Convection Heat Transfer

$$q = h\,A\,(T_{surface} - T_{fluid})$$

q: W, h: $\frac{W}{m^2 K}$, A: m^2, $(T_{surface} - T_{fluid})$: K

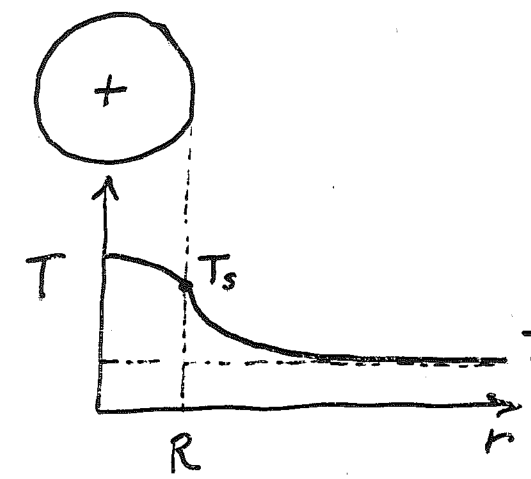
h depends on fluid properties, flow pattern and velocity.

We will use this equation for forced convection, free convection, external flows, internal flows.

Goal is to find h Chapters 7, 8, 9 emperical correlations for h

First some fundamentals on convection Chapter 6

Consider a sphere where $T_s > T_\infty$



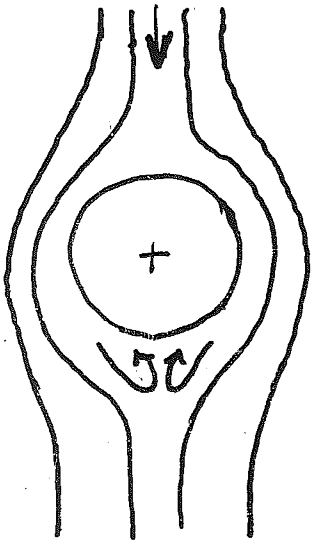
In terms of the temperature gradient <u>in the fluid</u>, what is the heat loss rate from the sphere?

$$q = -k_f\,A\left[\frac{dT}{dr}\right]_{\substack{\text{fluid at}\\ r=R}}$$

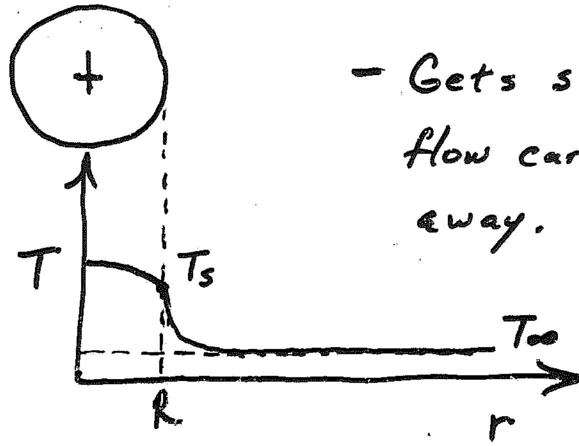
temperature gradient at surface

$$\left(\text{Also,}\quad q = -k_s\,A\left[\frac{dT}{dr}\right]_{\substack{\text{solid at}\\ r=R}}\right)$$

Now force air across the sphere



What happens to temperature gradient?



- Gets steeper because flow carries heated fluid away.

$$q = -k_f A \left[\frac{dT}{dr}\right]_{\substack{\text{fluid at} \\ r=R}}$$ increases.

Forced air thins boundary layer.

q is also given in terms of h: $q = hA(T_s - T_\infty)$

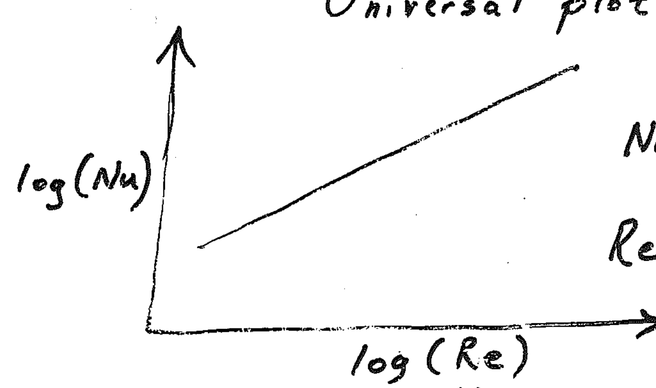
Equating the two, $$q = -k_f A \left[\frac{dT}{dr}\right]_{\substack{\text{fluid at} \\ r=R}} = hA(T_s - T_\infty)$$

gives definition of <u>local</u> heat transfer coefficient:

$$h \equiv \frac{-k \left[\frac{dT}{dr}\right]_{\text{fluid at } r=R}}{T_s - T_\infty}$$

For fixed T_s, T_∞

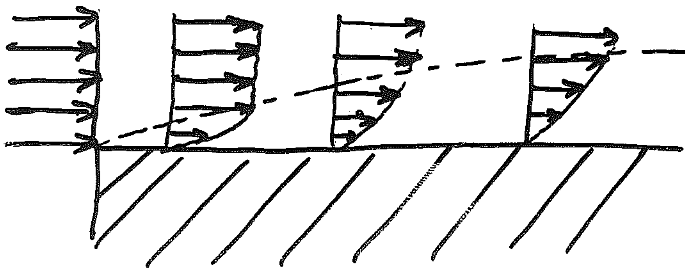
Universal plot



$$Nu = \frac{h L_c}{k_{fluid}}$$

$$Re = \frac{V L_c}{\nu}$$

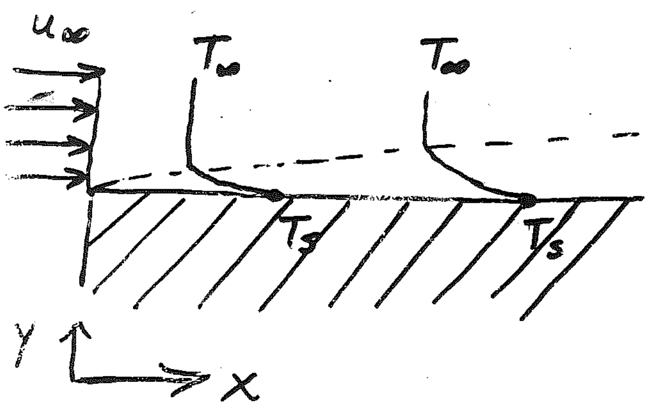
Flow over flat plate



$\delta(x)$ Boundary Layer Thickness

no slip condition at solid boundary

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$ Surface shear stress for Newtonian fluid.

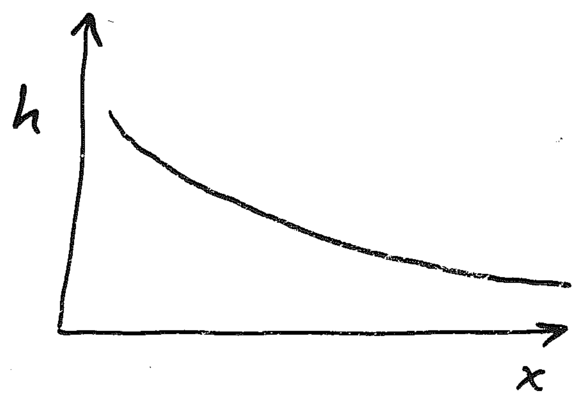


$\delta_t(x)$ Thermal Boundary Layer Thickness

heated plate $T_s > T_\infty$

What happens to local heat transfer coefficient h as a function of x?

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$$



Reynold's number

ratio of "inertia forces" to "viscous forces"

$$Re_L = \frac{(\rho V^2)}{\left(\mu \frac{V}{L}\right)}$$

(ρV^2) ~momentum flux

$\left(\mu \frac{V}{L}\right)$ ~shear force

$$Re_L = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

Prandtl number

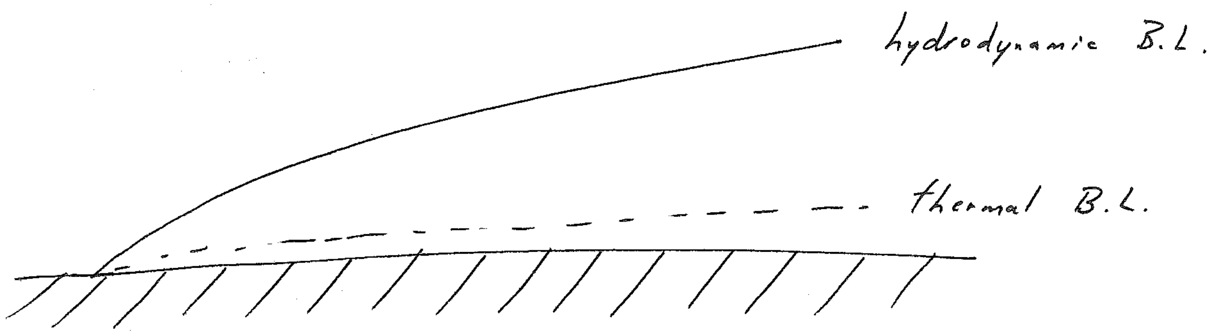
ratio of "momentum diffusivity" to "thermal diffusivity"

$$Pr = \frac{\nu}{\alpha}$$

$\nu = \frac{\mu}{\rho}$

$\alpha = \frac{k}{\rho c_p}$

Example of high Prandtl number? Oil



Low Prandtl number? Mercury

$Pr \sim 1$? Air

h is a local heat transfer coefficient.

$$h \equiv \frac{-k_f \left.\frac{dT}{dy}\right|_{y=0}}{T_s - T_\infty} \quad \text{and} \quad q_s'' = h(T_s - T_\infty)$$

We can also define an average convection coefficient $\bar{h}$ for an entire surface at T_s.

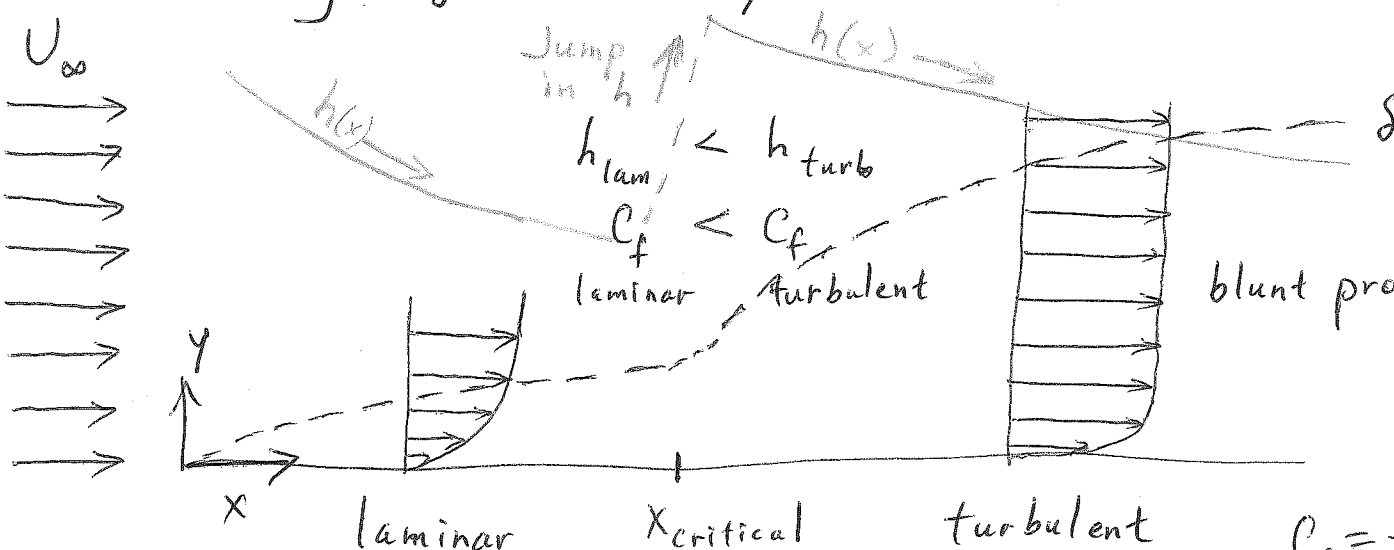
$$\bar{h} \equiv \frac{1}{A_s}\int_{A_s} h \, dA_s \quad \text{and} \quad q = \bar{h} A_s (T_s - T_\infty)$$

In many practical applications, the hydrodynamic boundary layer transitions from laminar to turbulent.

For a flat plate, this transition occurs at

$$Re_{x,\,critical} = \frac{U_\infty x}{\nu} \simeq 5 \times 10^5$$

where x is the distance measured from the leading edge of the plate.



$$C_f = \frac{\tau_s}{\rho U_\infty^2 / 2}$$

Most correlations from Chapters 7, 8 are of this form:

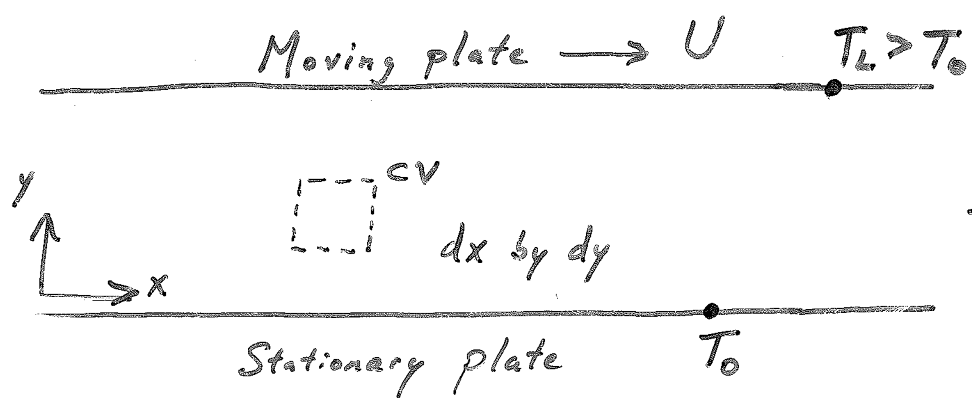
$$\overline{Nu}_L = \frac{\bar{h} L}{k_{fluid}} = \text{function}\left(Re_L,\ Pr\right)$$

From Chapter 9, $\overline{Nu}_L = \dfrac{\bar{h} L}{k_{fluid}} = \text{function}\left(\text{Rayleigh \#},\ Pr\right)$

None of the correlations from Chapters 7, 8, 9 are applicable for compressible flows in which Mach # $\gtrsim 0.3$

Let's look at one of a few special cases for which there is an exact analytical solution for both the velocity and temperature fields in a convection heat transfer problem.

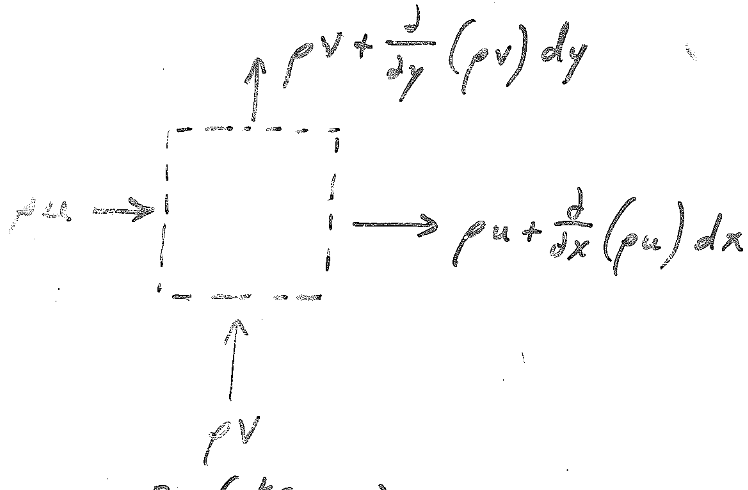
Convective Transfer Equations for Couette Flow



Laminar flow between two flat plates.

- What does temperature profile look like for $U=0$?

Conservation of Mass for Differential Control Volume



Steady state:

$$\left[\rho u + \frac{\partial}{\partial x}(\rho u)\,dx - \rho u\right] dy \cdot 1 + \left[\rho v + \frac{\partial}{\partial y}(\rho v)\,dy - \rho v\right] dx \cdot 1 = 0$$

units? $\left(\frac{kg}{m^3}\,\frac{m}{s}\right)$ mass flux

$$\frac{\partial}{\partial x}(\rho u)\,dx\,dy + \frac{\partial}{\partial y}(\rho v)\,dy\,dx = 0$$

For constant ρ, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

For laminar, fully developed flow, $v=0$, $\frac{\partial u}{\partial x} = 0$

Integrate $\frac{\partial u}{\partial x} = 0$ $\quad u = f(y)$

u is at most a function of y.

Fully developed means flow velocity is independent of x.

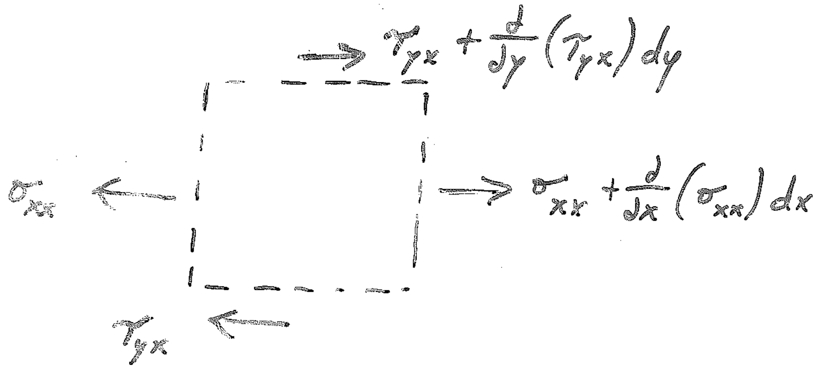
Newton's Second Law of Motion

Vector equation Sum of all forces acting on control volume must equal net rate at which momentum leaves control volume

For fully developed flow, rate at which momentum enters CV is equal to rate at which momentum leaves CV.

Sum of Forces = 0

Viscous Stresses (Normal & Shear Stresses) x-direction



No pressure gradient for Couette Flow

Net surface force in x-direction

$$F_{S,x} = \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}\right) dx\,dy$$

Body force in the x-direction is 0.

For Newtonian fluid,
$$\sigma_{xx} = 2\mu \cancel{\frac{\partial u}{\partial x}}^{0} - \frac{2}{3}\mu\left(\cancel{\frac{\partial u}{\partial x}}^{0} + \cancel{\frac{\partial v}{\partial y}}^{0}\right) = 0$$

$$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}^{0}\right) = \mu \frac{\partial u}{\partial y}$$