

3.11 Tension mbr., square with side h .

Required X and maximum ΔL .

First, look at mass and cost for given X .

$$\sigma = \frac{\sigma_c}{X} = \frac{P}{h^2}, \quad h^2 = \frac{PX}{\sigma_c} \quad \left(\begin{array}{l} h \text{ is geom.} \\ \text{that varies} \end{array} \right)$$

$$m = h^2 L \rho = \frac{PXL\rho}{\sigma_c} = [PXL] \left[\frac{\rho}{\sigma_c} \right] = f_1 f_2$$

Compare materials: $f_2 = \frac{\rho}{\sigma_c}$, $\frac{C_m \rho}{\sigma_c}$

| Material | Modulus E , GPa | Strength σ_c , MPa | Density ρ , g/cm ³ | Rel Cost C_m |
|------------|----------------------|------------------------------|---------------------------------------|-------------------|
| 1020 steel | 203 | 260 | 7.9 | 1 |
| 4340 steel | 207 | 1103 | 7.9 | 3 |
| 7075 Al | 71 | 469 | 2.7 | 6 |
| Ti-6-4 | 117 | 1185 | 4.5 | 45 |
| PC | 2.4 | 62 | 1.2 | 5 |
| Pine | 12.3 | 88 | 0.51 | 1.5 |
| GFRP | 21 | 380 | 2 | 10 |
| CFRP | 76 | 930 | 1.6 | 200 |

| Material | Mass f_2 ρ/σ_c | Mass Rank | Cost f_2 $C_m \rho/\sigma_c$ | Cost Rank |
|------------|-------------------------------|--------------|-----------------------------------|--------------|
| 1020 steel | 0.03038 | 8 | 0.0304 | 3 |
| 4340 steel | 0.00716 | 6 | 0.0215 | 2 |
| 7075 Al | 0.00576 | 4 | 0.0345 | 4 |
| Ti-6-4 | 0.00380 | 2 | 0.1709 | 7 |
| PC | 0.01935 | 7 | 0.0968 | 6 |
| Pine | 0.00580 | 5 | 0.0087 | 1 |
| GFRP | 0.00526 | 3 | 0.0526 | 5 |
| CFRP | 0.00172 | 1 | 0.3441 | 8 |

(3.11, p.2)

Second, look at mass and cost for given maximum ΔL . From Fig. A.1(a):

$$\Delta L = \frac{PL}{AE} = \frac{PL}{h^2 E}, \quad h^2 = \frac{PL}{\Delta L E}$$

h is again the geometry that varies.

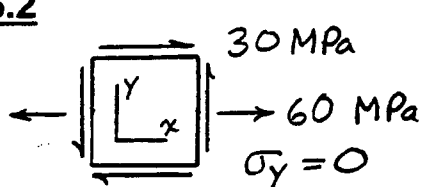
$$m = h^2 L \rho = \frac{PL^2 \rho}{\Delta L E} = \left[\frac{PL^2}{\Delta L} \right] \left[\frac{\rho}{E} \right] = f_1 f_2$$

Compare materials: $f_2 = \frac{\rho}{E}$, $\frac{C_m \rho}{E}$

| Material | Mass f_2 ρ/E | Mass Rank | Cost f_2 $C_m \rho/E$ | Cost Rank |
|------------|------------------------|--------------|----------------------------|--------------|
| 1020 steel | 0.0389 | 5 | 0.0389 | 1 |
| 4340 steel | 0.0382 | 3 | 0.1145 | 3 |
| 7075 Al | 0.0380 | 2 | 0.2282 | 4 |
| Ti-6-4 | 0.0385 | 4 | 1.7308 | 6 |
| PC | 0.5000 | 8 | 2.5000 | 7 |
| Pine | 0.0415 | 6 | 0.0622 | 2 |
| GFRP | 0.0952 | 7 | 0.9524 | 5 |
| CFRP | 0.0211 | 1 | 4.2105 | 8 |

If cost is unimportant, CFRP is the clear choice. 7075 Al is a good compromise considering cost, and is resistant to corrosion and rot.

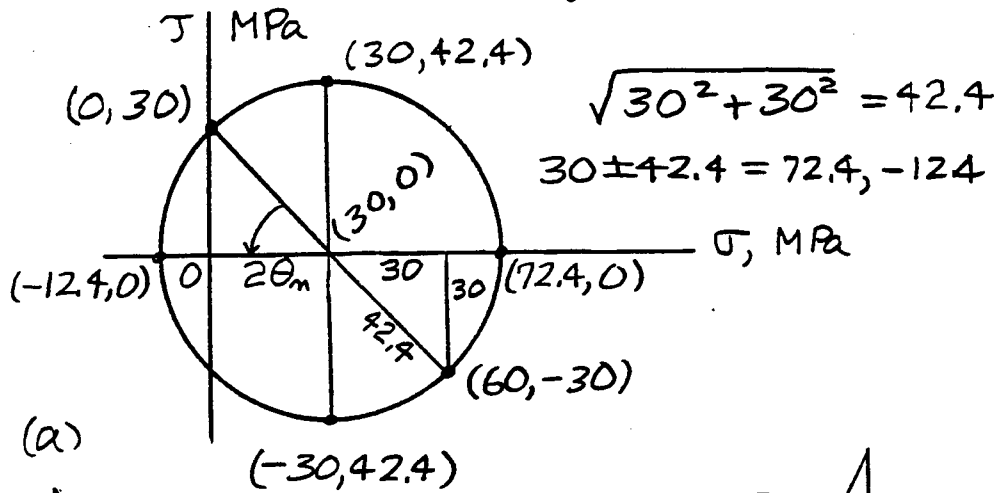
6.2



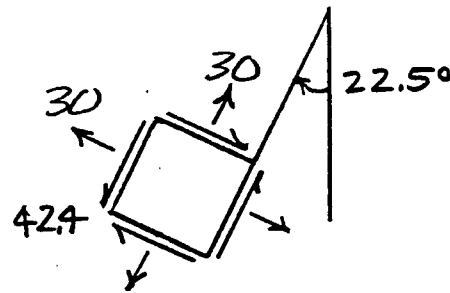
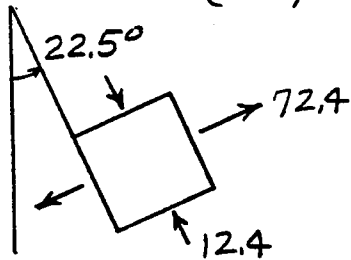
$$\tan 2\theta_m = \frac{30}{30}$$

$$\theta_m = 22.5^\circ \uparrow$$

$$\theta_s = 45 - \theta_m = 22.5^\circ \downarrow$$



(a)



$$\sigma_1, \sigma_2 = 72.4, -12.4 \text{ MPa}, \tau_3 = 42.4 \text{ MPa} \blacktriangleleft$$

(b) $\sigma_3 = 0$

$$\tau_{max} = \text{MAX} \left(\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2} \right)$$

$$\tau_{max} = \text{MAX} (42.4, 6.2, 36.2) = 42.4 \text{ MPa} \blacktriangleleft$$