Heat Exchanger Lab Experiment

# **SOLUTIONS**

May. 19<sup>th</sup>, 2017

ME 331 – Introduction to Heat Transfer

#### Abstract

An experiment was conducted to measure the performance of a simple, concentric tube heat exchanger operating in both parallel and counter-flow configurations. Separate cold and hot water supplies were fed into the heat exchanger, and measurements were made of the inlet temperature, outlet temperature, and flow rate for both streams and for both flow configurations. For the parallel flow configuration, heat exchanger performance metrics were calculated to be the following: NTU = 1.20,  $\varepsilon = 51\%$ ,  $U_{i,NTU} = 7.01 \times 10^3$  W/(m<sup>2</sup> K). For the counter-flow configuration, the same metrics were calculated to be: NTU = 1.38,  $\varepsilon = 63\%$ ,  $U_{i,NTU} = 8.17 \times 10^3$  W/(m<sup>2</sup> K). The overall heat transfer coefficient U based on the sum of predicted conductive wall resistance and two convective resistances from empirical correlations was within 30% of the actual value  $U_{NTU}$  for the parallel flow heat exchanger. For the counter flow heat exchanger, the overall heat transfer coefficient U based on the sum of predicted conductive and convective resistances was within 15% of the actual value. As expected, the counter-flow configuration was more effective than the parallel-flow configuration, for the same heat exchanger geometry.

## Introduction

In this heat exchanger laboratory experiment, temperature and flow rate measurements were taken for a simple concentric tube heat exchanger in order to analyze the heat exchanger's performance in both parallel and counter flow configurations. All of the following equations are taken from the 6<sup>th</sup> edition of Incropera et. al.'s "Introduction to Heat Transfer." The derivation of these equations, along with a more complete description of heat exchanger design and analysis can be found in the ME 331 textbook [1].

Heat exchanger analysis starts from the application of an energy balance on each of the fluid streams. The heat gained by the cold fluid is given by

$$q_c = \dot{m}_c c_{p,c} \left( T_{c,o} - T_{c,i} \right),$$
 (Eq. 11.7b)

and the heat lost by the hot fluid is given by

$$q_h = \dot{m}_h c_{p,h} \left( T_{h,i} - T_{h,o} \right)_{i}$$
 (Eq. 11.6b)

The subscripts *i* and *o* are use to distinguish between inlet and outlet temperatures. In an ideal heat exchanger,  $q_h$  and  $q_c$  are equal to each other for the case of zero heat exchange with the environment.

The "heat capacity rate" (*C*) is defined as the product of the mass flow rate and the specific heat. For the hot fluid,  $C_h = \dot{m}_h c_{n,h}$ 

and for the cold fluid,

$$C_c = \dot{m}_c \, c_{p,c}$$

One measure of the performance of a heat exchanger is its "effectiveness" ( $\varepsilon$ ), but in order to define the effectiveness one must first determine the heat exchanger's "maximum possible heat transfer rate" ( $q_{max}$ ):

$$Q_{max} = C_{min} \left( T_{h,i} - T_{c,i} \right)_{i}$$
 (Eq. 11.18)

 $C_{min}$  is the minimum of the hot and cold heat capacity rates, since this is the fluid stream that limits the heat exchange process thereby determining the maximum possible heat transfer rate.

The heat exchanger effectiveness is then the ratio between the actual heat transfer rate and the maximum possible heat transfer rate:

$$\epsilon = \frac{q_c}{q_{max}} = \frac{q_h}{q_{max}}.$$
(Eq. 11.19)

The number of transfer units (NTU) is a metric used for quantifying heat exchanger performance. It is defined as:

$$NTU \equiv \frac{UA}{C_{min}}$$
(Eq. 11.24)

where *U* is a overall heat transfer coefficient and *A* is the corresponding surface area. For parallel flow heat exchangers, NTU can be found from the following equation:

$$NTU = -\frac{\ln\left[1 - \epsilon(1 + C_r)\right]}{1 + C_r},$$
 (Eq. 11.28b)

and for counter flow heat exchangers

$$NTU = \frac{1}{C_r - 1} \ln \left( \frac{\epsilon - 1}{\epsilon C_r - 1} \right),$$
 (Eq. 11.29b)

where in both equations

$$C_r = \frac{C_{min}}{C_{max}}.$$

An alternative approach for determining the overall heat transfer coefficient for a heat exchanger is to use a resistance network to model the heat transfer from the hot to the cold fluid. Solving for *U* using the following equation for a concentric tube geometry:

$$R_{total} = \frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln\left(D_o/D_i\right)}{2\pi kL} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}.$$
 (Eq. 11.5)

In this equation, *R*<sub>total</sub> is the total resistance to the heat transfer between the hot and cold fluids and is equal to the sum of the following (from left to right): convection resistance on the inner surface of the inner tube, resistance due to fouling on the same inner surface, conduction resistance through the tube wall separating the hot and cold fluids, resistance due to fouling on the outer surface of the inner tube, and finally, convection resistance on the same outer surface of the inside tube.

In order to determine the convection coefficients on the inner and outer surfaces of the inside tube, convection correlations for internal turbulent flow were used from the textbook [1]. The hot water stream flows through a standard circular tube geometry. Since the flow was turbulent ( $Re_D = 1.2 \times 10^4 > 2300$ ) and each of the four straight sections of the tubing were long enough to model the entire heat exchanger as being fully developed (L/D = 150 > 10), determining the appropriate correlation was straightforward. The convection coefficient for the hot water flow on the inner surface of the inner pipe was found using the Dittus-Boelter equation for cooling since Ts < Tm (See footnote below Eq. 8.60).

$$Nu_D = 0.0265 Re_D^{4/5} Pr^{0.3}$$

The cold water flows through an annular geometry. At the end of Section 8.6 of the textbook, it states that for fully developed turbulent flows, the convection coefficient for annular geometries may be found using the hydraulic diameter and the Dittus-Boelter equation. The hydraulic diameter for annular tubes is:

$$D_h = D_o - D_{i_i} \tag{Eq. 8.71}$$

and for the cold flow, since *Ts* > *Tm*, the Dittus-Boelter equation for heating was used.

$$Nu_{D_h} = 0.0243 Re_{D_h}^{4/5} Pr^{0.4}$$

# Procedure

### Part A: Parallel-Flow Configuration

- 1. Verify that the heat exchanger is running in the parallel-flow configuration with the cold water flowing through the outer tube.
- 2. Verify that the heat exchanger has reached a steady-state by inspecting the temperature time history plot on the computer's data acquisition program.
- 3. If the heat exchanger is at steady-state, record data: five thermocouple temperatures (T<sub>h,in</sub>, T<sub>h,out</sub>, T<sub>c,in</sub>, T<sub>c,out</sub>, T<sub>ambient</sub>) and two volumetric flow rates for both the hot and cold water streams.

### Part B: Switching Flow Configuration

- 1. Turn off the cold water pump.
- 2. Unscrew the cold water supply and outlet hoses and let drain.
- 3. Reattach the hoses to the opposite connections.

### Part C: Counter-Flow Configuration

- 1. Verify that the heat exchanger is running in the counter-flow configuration with the cold water flowing through the outer tube.
- 2. Turn on the cold water pump and wait for the heat exchanger to reach steady-state.
- 3. Verify that the heat exchanger has reached a steady-state by inspecting the temperature time history plot on the computer's data acquisition program.
- 4. If the heat exchanger is at steady-state, record data: five thermocouple temperatures (T<sub>h,in</sub>, T<sub>h,out</sub>, T<sub>c,in</sub>, T<sub>c,out</sub>, T<sub>ambient</sub>) and two volumetric flow rates for both the hot and cold water streams.

# Results

#### **Experimental Data:**

Flow Configuration	T <sub>cold,in</sub> (°C)	T <sub>hot,in</sub> (°C)	T <sub>cold,out</sub> (°C)	$T_{hot,out(^{\circ}C)}$	T <sub>ambient</sub> (°C)	Q <sub>cold</sub> (gal /min)	Q <sub>hot</sub> (gal /min)
Parallel	20.0	43.5	32.1	34.6	24.6	3.81	5.50
Counter	23.7	53.1	42.2	39.7	24.4	3.87	5.54

#### Dimensions and material properties

ID of inner	OD of inner	ID of outer	OD of outer	Length(")	$k_w(W/m \cdot K)$
tube(")	tube(")	tube(")	tube('')		
3/8	1/2	5/8	3/4	225	52

Calculations:

1. Flow properties:

#### Parallel flow:

	T <sub>mean</sub> (K)	$\rho(kg/m^3)$	c <sub>p</sub> (J/kg⋅K)	$\mu \cdot 10^6 (N \cdot s/m^2)$	Pr	$k(W/m \cdot K)$	
Cold	299.2	997.2	4179	872	5.96	0.6119	
Hot	312.2	992.2	4178	667	4.42	0.6307	

#### Counter flow:

	T <sub>mean</sub> (K)	$\rho(kg/m^3)$	c <sub>p</sub> (J/kg⋅K)	$\mu \cdot 10^6 (N \cdot s/m^2)$	Pr	$k(W/m \cdot K)$
Cold	306.1	994.6	4178	752	5.07	0.6218
Hot	319.5	983.4	4180	582	3.81	0.6394

2. Reynolds number for both flows: Parallel flow configuration:

$$\operatorname{Re}_{D,hot} = \frac{4 \rho_{h} \dot{\forall}_{h}}{\pi D \mu} = 6.90 \times 10^{4}, \quad \operatorname{Re}_{D,cold} = \frac{4 \rho_{c} \dot{\forall}_{c}}{\pi (Do + Di)\mu} = 1.23 \times 10^{4}$$

Counter flow configuration:

$$\operatorname{Re}_{D,hot} = \frac{4 \rho_{h} \dot{\forall}_{h}}{\pi D \mu} = 7.94 \times 10^{4}, \quad \operatorname{Re}_{D,cold} = \frac{4 \rho_{c} \dot{\forall}_{c}}{\pi (\text{Do} + \text{Di})\mu} = 1.44 \times 10^{4}$$

3. Calculate q (actual) from the cold flow stream. Calculate q (actual) from the hot flow stream.

Compare by calculating percent difference. Parallel flow configuration:

$$\begin{split} q_{cold} &= \rho_c c_{p,c} \dot{\Psi}_c \big( T_{c,out} - T_{c,in} \big) = 1.20 \times 10^4 W; \\ q_{hot} &= \rho_h c_{p,h} \dot{\Psi}_h \big( T_{h,in} - T_{h,out} \big) = 1.28 \times 10^4 W; \ difference = 6\% \end{split}$$

Counter flow configuration:

$$q_{cold} = \rho_c c_{p,c} \dot{\forall}_c (T_{c,out} - T_{c,in}) = 1.88 \times 10^4 \text{W};$$

$$q_{hot} = \rho_h c_{p,h} \dot{\Psi}_h (T_{h,in} - T_{h,out}) = 1.95 \times 10^4 W$$
; difference = 3%

4. Calculate q (max), heat exchanger effectiveness *ε*, and number of transfer units NTU. Parallel flow configuration:

$$\begin{split} C_{\min} &= C_{c} = \rho_{c} c_{p,c} \dot{\forall}_{c} = 1002 \text{W/K} \\ C_{\max} &= C_{h} = \rho_{h} c_{p,h} \dot{\forall}_{h} = 1438 \text{W/K} \\ q_{max} &= C_{min} (T_{h,\text{in}} - T_{c,\text{in}}) = 2.35 \times 10^{4} \text{W}; \\ \epsilon &= \frac{q}{qmax} = 51\%, \text{ NTU} = 1.20 \end{split}$$

Counter flow configuration:

$$\begin{split} C_{\min} &= C_{c} = \rho_{c} c_{p,c} \dot{\forall}_{c} = 1014 \text{W/K} \\ C_{\max} &= C_{h} = \rho_{h} c_{p,h} \dot{\forall}_{h} = 1344 \text{W/K} \\ q_{max} &= C_{min} (T_{h,in} - T_{c,in}) = 2.98 \times 10^{4} \text{W}; \\ \epsilon &= \frac{q}{qmax} = 63\%, \text{ NTU} = 1.38 \end{split}$$

5. Calculate the overall heat transfer coefficient U based on NTU. Parallel flow configuration:

 $U_{i,NTU} = 7.01 \times 10^{3} W/m^{2} \cdot K, \qquad U_{o,NTU} = 5.26 \times 10^{3} W/m^{2} \cdot K$ Counter flow configuration:  $U_{i,NTU} = 8.17 \times 10^{3} W/m^{2} \cdot K, \qquad U_{o,NTU} = 6.13 \times 10^{3} W/m^{2} \cdot K$ 

6. Calculate the overall heat transfer coefficient U based on conduction resistance and convection resistances from empirical correlations. Compare this overall heat transfer coefficient with U based on the NTU method.

Equation 11.5 is used, but without fouling resistances.

$$R_{w} = \frac{ln(D_{2}/D_{1})}{2\pi k_{w}L} = 1.54 \times 10^{-4} \cdot \frac{K}{W}; \ k_{w} = 52W/(m^{2} \cdot K)$$

Parallel flow configuration:

	Nu	$h(W/m^2 \cdot K)$	A(m <sup>2</sup> )	$U(W/m^2 \cdot K)$
Cold(outer)	90.0	17342	0.228	6690
Hot(inner)	355.2	23520	0.171	8920

$$U_{i,\Sigma \text{ resistances}} = 8.92 \times 10^3 W/m^2 \cdot K$$
,

 $U_{o,\Sigma \text{ resistances}} = 6.91 \times 10^3 W/m^2 \cdot K$ 

Counter flow configuration:

	Nu	$h(W/m^2 \cdot K)$	A(m <sup>2</sup> )	$U(W/m^2 \cdot K)$
Cold(outer)	97.5	19091	0.228	7107
Hot(inner)	373	25068	0.171	9476

$$U_{i,\Sigma \text{ resistances}} = 9.47 \times 10^3 W/m^2 \cdot K,$$

 $U_{o,\Sigma \text{ resistances}} = 7.11 \times 10^3 W/m^2 \cdot K$ 

### Discussion

- The accuracy of temperature and flow rate measurements. Type K thermocouples have standard error of 2.2°C according to the official OMEGA site [2]. The flow rate uncertainty is estimated to be 0.05 gal/min based on a bucket test calibration.
- 2. The accuracy of the q values calculated based on temperature and flow rate measurement errors.

Assuming no error in the property terms,

$$\frac{\Delta q}{q} = \sqrt{\left(\frac{\Delta \dot{\forall}}{\dot{\forall}}\right)^2 + \left(\frac{\sqrt{2}\Delta T}{T_{out} - T_{in}}\right)^2} = 0.12, \qquad \Delta q = 1.4 \times 10^3 W$$

- 3. The effect of neglecting heat exchange with the environment. Discuss differences between q values calculated from the cold and hot streams.
  - In both cases,  $q_{cold} < q_{hot}$ . In parallel flow configuration,  $q_{hot} q_{cold} = 761 W$ ; in counter flow configuration,  $q_{hot} q_{cold} = 648 W$ . Considering the error estimated in part 2 (above) far exceeds these differences, it cannot be determined whether this difference is physical or due to measurement error. Therefore the effect of neglecting heat exchange with the environment cannot be investigated with the data acquired. However, since the average mean temperature of the cold side is well above ambient temperature for both flow configurations, the "cold" stream will lose heat to the ambient environment. Therefore, the magnitude of  $q_{cold}$  is expected to be less than the magnitude of  $q_{hot}$ .
- 4. The accuracy of your estimates of h.

According to the textbook correlations, the estimates of h are expected to be within 30% of the actual values. However, the estimated h values and associated convective resistances are for clean tubes, and therefore they do not include the effects of fouling resistance.

5. Discuss differences in the calculated overall heat transfer coefficient U using the NTU method and the resistance method.

The values of U calculated using the NTU method are based on the actual measured data; therefore, the values of 1/(UA) from the NTU method represent an actual overall resistance to heat exchange between the hot and cold streams.

On the other hand, values of 1/(UA) that are based on a sum of resistances account only for the conductive resistance and two convective resistances on clean tube surfaces. Using the sum of resistances method, the only way to independently determine an accurate value of the overall resistance is to specifically account for and include the resistances due to fouling. Unfortunately, independent determination of the fouling resistances is not possible because no detailed information about the interior tube surfaces is available.

The overall resistance 1/(UA), where the value of U is based on the NTU method, is expected to be greater than the resistance based on the sum of conduction and convection resistances, since this latter sum does not include fouling resistance.

However, using the sum of resistances, it is possible to estimate the total resistance that occurs due to fouling. Based on the actual overall resistance from the NTU

method, the total of the two fouling resistances can be obtained by difference, using equation 11.5.

$$R_{total} = \frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln\left(D_o/D_i\right)}{2\pi kL} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}.$$
 (Eq. 11.5)

In the following equation, fouling resistance is estimated by difference.

$$\frac{\mathbf{R}_{f,i}''}{Ai} + \frac{\mathbf{R}_{f,o}''}{Ao} = \frac{1}{(\mathbf{UA})_{\mathbf{NTU}}} - \frac{1}{hiAi} - \frac{1}{hoAo} - \frac{\ln(D_2/D_1)}{2\pi k_w L}$$

For parallel flow,  $R_{f,i}'' = R_{f,0}'' = 9.4 \times 10^{-6} (m^2 \cdot K/W)$ ; for counter flow,  $R_{f,i}'' = R_{f,0}'' = 7.6 \times 10^{-6} (m^2 \cdot K/W)$ . This could be realistic since the representative fouling factor in Table 11.1 from the text for treated boiler feedwater below 50°C is  $0.0001(m^2 \cdot K/W)$ . This value from Table 11.1 is about 10~13 times the value of a fouling factor needed to explain the difference in the two values of (UA).

6. Discuss differences in heat exchanger effectiveness for parallel flow and counter flow configurations.

The heat exchanger effectiveness for counter flow is approximately 20% greater than that for parallel flow. This is not surprising since the counter flow configuration yields greater log mean temperature difference. With U values that are approximately the same, the counter flow configuration has greater actual q, thus greater effectiveness.

# Conclusions

The heat exchanger lab investigated the performance of an unfinned, concentric tube heat exchanger under parallel and counter flow configurations with NTU method and resistance method. The temperatures of the inlet and outlet of both fluid steams are measured; the flow rates are measured. The heat exchanger effectiveness  $\mathcal{E}$  and the number of transfer units NTU are calculated directly from the experimental data. Overall heat transfer coefficients U are calculated using both the NTU method and the resistance method. In the latter method, estimates of h from empirical correlations are used. Measured differences in q values between the cold and hot flows were small and well below the uncertainty in q. Therefore, heat exchange with the environment was neglected. The counter flow heat exchanger showed better performance in terms of its effectiveness compared to the parallel flow.

Differences in the calculated *U*'s from both methods are discussed. The U values calculated with the resistance method assume perfectly clean pipes and therefore neglect any fouling that might be present on the heat exchanger tubing. A fouling resistance is calculated that accounts for the differences between the U values calculated with the two different methods and it was found to be  $10 \sim 13$  times smaller than the fouling factors provided in the textbook. Predicted values of the overall heat transfer coefficient U are based on empirical correlations and conductive wall resistance. For counter flow configurations, the predicted values of U were within 15% of the actual values of U<sub>NTU</sub>; for parallel flow

configurations, the discrepancy is less than 30%. This is consistent with the statement that convection coefficients calculated from empirical correlations are typically within 30% of the actual value.

# Reference

[1] F. P. Incropera et. al., Introduction to Heat Transfer, 6<sup>th</sup> ed. Hoboken, NJ: Wiley, 2011.
[2] Thermocouple. (2017, April 14). Retrieved from

http://www.omega.com/prodinfo/thermocouples.html.