32.73	C
Thermal Conductivity:	0.040	W/m-K
% Difference:	9.4	

b.) Average Temperatures Between Points

Position from End:	0.044	m
Average TC1 Temp:	68.57	C
Average TCo1 Temp:	30.08	C
Average Temp Delta:	38.49	C
Thermal Conductivity:	0.034	W/m-K
% Difference:	6.8	
Position from End:	0.107	m
Average TC2 Temp:	66.35	C
Average TCo2 Temp:	30.48	C
Average Temp Delta:	35.87	C
Thermal Conductivity:	0.037	W/m-K
% Difference:	0.3	
Position from End:	0.166	m
Average TC3 Temp:	62.22	C
Average TCo3 Temp:	29.89	C
Average Temp Delta:	32.33	C
Thermal Conductivity:	0.041	W/m-K
% Difference:	10.6	
Position from End:	0.229	m
Average TC4 Temp:	59.30	C
Average TCo4 Temp:	29.54	C
Average Temp Delta:	29.76	C
Thermal Conductivity:	0.044	W/m-K
% Difference:	18.9	
Position from End:	0.291	m
Average TC5 Temp:	56.07	C
Average TCo5 Temp:	28.87	C
Average Temp Delta:	27.19	C
Thermal Conductivity:	0.048	W/m-K
% Difference:	27.8