5.12 Plane stress, τ_{xy} is nonzero, but

$\sigma_x = \sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$

From Eq. 5.27(a): $\gamma_{xy} = \dfrac{\tau_{xy}}{G}$

Fig. 5.10 gives principal strains,

$$\varepsilon_1 = \frac{\gamma_{xy}}{2} = \frac{\tau_{xy}}{2G} = -\varepsilon_2 \qquad \blacktriangleleft$$

The third one is

$$\varepsilon_3 = \varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = 0 \qquad \blacktriangleleft$$

Eq. 5.35 gives volumetric strain.

$$\varepsilon_v = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = 0 \qquad \blacktriangleleft$$

To rationalize this $\sigma_1 = \tau$ and $\sigma_2 = -\tau$ give cancelling Poisson strains in the thickness direction (z), and so there is no thickness change, $\varepsilon_3 = \varepsilon_z = 0$. Strain ε_1 considered alone gives a volume increase, but $\varepsilon_2 = -\varepsilon_1$ gives a decrease by the same amount, and with $\varepsilon_3 = 0$, we thus have $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$.

5.21 $\sigma_y = \lambda\sigma_x, \quad \varepsilon_z = 0$

(a) Yes

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E}(1+\lambda)$$

$$\sigma_z = \nu\sigma_x(1+\lambda) \quad \blacktriangleleft$$

(b) $\varepsilon_x E = \sigma_x - \nu(\sigma_y + \sigma_z)$

$$\varepsilon_x E = \sigma_x - \nu\lambda\sigma_x - \nu^2\sigma_x(1+\lambda)$$

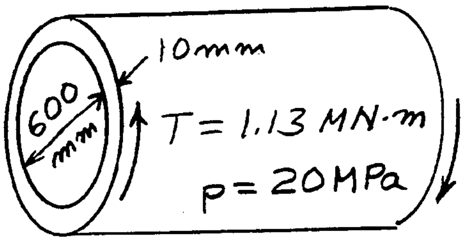
$$\frac{\sigma_x}{\varepsilon_x} = E' = \frac{E}{1-\nu\lambda-\nu^2(1+\lambda)} \quad \blacktriangleleft$$

λ	-1	0	$+1$
E'	$0.77E$	$1.10E$	$1.92E$

(c) ◀

The effect of λ on E' for $\nu = 0.3$ is substantial.

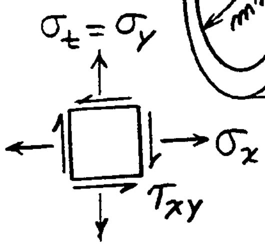
6.7



$r_1 = 300$ mm

$r_2 = 310$ mm

$r_{avg} = 305$ mm



$\sigma_1, \sigma_2, \sigma_3 = ?$

$\tau_{max} = ?$

Since $t/r_1 = 10/300 = 0.033$ is small, treat as a thin-walled tube. Combine τ_{xy}, max at r_2, and pressure stresses at r_1, as an approximate and conservative approach.

$$\sigma_y = \frac{p r_1}{t} = \frac{(20\ \text{MPa})(300\ \text{mm})}{10\ \text{mm}} = 600\ \text{MPa}$$

$$\sigma_x = \frac{p r_1}{2t} = 300\ \text{MPa}, \quad \sigma_z = -p = -20\ \text{MPa}$$

$$\tau_{xy} = \frac{T r_2}{J} = \frac{T r_2}{2\pi r_{avg}^3 t} = \frac{1.13\ \text{MN·m}\ (0.310\ \text{m})}{2\pi (0.305\ \text{m})^3\ 0.01\ \text{m}}$$

$$\tau_{xy} = 196.5\ \text{MPa}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = 450 \pm 247 = 697, 203\ \text{MPa}$$ ◀

$$\sigma_3 = \sigma_z = -20\ \text{MPa}$$ ◀

$$\tau_{max} = \text{MAX}\left(\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}\right)$$

$$\tau_{max} = 359\ \text{MPa}$$ ◀