ME 354
Hmwk 3 Solutions

4.12 Explain in your own words, without using equations the difference between engineering stress and true stress and true and the difference between engineering strain and true strain.

(10 pts)

Engineering stress is the ratio of the instantaneous applied force to the original, undeformed cross section. True stress is the ratio of the instantaneous applied force to the corresponding instantaneous deformed cross section.

Engineering strain is the summation of the incremental strains with respect to the fixed (and constant) original gage length. True strain is the summation of the incremental strains with respect of the instantaneous gage length.

4.18 For the initial portion of a tension test on 1100-O aluminum, a plot is shown in Fig. P4.18 of change in length of the gage section versus load. At the beginning of the test, the gage length was 50 mm and the diameter was 9.07 mm. During the large strain portion of the test, additional data as listed in Table P4.18 were obtained. Diameters were periodically measured, and once necking started, the particular diameter measured was the minimum in the neck. Compute true stresses and strains for both the initial and later portions of the test. Then plot true stress versus true strain on linear coordinates. Also, on the same graph, plot the engineering stress-strain curve. Comment on the comparison between the two curves.

(20 pts)

4.18 1100-O Al engineering and true stress-strain curves. Read several points from Fig. P4.18 and combine with the Table P4.18 data. For each point calculate $\sigma = P/A_i$ and $\varepsilon = \Delta L/L_i$.

L_i mm	d_i mm	A_i mm^2
50	9.07	64.61

Notes:
1 Ultimate
2 Fracture

P kN	ΔL mm	d mm	σ MPa	ε	True σ MPa	True ε	True σ_B MPa
4.4	0.05	~~~	68.1	0.0010	68.2	0.0010	68.2
6.8	0.1	~~~	105.2	0.0020	105.5	0.0020	105.5
8.15	0.2	~~~	126.1	0.0040	126.6	0.0040	126.6
8.55	0.4	~~~	132.3	0.0080	133.4	0.0080	133.4
9.12[1]	1.81	8.89	141.2	0.0362	146.9	0.0401	146.9
8.93	4.11	8.38	138.2	0.0822	161.9	0.1582	158.5
7.94	5.93	7.11	122.9	0.1186	200.0	0.4869	177.6
6.18	7.36	5.59	95.6	0.1472	251.8	0.9680	209.7
4.00[2]	8.50	4.06	61.9	0.1700	309.0	1.6076	244.6

Calculate true stress and strain:

$\tilde{\sigma} = \sigma(1+\varepsilon), \quad \tilde{\varepsilon} = \ln(1+\varepsilon) \qquad (\sigma < \sigma_u)$

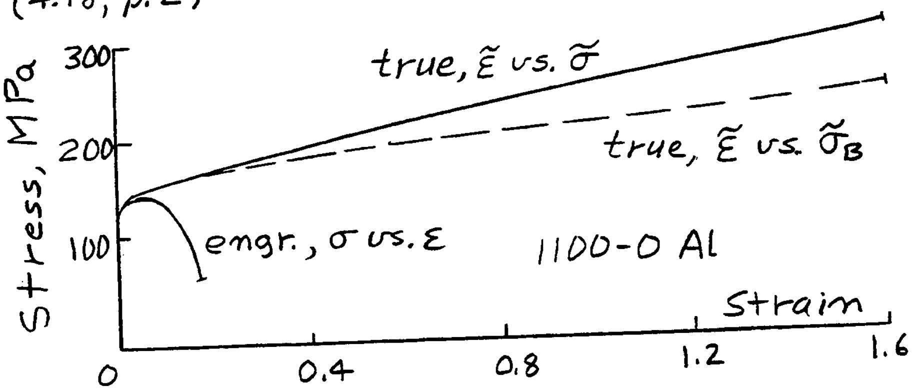
$\tilde{\sigma} = \frac{P}{A}, \quad A = \frac{\pi d^2}{4} \qquad (\sigma \geq \sigma_u;\ d \text{ known})$

$\tilde{\varepsilon} = \ln\frac{A_i}{A} = 2\ln\frac{d_i}{d} \qquad (\quad " \quad)$

$\tilde{\sigma}_B = \tilde{\sigma}(0.83 - 0.186 \log \tilde{\varepsilon}) \qquad (\tilde{\varepsilon} \geq 0.15)$

$\tilde{\sigma}_B = \tilde{\sigma} \qquad (\tilde{\varepsilon} < 0.15)$

(4.18, p. 2)



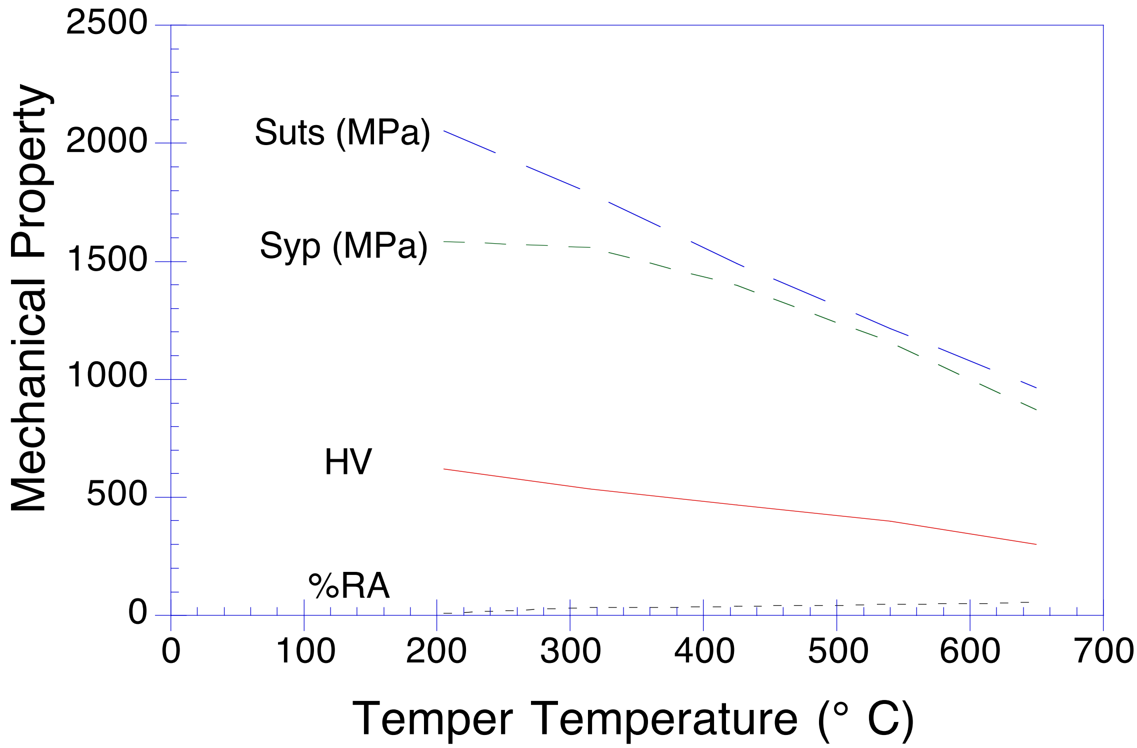
The true and engineering stress-strain curves differ drastically due to the large strains involved. The Bridgman correction, estimated from the equation for steels, also gives values $\tilde{\sigma}_B$ considerably below $\tilde{\varepsilon}$ at the larger strains.

4.29 Vickers hardness and tensile data are listed in Table P4.29 for AISI 4140 steel that has been heat treated to various strength levels by varying the tempering temperature. Plot the hardness and the various tensile properties all as a function of tempering temperature. Then discuss the trends observed. How do the various tensile properties vary with hardness?

TABLE P4.29

Temper, °C	205	315	425	540	650
Hardness, *HV*	619	535	468	399	300
Ultimate, σ_u, MPa	2053	1789	1491	1216	963
Yield, σ_o, MPa	1583	1560	1399	1158	872
Red. in Area, %*RA*	7	33	38	48	55

(10 pts)



As tempering temperature is increased the strengths and hardness decrease but ductility increases. These treads are consistent with the greater removal of carbon from the martensite and grater formation of dispersed cementite at higher temperatures. Decreasing martensite leads to decreasing strength and hardness.

Tensile strength decreases with decreasing hardness, but %RA (ductility) increases with decreasing hardness.

4.30 Explain in your own words why notch-impact fracture tests are widely used for metals and also why caution is needed in applying the results to real engineering siutations.

(10 pts)

Notch impact tests are of interest because they provide information (e.g., strain rate, notch sensitivity and temperature sensitivity) not available from other mechanical tests. In addition, notch impact tests can be quick and inexpensive.

Unfortunately, the notch impact energies obtained are test specimen specific (size, geometry, notch tip radius) and test machine specific (e.g., support, loading configuration, mass and velocity of the pendulum). Thus, notch impact energy is a "qualitative" measure of mechanical performance because it is specific to the test conditions and is therefore not a "quantitative" mechanical property such as ultimate tensile strength.

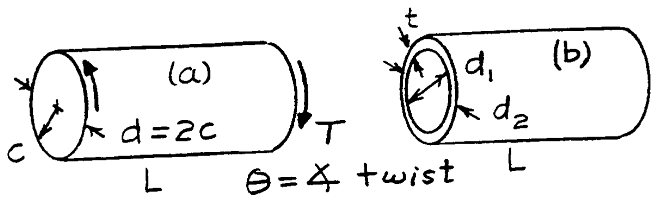
4.35 A solid circular shaft of length L and diameter d is to be used in materials testing in torsion.

(a) Develop equations for calculating the shear fracture stress τ_f and shear modulus G from torque T and angle of twist θ. What are the limitations of your equations?

(b) Modify these equations for specimens that are hollow tubes of inner diameter d_1 and outer diameter d_2. Can you think of any advantages or disadvantages of a tubular versus a solid torsion specimen?

(15)

4.35



(a) Develop equations for τ_f and G.

$$\tau_f = \frac{T_f c}{J} = \frac{T_f \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16\, T_f}{\pi d^3}$$

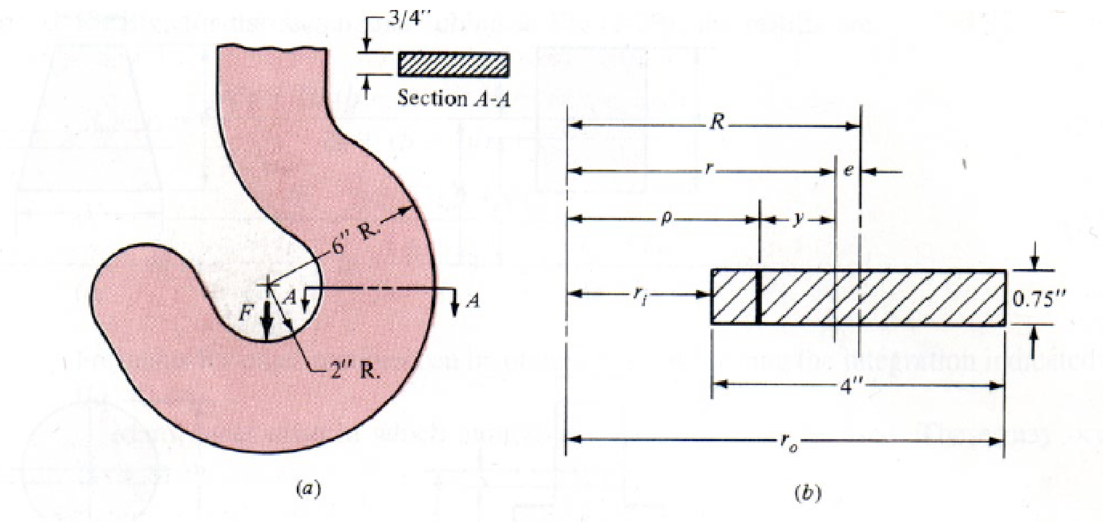
$$\theta = \frac{TL}{JG} = \frac{TL}{\frac{\pi d^4}{32} G}, \quad G = \frac{32L}{\pi d^4}\left(\frac{dT}{d\theta}\right)$$

(b) $J = \frac{\pi}{32}(d_2^4 - d_1^4)$

$$\tau_f = \frac{T_f c}{J} = \frac{T_f \frac{d_2}{2}}{\frac{\pi}{32}(d_2^4 - d_1^4)} = \frac{16\, T_f\, d_2}{\pi (d_2^4 - d_1^4)}$$

$$\theta = \frac{TL}{JG}, \quad G = \frac{32L}{\pi (d_2^4 - d_1^4)}\left(\frac{dT}{d\theta}\right)$$

In a thin-walled tube, τ is nearly uniform through the wall, so there is minimal error in τ_f even if nonlinear behavior occurs. However, for a solid member as in (a), yielding can occur at substantial depth, making $\tau = Tc/J$ quite inaccurate. (See Section 13.4 for more general analysis methods.)



In this case, r_o=6 in, r_i=2 in, h=4 in, b=0.75 in and F=5000 lbf

To use the formula for curved beam (see table), we need to know the radius of the neutral axis, r_n and the radius of the centroid, R, the moment, M, the cross sectional area, A, and the distance from the neutral axis, y. The equations for these parameters are given in the table.

Table Some Useful Equations for Curved Beams with Rectangular Cross Sections

Bending Stress	N/A radius	Centroid radius	Distance fromN/A	Moment	Area
$\sigma = \dfrac{My}{Ae(r_n - y)}$	$r_n = \dfrac{h}{\ln(r_o / r_i)}$	$R=(r_o+r_i)/2$	$y=(r_n-r)$	$M=F \bullet R$	$A=bh$

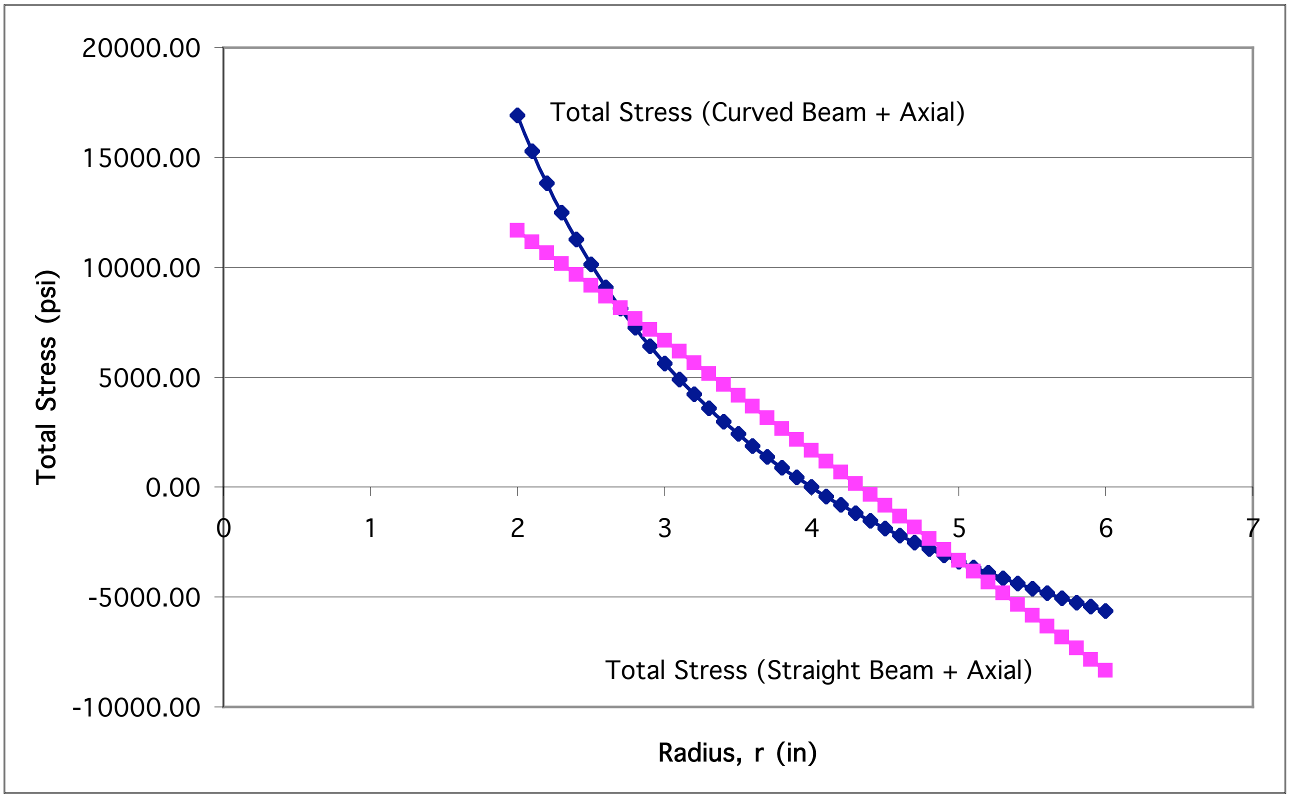
Substituting values into these equations gives r_n=3.64 in, R=4.0 in, M=20,000 lbf-in, and A=3 in^2.

For part a, a table of values constructed in spreadsheet with the curved beam bending stress and axial stress as functions of the variable r. The plot of total stress (curved and axial) is plotted as a function of radius along with the total stress (straight and axial).

For part b, stresses are as follows. Note that for this case R/h=1.5 which is less than 5 meaning that the curved beam equations definitely need to be used.

Analysis Type	Bending Stress at inner radius, r_i (psi)	Bending Stress at outer radius,r_o (psi)
Curved Beam	15, 235	+10,000
Straight Beam	-7,300	-10,000

For part c, the N/A shifts toward the center of curvature. This is consistent with the equations and the idea that the radius of curvature decreases as on moves toward the center of curvature.


Total Stress (psi)
20000.00
15000.00
10000.00
5000.00
0.00
-5000.00
-10000.00
0
1
2
3
4
5
6
7
Total Stress (Curved Beam + Axial)
Total Stress (Straight Beam + Axial)
Radius, r (in)

	r (mm)	y (mm)	Curve Beam sig bend (psi)	sig axial (psi)	Total Stress (psi)	Straight Beam sig bend (psi)	sig axial (psi)	Total Stress (psi)
ri	2	1.64	15234.54	1666.67	16901.21	10000	1666.67	11666.67
	2.1	1.54	13624.90	1666.67	15291.57	9500	1666.67	11166.67
	2.2	1.44	12161.59	1666.67	13828.26	9000	1666.67	10666.67
	2.3	1.34	10825.53	1666.67	12492.20	8500	1666.67	10166.67
	2.4	1.24	9600.80	1666.67	11267.47	8000	1666.67	9666.67
	2.5	1.14	8474.06	1666.67	10140.72	7500	1666.67	9166.67
	2.6	1.04	7433.98	1666.67	9100.65	7000	1666.67	8666.67
	2.7	0.94	6470.95	1666.67	8137.62	6500	1666.67	8166.67
	2.8	0.84	5576.71	1666.67	7243.37	6000	1666.67	7666.67
	2.9	0.74	4744.14	1666.67	6410.80	5500	1666.67	7166.67
	3	0.64	3967.07	1666.67	5633.74	5000	1666.67	6666.67
	3.1	0.54	3240.14	1666.67	4906.80	4500	1666.67	6166.67
	3.2	0.44	2558.63	1666.67	4225.30	4000	1666.67	5666.67
	3.3	0.34	1918.44	1666.67	3585.10	3500	1666.67	5166.67
	3.4	0.24	1315.90	1666.67	2982.57	3000	1666.67	4666.67
	3.5	0.14	747.79	1666.67	2414.46	2500	1666.67	4166.67
	3.6	0.04	211.25	1666.67	1877.91	2000	1666.67	3666.67
	3.7	-0.06	-296.30	1666.67	1370.37	1500	1666.67	3166.67
	3.8	-0.16	-777.13	1666.67	889.54	1000	1666.67	2666.67
	3.9	-0.26	-1233.30	1666.67	433.36	500	1666.67	2166.67
	4	-0.36	-1666.67	1666.67	0.00	-8.8818E-12	1666.67	1666.67
	4.1	-0.46	-2078.89	1666.67	-412.22	-500	1666.67	1166.67
	4.2	-0.56	-2471.49	1666.67	-804.82	-1000	1666.67	666.67
	4.3	-0.66	-2845.82	1666.67	-1179.15	-1500	1666.67	166.67
	4.4	-0.76	-3203.14	1666.67	-1536.47	-2000	1666.67	-333.33
	4.5	-0.86	-3544.58	1666.67	-1877.91	-2500	1666.67	-833.33
	4.6	-0.96	-3871.17	1666.67	-2204.51	-3000	1666.67	-1333.33
	4.7	-1.06	-4183.87	1666.67	-2517.20	-3500	1666.67	-1833.33
	4.8	-1.16	-4483.53	1666.67	-2816.87	-4000	1666.67	-2333.33
	4.9	-1.26	-4770.97	1666.67	-3104.30	-4500	1666.67	-2833.33
	5	-1.36	-5046.91	1666.67	-3380.24	-5000	1666.67	-3333.33
	5.1	-1.46	-5312.02	1666.67	-3645.36	-5500	1666.67	-3833.33
	5.2	-1.56	-5566.95	1666.67	-3900.28	-6000	1666.67	-4333.33
	5.3	-1.66	-5812.25	1666.67	-4145.58	-6500	1666.67	-4833.33
	5.4	-1.76	-6048.46	1666.67	-4381.79	-7000	1666.67	-5333.33
	5.5	-1.86	-6276.09	1666.67	-4609.42	-7500	1666.67	-5833.33
	5.6	-1.96	-6495.58	1666.67	-4828.92	-8000	1666.67	-6333.33
	5.7	-2.06	-6707.38	1666.67	-5040.71	-8500	1666.67	-6833.33
	5.8	-2.16	-6911.87	1666.67	-5245.20	-9000	1666.67	-7333.33
	5.9	-2.26	-7109.43	1666.67	-5442.76	-9500	1666.67	-7833.33
ro	6	-2.36	-7300.40	1666.67	-5633.74	-10000	1666.67	-8333.33