ME 354
Hmwk 5 Solutions

9.4 Hot-rolled and normalized AISI 1045 steel can be assumed to have an S-N curve of the form of Eq. 9.6. Some test data for unnotched specimens under axial stress with zero mean are given in Table P9.4.

(a) Plot these data on log-log coordinates and use the graph to obtain approximate values for the constants A and B.

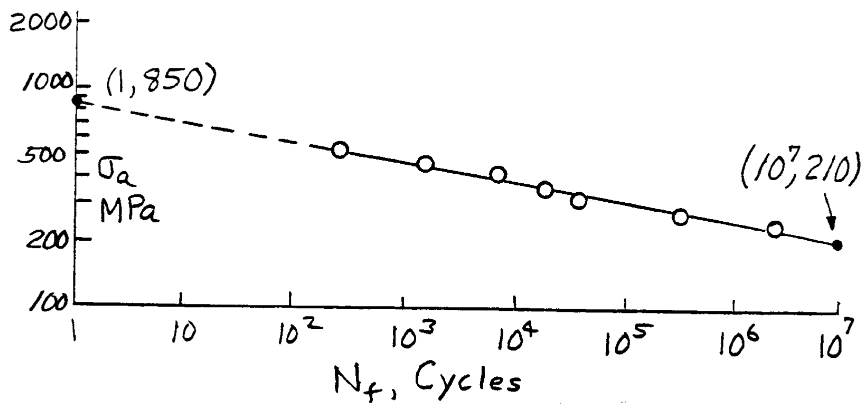
(b) Obtain refined values for A and B using a linear least-squares fit to $\log N_f$ versus $\log \sigma_a$.

(c) Calculate corresponding values of σ_f' and b for Eq. 9.7 from one of your results above, preferably from (b).

(20 pts)

9.4 $\sigma_a = A N_f^{B}$ HR & Norm AISI 1045

(a) Plot data on log-log coordinates and draw a straight line through the data.



$A = 850$ is σ_a intercept at $N_f = 1$.
Calculate B from a second pt. on line.

$$\frac{\sigma_a}{A} = N_f^{B}, \quad B = \frac{\log(\sigma_a/A)}{\log N_f}$$

$$B = \frac{\log(210/850)}{\log(10^7)} = -0.0867$$

$$\sigma_a = 850(N_f)^{-0.0867} \text{ MPa}$$

(b) Do a least-squares fit; N_f dependent

$$N_f = \left(\frac{\sigma_a}{A}\right)^{1/B}$$

(9.4, p. 2)

$$\underbrace{\log N_f}_{y} = \frac{1}{B}\underbrace{\log \sigma_a}_{} + \underbrace{\log \frac{1}{A^{1/B}}}_{}$$

$$= m \quad x \quad + \quad c$$

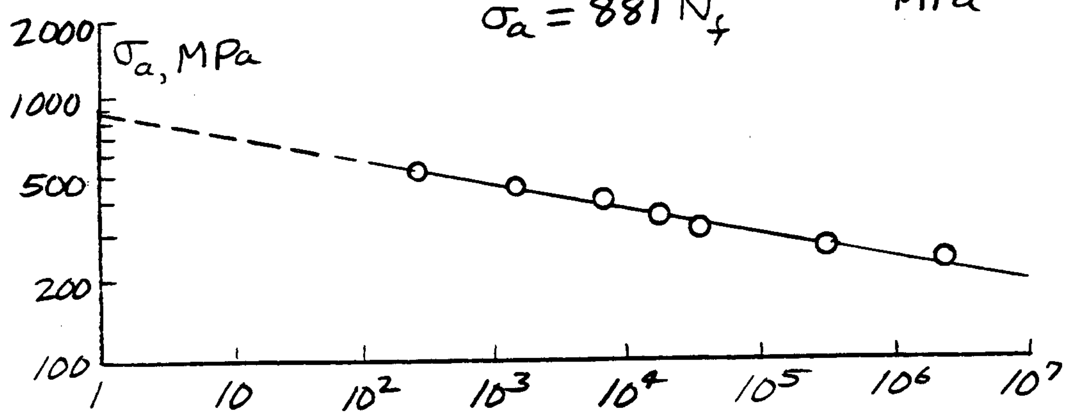
The resulting constants are

$m = -10.916$, $B = 1/m = -0.09161$

$c = 32.15 = \log \frac{1}{A^{1/B}}$

$10^{-cB} = A = 881$ MPa

$\sigma_a = 881\, N_f^{-0.0916}$ MPa ◀



(c) $\sigma_f' = A/2^b = 881/2^{-0.09161} = 939$ MPa

$b = B = -0.0916$

$\sigma_a = 939\,(2N_f)^{-0.0916}$ MPa ◀

9.8 Describe the likely sources of cyclic loading for a sailboat rudder. Consider static loads, working loads, vibratory loads and accidental loads.

(10 pts)

See Figure 9.9 for examples of these types of loads for an aircraft.

For the rudder, static loads could be the weight and bouyancy of the rudder itself. Working loads could arise from turns associated with tacking. Vibratory loads come from high speed sailing and interactions with the waves. Accidental loads could come from sudden hard over or strking the bottom.

10.1 Define the following terms in your own words: a) elastic stress concentration factor, kt; b) stress intensity factor, K; c) fatigue notch factor, kf. and d) notch sensitivity factor, q. You may use equations to supplement but not replace your word definitions.

(10 pts)

a) elastic stress concentration factor is the ratio of the local stress to the remote stress

b) stress intensity factor is stress/geometry based term that uniquely defines the stress state at a crack tip.

c) fatigue notch factor is ratio of the endurance limit for an unnotched test piece and the endurance limit for a notched test piece of the same material

d) the notch sensitivity factor is a combination of material properties and geometry

10.5 A shaft with a step-down in diameter has dimensions, as defined in Fig. A.9(b), of $d_1 = 50$, $d_2 = 55$, and $\rho = 1.3$ mm. The shaft is subjected to bending and is made of a quenched and tempered low-alloy steel having an ultimate tensile strength of 1100 MPa. This steel has a completely reversed ($\sigma_m = 0$), smooth specimen, fatigue limit of $\sigma_e = 500$ MPa.

(a) Determine k_t and then estimate k_f.

(b) What completely reversed bending moment amplitude M_a can be applied to the notched shaft for 10^6 cycles before fatigue failure is expected? A safety factor of 1.8 on stress is required.

(20 pts)

10.5 Shaft with diameter step, in bending.

Fig. A.9(b): $d_1 = 50$, $d_2 = 55$, $\rho = 1.3$ mm

$\sigma_u = 1100$ MPa, low-alloy Q & T steel

(a) Find k_t, k_f $\quad \sigma_e = 500$ MPa,

$d_2/d_1 = 1.1$, $\rho/d_1 = 0.026$, $k_t = 2.1$

$$\alpha = 0.025\left(\frac{2070 \text{ MPa}}{\sigma_u}\right)^{1.8} = 0.078 \text{ mm}$$

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho} = 2.04$$

(b) $\sigma_{ar} = 500$ MPa at $N_f = 10^6$, $X_S = 1.8$

$M_a = ?$ for $N_f = 10^6$, with $M_m = 0$

$$S_{ar} = \frac{\sigma_{ar}}{k_f} = \frac{32 M_a}{\pi d_1^3}, \quad M_a = \frac{\pi d_1^3 \sigma_{ar}}{32 k_f}$$

$$M_a = \frac{\pi (0.05 \text{ m})^3 \, 500 \text{ MN/m}^2}{32 (2.04)} \times \frac{10^6 \text{ N}}{\text{MN}} = 3008 \text{ N·m}$$

Design allowable value: $\hat{M}_a = \dfrac{M_a}{X_S} = 1671$ N·m

11.20 A center-cracked plate made of 2024-T3 aluminum has dimensions, as defined in Fig. 8.12(a), of $b = 50$, $t = 4$ mm, and large h, and an initial crack length of $a_i = 2$ mm. How many cycles of loading in tension between $P_{min} = 18$ and $P_{max} = 60$ kN are required to grow the crack to failure by either fully plastic yielding or brittle fracture?

(20 pts)

11.20 2024-T3 Al center-cracked plate,

$b = 50$, $t = 4$, $a_i = 2$ mm, $N_{if} = ?$ for

$P_{min} = 18$, $P_{max} = 60$ kN. $a_f = a_c$ or a_o.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\Delta S\sqrt{\pi})^m(1-m/2)} \qquad (F \text{ constant})$$

Table 11.2: $\sigma_o = 353$ MPa, $K_{Ic} = 34$ MPa$\sqrt{m}$

$C_1 = 1.42 \times 10^{-11} \frac{m/cyc}{(MPa\sqrt{m})^m}$, $m_1 = 3.59$, $\gamma = 0.68$

$$S_{max} = \frac{P_{max}}{2bt} = \frac{60{,}000\text{ N}}{2(50)(4)\text{ mm}^2} = 150 \text{ MPa}$$

$$\Delta S = \frac{\Delta P}{2bt} = \frac{(60-18)1000}{2(50)(4)} = 105 \text{ MPa}$$

$R = S_{min}/S_{max} = P_{min}/P_{max} = 18/60 = 0.30$

Assume $F = 1.00$ ($a/b < 0.4$, Fig 8.12(a))

$$a_c = \frac{1}{\pi}\left(\frac{K_c}{FS_{max}}\right)^2, \quad \text{use } K_c = K_{Ic} = 34 \text{ MPa}\sqrt{m}$$

$$a_c = \frac{1}{\pi}\left(\frac{34\text{ MPa}\sqrt{m}}{(1)(150\text{ MPa})}\right)^2 = 0.01635\text{ m} = 16.35\text{ mm}$$

$a_c/b = 16.35/50 = 0.327 < 0.4$

The a_c calculation is OK and $F \approx 1.0$.

$$a_o = b\left(1 - \frac{P_{max}}{2bt\sigma_o}\right) \qquad (\text{Fig. A.13(a)})$$

(11.20, p. 2)

$$a_o = (50\ mm)\left[1 - \frac{60{,}000\ N}{(2)(50\ mm)(4\ mm)(353\ MPa)}\right]$$

$a_o = 28.75\ mm$

$a_o > a_c$, a_c controls, $a_f = 16.35\ mm$ ◁

For $R = 0.3$, C for $da/dN = C(\Delta K)^m$ is

$$C = \frac{C_1}{(1-R)^{m_1(1-\gamma)}}, \quad m_1 = m = 3.59$$

$$C = \frac{1.42 \times 10^{-11}}{(1-0.3)^{3.59(1-0.68)}} = 2.139 \times 10^{-11}\ \frac{m/cyc}{(MPa\sqrt{m})^m}$$

$1 - m/2 = -0.795$

Substitute into N_{if} eqn. with units of m, MPa, and $MPa\sqrt{m}$, as in Ex. 11.4.

$$N_{if} = \frac{0.01635^{-0.795} - 0.002^{-0.795}}{2.139 \times 10^{-11}\,(1 \times 105\sqrt{\pi})^{3.59}\,(-0.795)}$$

$N_{if} = 47{,}400$ cycles ◀

15.2 A 40% tin, 60% lead alloy solder wire of diameter 3.15 mm is subjected to creep by hanging weights from lengths of the wire. Length changes measured over a 254 mm gage length after various elapsed times are given in Table P15.2 for three different weights.

(a) Plot the family of strain vs. time curves that results.

(b) Characterize the behavior. Is it dominated by either transient or steady-state creep, or do significant amounts of both occur?

(c) Determine steady-state creep rates, $\dot{\varepsilon}_{sc}$, for each value of weight and plot these on log-log coordinates versus the corresponding stresses. Does a straight line provide a reasonable fit to the data? If so, find values of constants B and m for the relationship $\dot{\varepsilon}_{sc} = B\sigma^m$.

TABLE P15.2

	Length change, mm		
Time, min	4.54 kg	6.80 kg	9.07 kg
0	0	0	0
0.25	0.28	0.46	0.69
0.5	0.36	0.66	0.94
1	0.48	0.91	1.45
2	0.71	1.40	2.36
4	1.09	2.24	4.09
6	1.47	3.00	5.72
8	1.83	3.38	7.26
12	2.54	4.90	10.41
16	3.23	6.38	13.64
20	3.91	7.82	16.74

(20pts)

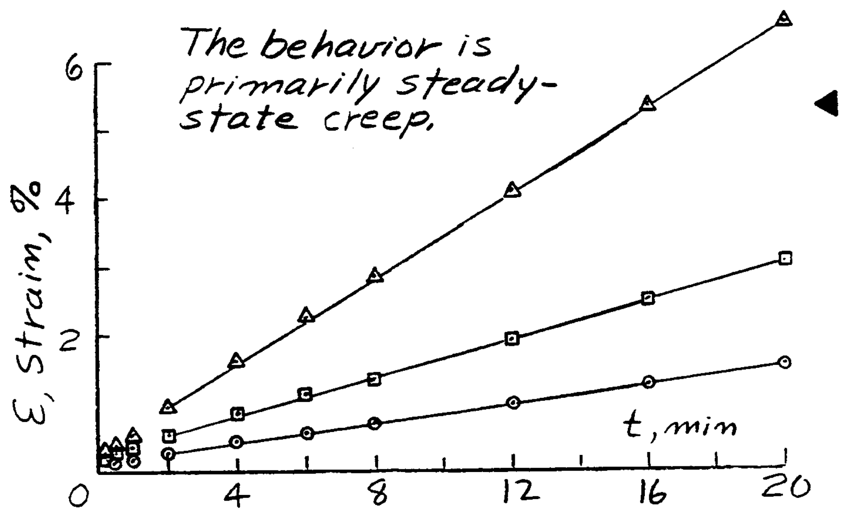
15.2 Solder wire, $d = 3.15\,mm$, with $L_o = 254\,mm$

(a) Plot ε vs. t, and (b) comment.

$$\varepsilon = \frac{\Delta L}{L_o}, \quad \sigma = \frac{P}{A} = \frac{mg}{\pi d^2/4} = \frac{(m, kg)\,9.807\,\frac{m}{s^2}}{\pi\,(3.15\,mm)^2/4}$$

$$\sigma, \frac{N}{mm^2} = 1.258\,m = 5.71,\ 8.56,\ 11.41\ MPa$$

	$\varepsilon = \Delta L/L_o$, $10^{-2}\,\frac{m}{m} = \%$		
t, min	$\sigma = 5.71$	8.56	11.41 MPa
0.25	0.110	0.181	0.272
0.5	0.142	0.260	0.370
1	0.189	0.358	0.571
2	0.280	0.551	0.929
4	0.429	0.882	1.610
6	0.579	1.181	2.25
8	0.720	1.331	2.86
12	1.000	1.929	4.10
16	1.272	2.51	5.37
20	1.539	3.08	6.59



(15.2, p.2)

(b) Evaluate the $\dot{\varepsilon}_{sc}$, and fit $\dot{\varepsilon}_{sc} = B\sigma^m$

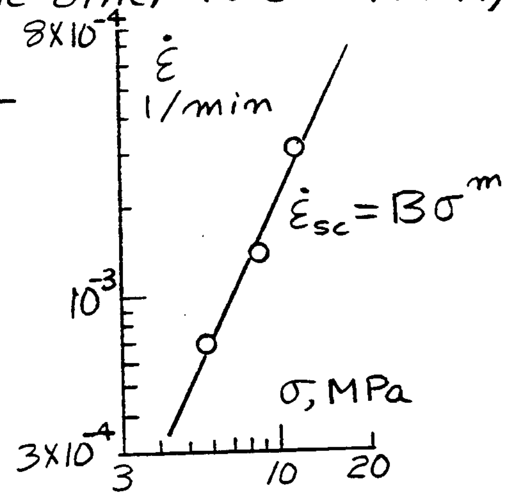
As an approximate procedure, find slopes of the straight lines shown between the data points at t = 2 and 20 min.

$$\dot{\varepsilon}_{sc} = \frac{\varepsilon_{20} - \varepsilon_2}{t_{20} - t_2} = \frac{(1.539 - 0.280) \times 10^{-2}}{20 - 2} = 6.99 \times 10^{-4} \ \text{1/min}$$

for $\sigma = 5.71$ MPa, the other two similarly.

σ, MPa	$\dot{\varepsilon}_{sc}$, 1/min
5.71	6.99×10^{-4}
8.56	1.405×10^{-3}
11.41	3.15×10^{-3}

A straight line fit is reasonable.



$$\underbrace{\log \dot{\varepsilon}_{sc}}_{y} = \underbrace{\log B}_{b} + m \underbrace{\log \sigma}_{x}$$

A least squares fit gives: $m = 2.145$
$b = -4.8004$, $B = 10^b = 1.584 \times 10^{-5}$

$$\dot{\varepsilon}_{sc} = 1.584 \times 10^{-5} \ \sigma^{2.145}$$ ◀