ME 354
Hmwk 4 Solutions

7.1 The most severely stressed point in an engineering component occurs on a free surface, where the stresses are $\sigma_x = 60$, $\sigma_y = 0$, and $\tau_{xy} = 30$ MPa. What is the safety factor against failure at this point if:

(a) The material is AISI 1020 steel (as rolled) and yielding is considered failure?

(b) The material is the ceramic silicon carbide (SiC) and fracture is considered failure?

(20 pts)

7.1 $\sigma_x = 60, \quad \sigma_y = 0, \quad \tau_{xy} = 30$

(a) $X_o = ?$ if AISI 1020 steel, $\sigma_o = 260$ MPa

(b) $X_u = ?$ if SiC, $\sigma_{ut} = 307$ MPa

(Tables 3.10 and 4.2)

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = 30 \pm 42.43 = 72.43, \ -12.43 \text{ MPa}$$

$$\sigma_3 = \sigma_z = 0$$

(a) $\bar{\sigma}_S = MAX(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|)$

$\bar{\sigma}_S = 84.85$ MPa

$X_o = \sigma_o / \bar{\sigma}_S = 260/84.85 = 3.06$ ◀

(b) $\bar{\sigma}_N = MAX(|\sigma_1|, |\sigma_2|, |\sigma_3|) = 72.43$ MPa

$X_u = \sigma_{ut} / \bar{\sigma}_N = 307/72.43 = 4.24$ ◀

7.9 Consider a solid circular shaft subjected to bending and torsion so that the state of stress of interest involves only a normal stress σ_x and a shear stress τ_{xy}, with all other stress components being zero. (See Figure P7.9.)

(a) Determine the safety factor against yielding as a function of the yield strength of the ductile material and the applied stresses.

(b) Use the results of (a) to develop a design equation for the shaft giving diameter d as a function of yield strength, safety factor, bending moment M, and torque T.

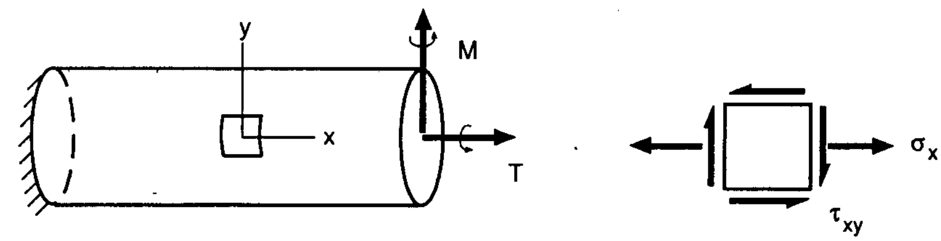
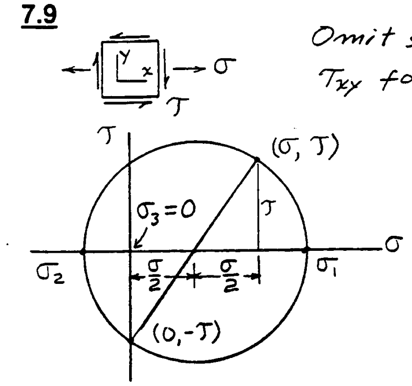


Figure P7.9

(20 pts)

7.9



Omit subscripts on σ_x and τ_{xy} for brevity.

$$\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\tau^2 + \left(\frac{\sigma}{2}\right)^2}$$

$r = d/2$, d = diam.

(a) $X = \dfrac{\sigma_o}{\bar{\sigma}_S}$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right) = |\sigma_1 - \sigma_2|$$

$$\bar{\sigma}_S = \left(\frac{\sigma}{2} + \sqrt{\tau^2 + \left(\frac{\sigma}{2}\right)^2}\right) - \left(\frac{\sigma}{2} - \sqrt{\tau^2 + \left(\frac{\sigma}{2}\right)^2}\right)$$

$$\bar{\sigma}_S = \sqrt{4\tau^2 + \sigma^2}$$

$$X = \frac{\sigma_o}{\sqrt{4\tau^2 + \sigma^2}} \quad \blacktriangleleft$$

(b) $$\sigma = \frac{Mr}{I} = \frac{Mr}{\pi r^4/4} = \frac{4M}{\pi r^3} = \frac{4M}{\pi\left(\frac{d}{2}\right)^3} = \frac{32M}{\pi d^3}$$

$$\tau = \frac{Tr}{J} = \frac{Tr}{\pi r^4/2} = \frac{2T}{\pi r^3} = \frac{16T}{\pi d^3}$$

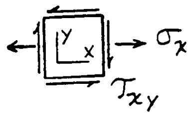
$$\sigma_o = X\sqrt{4\tau^2 + \sigma^2} = X\sqrt{4\left(\frac{16T}{\pi d^3}\right)^2 + \left(\frac{32M}{\pi d^3}\right)^2}$$

$$\sigma_o = \frac{32X}{\pi d^3}\sqrt{T^2 + M^2}, \quad d = \left(\frac{32X}{\pi\sigma_o}\sqrt{T^2 + M^2}\right)^{1/3} \quad \blacktriangleleft$$

(7.9, p.2) Second solution:

r = radius

d = diam.



$$\sigma_x = \frac{Mr}{I} = \frac{32M}{\pi d^3} = \sigma$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \tau$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{\sigma_x^2+\sigma_x^2+6\tau_{xy}^2} = \sqrt{\sigma^2+3\tau^2} = \frac{\sigma_o}{X}$$

$$X = \frac{\sigma_o}{\sqrt{\sigma^2+3\tau^2}} \quad \blacktriangleleft$$

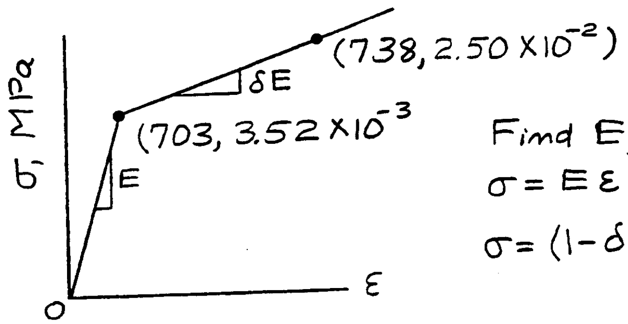
$$\sigma_o = X\sqrt{\sigma^2+3\tau^2} = X\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 3\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\sigma_o = \frac{32X}{\pi d^3}\sqrt{M^2+\frac{3}{4}T^2}, \quad d = \left(\frac{32X}{\pi\sigma_o}\sqrt{M^2+\frac{3T^2}{4}}\right)^{1/3} \quad \blacktriangleleft$$

12.1 The monotonic stress-strain curve of RQC-100 steel under uniaxial stress can be approximated by an elastic, linear-hardening relationship. Two points on this curve are given in Table P12.1, with the first point corresponding to the beginning of yielding. Plot the curve and write its equation in the form of Eq. 12.4, with numerical values substituted for all constants.

(20 pts)

12.1 *Monotonic σ-ε curve, RQC-100 steel*



Find E, σ_o, δ for:

$\sigma = E\varepsilon \quad (\sigma \leq \sigma_o)$

$\sigma = (1-\delta)\sigma_o + \delta E \varepsilon \quad (\sigma \geq \sigma_o)$

$\sigma_o = 703 \text{ MPa}$, $E = \dfrac{\sigma_o}{\varepsilon_o} = \dfrac{703 \text{ MPa}}{3.52 \times 10^{-3}} = 199{,}700 \text{ MPa}$

$$\delta E = \frac{\sigma - \sigma_o}{\varepsilon - \varepsilon_o} = \frac{(738-703)\text{ MPa}}{2.5\times10^{-2} - 3.52\times10^{-3}} = 1629 \text{ MPa} \quad ◀$$

$\delta = 0.00816$

$$\sigma = 0.99184(703) + 1629\varepsilon = 697 + 1629\varepsilon \quad ◀$$

$(\text{MPa}, \sigma \geq \sigma_o)$

8.1 Look ahead to Fig. 8.31, and:

(a) Obtain approximate values of fracture toughness K_{Ic} for AISI 4340 steel heat treated to yield strengths of 800 and 1600 MPa.

(b) For each of these yield strengths, calculate the transition crack length a_t, and comment on the significance of the values obtained.

(20 pts)

8.1 AISI 4340 steel (Fig. 8.31), $\sigma_o = 800$ and 1600 MPa.

(a) $K_{Ic} = ?$ (b) $a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2$

σ_o, MPa	K_{Ic}, MPa$\sqrt{m}$	a_t, mm
800	185	17.0
1600	40	0.20

The much smaller a_t for the higher σ_o indicates a greater sensitivity to flaws, so that brittle fracture would be an important design consideration.

8.3 For each metal in Table 8.1:

(a) Calculate the transition crack length a_t.

(b) Plot these as data points on a logarithmic scale, versus yield strength σ_o on a linear scale, using different symbols for steels, aluminum alloys, and titanium alloys.

(c) Comment on the values obtained and on any trends with yield strength.

8.3 For each metal in Table 8.1, calculate a_t and plot on log scale vs. σ_o.

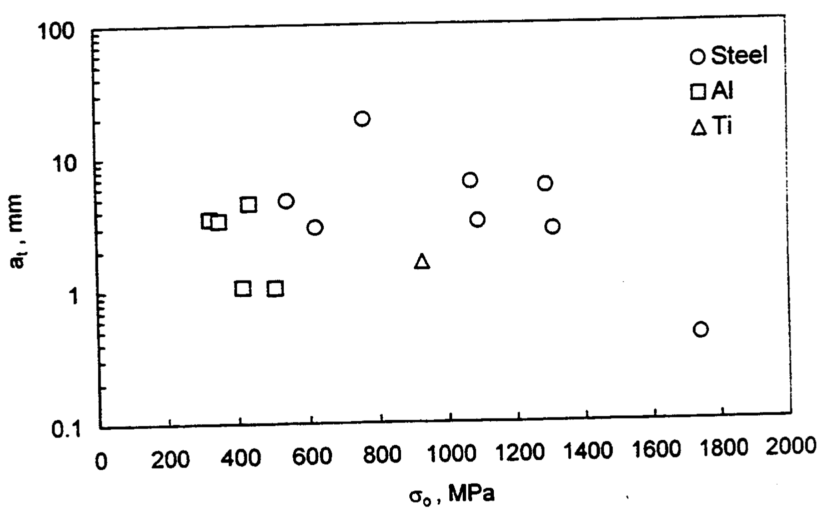
$$a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2 = \frac{1}{\pi}\left(\frac{66\ \mathrm{MPa}\sqrt{m}}{540\ \mathrm{MPa}}\right)^2$$

$$a_t = 4.75 \times 10^{-3}\ m = 4.75\ mm$$

(for AISI 1144 steel, others similarly)

K_{Ic} MPa $m^{0.5}$	σ_o MPa	a_t mm
(a) Steels		
66	540	4.75
60	620	2.98
187	760	19.27
110	1090	3.24
123	1310	2.81
176	1290	5.93
152	1070	6.42
65	1740	0.44

K_{Ic} MPa $m^{0.5}$	σ_o MPa	a_t mm
(b) Aluminum Alloys		
24	415	1.06
34	325	3.48
36	350	3.37
29	505	1.05
52	435	4.55
(c) Titanium Alloy		
66	925	1.62



Although there is considerable scatter, it appears that a_t decreases with increasing σ_o for steels. Aluminum alloys show a similar trend, but with a_t smaller than for steels of similar strength.