8.5 A center-cracked plate of AISI 1144 steel has dimensions, as defined in Fig. 8.12(a), of $b = 40$ mm and $t = 15$ mm. For a safety factor of three against brittle fracture, what is the maximum permissible load on the plate if the crack half-length a is (a) 10 mm, and (b) 24 mm?

(20 pts)

8.5 Center-cracked plate, AISI 1144 steel, $b = 40$, $t = 15$ mm. $K_{Ic} = 66\ MPa\sqrt{m}$ (Tbl. 8.1)

$P = ?$ for $X = P_c/P = 3$ if $a = 10, 24$ mm

(a) $a = 10$ mm, $\alpha = a/b = 10/40 = 0.25$

$K = F S_g \sqrt{\pi a}$, $F \approx 1$ (Fig. 8.12)

$S_g = \dfrac{P_c}{2bt} = \dfrac{XP}{2bt}$, P = allowable design load

$$K_{Ic} = F\,\frac{XP}{2bt}\sqrt{\pi a}$$

$$66\ MPa\sqrt{m} = \frac{3\,(P, N)}{2(40)(15)\ mm^2}\sqrt{\pi(0.010\ m)}$$

$P = 148{,}900\ N = 148.9\ kN$ ◀

(b) $a = 24$ mm, $\alpha = a/b = 0.6$

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} = 1.292 \quad \text{(Fig. 8.12)}$$

$$66 = 1.292\,\frac{3P}{2(40)(15)}\sqrt{\pi(0.024)}$$

$P = 74{,}400\ N = 74.4\ kN$ ◀

8.13 A cylindrical pressure vessel has an inner diameter of 150 mm and a wall thickness of 5 mm, and it contains a pressure of 20 MPa. The safety factor against yielding must be at least $X_o = 2$. Also, a leak-before-break criterion must be met with a safety factor of at least $X_a = 9$ on crack length; that is, the critical crack length c_c must be at least $X_a t$, where t is the thickness.

(a) Is the vessel safe if it is made from 300-M steel (300°C temper)?

(b) From ASTM A517-F steel?

(c) What minimum fracture toughness is required for the material in this application?

(d) What safety factor on K relative to K_{Ic} is accomplished by the $X_a = 9$ requirement?

(20 pts)

8.13 Cylindrical pressure vessel

$d_1 = 150$, $t = 5$ mm, $p = 20$ MPa, $X_o = 2$

Leak-before-break, $X_a = 9$, i.e., $c_c = X_a t$

(a) Safe if 300-M steel (300°C)?

$K_{Ic} = 65\ \text{MPa}\sqrt{m}$, $\sigma_o = 1740$ MPa (Table 8.1)

$$\sigma_t = \frac{p r_1}{t} = \frac{(20\ \text{MPa})(75\ \text{mm})}{5\ \text{mm}} = 300\ \text{MPa}$$

$$\sigma_x = \frac{p r_1}{2t} = 150\ \text{MPa}, \quad \sigma_r = -p = -20\ \text{MPa}$$

$\sigma_t, \sigma_x, \sigma_r = \sigma_1, \sigma_2, \sigma_3$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} = 277\ \text{MPa}$$

$X_o = \sigma_o / \bar{\sigma}_H = 6.27$ OK ◁

$$c_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 = \frac{1}{\pi}\left(\frac{65\ \text{MPa}\sqrt{m}}{300\ \text{MPa}}\right)^2 = 0.01494\ m$$

$$X_a = \frac{c_c}{t} = \frac{14.9\ \text{mm}}{5\ \text{mm}} = 2.99, \quad 2.99 < 9 \quad \text{No!}$$ ◀

(b) Safe if A517-F steel?

$K_{Ic} = 187\ \text{MPa}\sqrt{m}$, $\sigma_o = 760$ MPa (Table 8.1)

$$X_o = \frac{\sigma_o}{\bar{\sigma}_H} = \frac{760\ \text{MPa}}{277\ \text{MPa}} = 2.74 \quad \text{OK}$$ ◁

$$c_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 = \frac{1}{\pi}\left(\frac{187\ \text{MPa}\sqrt{m}}{300\ \text{MPa}}\right)^2 = 0.1237\ m$$

$X_a = c_c/t = 123.7\ \text{mm}/5\ \text{mm} = 24.7$ Yes! ◀

(8.13, p.2)

(c) What minimum K_{Ic} required?

$$c_c = X_a t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2, \quad K_{Ic} = \sigma_t \sqrt{\pi X_a t}$$

$$K_{Ic} = 300\,\text{MPa}\sqrt{\pi\,(9)(0.005\,\text{m})}$$

$$K_{Ic} = 113\ \text{MPa}\sqrt{\text{m}}$$ ◀

(d) What $X_K = K_{Ic}/K$ achieved?

$K_{Ic} = \sigma_t \sqrt{\pi X_a t}$ (from above)

$$\frac{K_{Ic}}{\sqrt{X_a}} = \sigma_t\sqrt{\pi t} = \hat{K} = K \text{ at leak}$$

$$\frac{K_{Ic}}{\hat{K}} = \sqrt{X_a}\,; \quad X_K = \sqrt{X_a} = \sqrt{9} = 3$$ ◀

11.1 Estimate the constants C and m for the straight line portion of the data in Figure 11.3

(20 pts)

11.1 Estimate C and m for straight line portion in Fig. 11.3. Use $MPa\sqrt{m}$, $\frac{mm}{cycle}$.

$$\frac{da}{dN} = C(\Delta K)^m, \quad \log\frac{da}{dN} = \log C + m\log(\Delta K)$$

Two points on line are $(\Delta K, da/dN) =$
$(5, 3\times10^{-7})$, $(100, 4.2\times10^{-3})$

$$m = \frac{\log(4.2\times10^{-3}) - \log(3\times10^{-7})}{\log 100 - \log 5} = 3.19$$

$$C = 3\times10^{-7}/5^{3.19} = 1.77\times10^{-9} \quad \frac{mm/cycle}{(MPa\sqrt{m})^m}$$

$$\frac{da}{dN} = 1.77\times10^{-9}(\Delta K)^{3.19} \quad (10 < \Delta K < 100)$$

(For units of $MPa\sqrt{m}$, mm/cycle)

11.22 A bending member has a rectangular cross section of dimensions, as defined in Fig. 8.13, of depth b=60 and thickness t=12 mm. It is made of the AISI 4340 steel of Table 11.2 and is subjected to a cyclic moment between M_{min}=0.8 and M_{max}=4.0 kN•m. Failure occurred after 60,000 cycles of this loading by brittle fracture from a through-thickness edge crack extending 14 mm in the depth direction. Estimate the initial crack length present at the beginning of the cyclic loading.

(20 pts)

11.22 Bending member, AISI 4340 steel

$b = 60,\ t = 12$ mm, $M_{min} = 0.8$, $M_{max} = 4.0$ kN·m

$N_{if} = 60{,}000$ cycles, $a_f = 14$ mm

$a_i = ?$

$C_1 = 5.11 \times 10^{-10}$

$m_1 = 3.24,\ \gamma = 0.42$

(for MPa$\sqrt{m}$ and mm/cycle, $R=0$)

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m (1-m/2)}$$

$$C = \frac{C_1}{(1-R)^{m_1(1-\gamma)}} = \frac{5.11 \times 10^{-13}}{(0.8)^{3.24(1-0.42)}} = 7.77 \times 10^{-13}$$

(for m/cycle)

$R = M_{min}/M_{max} = 0.20$, $m = m_1$

$$\Delta S = \frac{6\,\Delta M}{b^2 t} = \frac{6\,(4.0-0.8) \times 1000\ \text{N·m}}{(60\ \text{mm})^2\,(0.012\ \text{m})} = 444\ \text{MPa}$$

$a_f/b = 0.25 < 0.4$, so $F \approx 1.12$ (Fig. 8.13)

Substitute $a_f = 0.014$ m, and solve for a_i

$a_i = 0.000475\ \text{m} = 0.475\ \text{mm}$ ◄