Example of a finite element analysis of a beam

A finite element model was constructed using plane 2-D elements. Over 700 nodes and 800 elements comprise the model of the simply supported beam which is constrained in the x and y directions at the LHS (key point 1) and in the y direction at the RHS (key point 2). Force of 250 N was applied at key point 6 which was 355 mm from the RHS.

The model was constructed by defining key points (1-7) as shown by the green picture in Figure 1. The shape of the model was then defined by curves which connected the key points. A region was defined for the curves. This region was auto-meshed resulting in the node pattern shown by the green picture and the element pattern (shown deformed) in the black picture of Figure 1. Note that in this case the beam cross section is 25.4 mm tall but 11.6 mm wide to give a rectangular cross section with a moment of inertia the same as the C-channel beam used in the lab. The length of the beam and the relative loading points are the same the beam used in the lab.

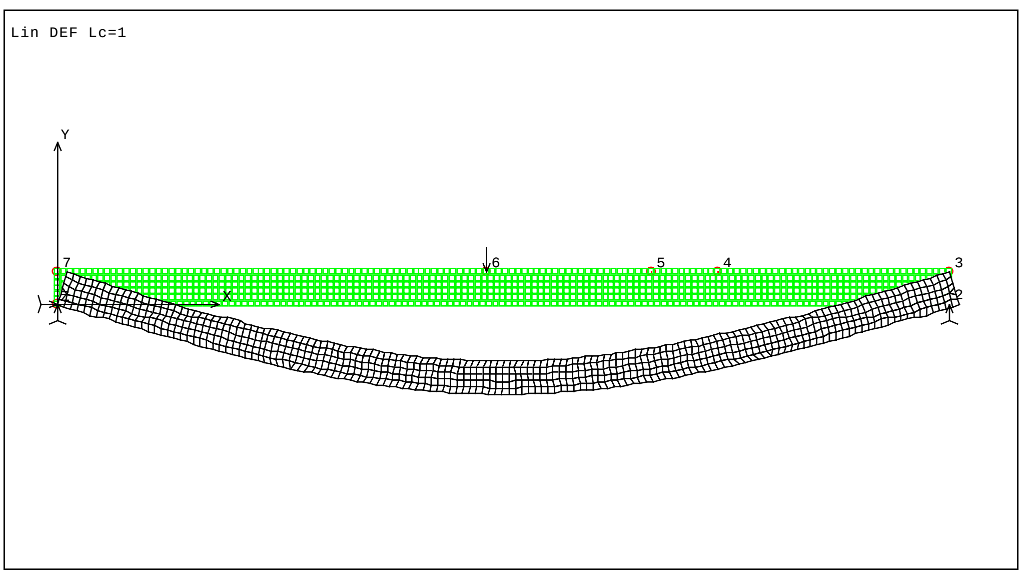

Figure 1 FEA model of a beam (green picture is the undeformed node pattern with the key points shown in black) in undeformed and deformed (black picture is the deformed element pattern) shapes for an applied force of 255 N. Note that the vertical deflection at key point 4 is -1.08 mm.

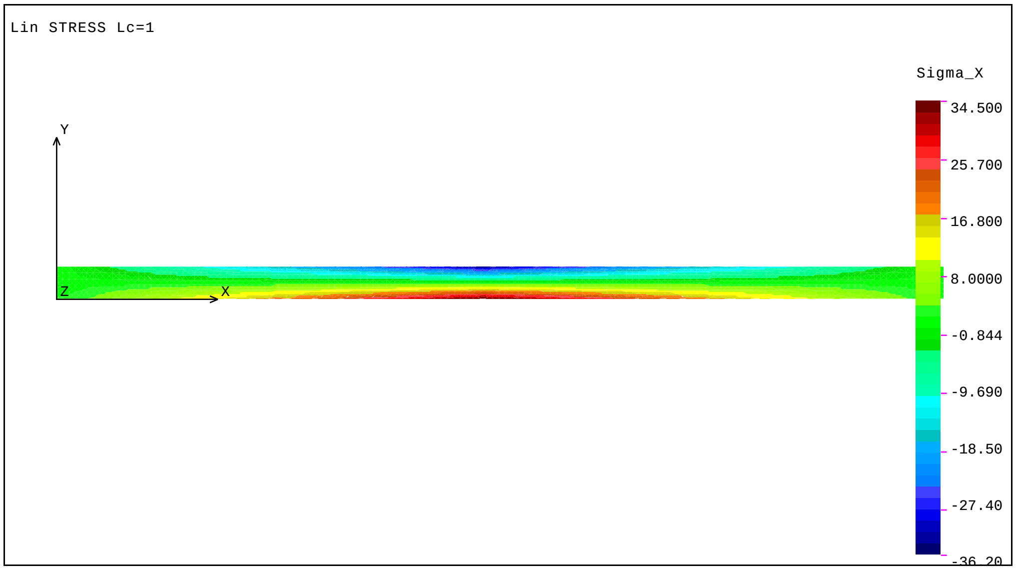

Figure 2 Normal stresses in the x direction. Note that the legend shows stresses in MPa.

The solution for the FEA model is found by solving for the displacements in the equation: $\{F\} = [k]\{x\}$ where F is the applied force vector and x is the resulting displacement vector. The model stiffness k is comprised of the individual stiffnesses for all the elements. Once the displacement vector is calculated, the strains can be calculated knowing the initial dimensions of the respective element. Using constitutive relations $\{\sigma\} = [C]\{\varepsilon\}$, the stresses are calculated from the strains.

Figure 2 shows the normal stress distribution in the x-direction (in this case the bending stress). Note the stress gradient across the cross section and along the length of the beam. Also note the change of sign of the stress from the top of the beam to the bottom, with a zero stress at the neutral axis. Compare these gradients to what you would expect from the beam bending equation and from the moment diagram.

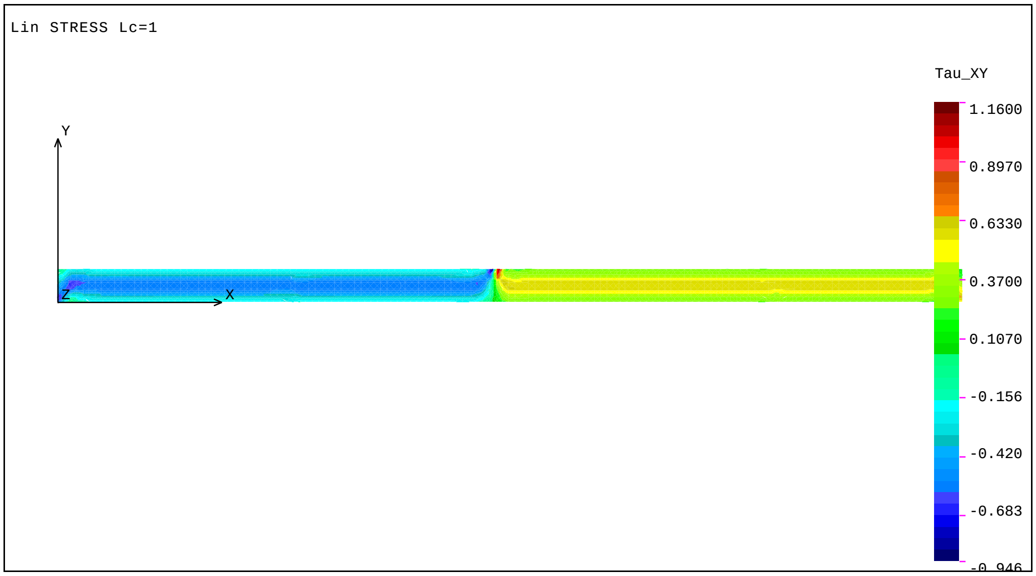

Figure 3 Shear stresses in the x-y direction. Note that the legend shows stresses in MPa.

Figure 3 shows the shear stress distribution in the x-y direction. Note the stress gradient across the cross section and along the length of the beam. Also note the change of sign of the stress from the LHS to the RHS, with a zero stress at the top and bottom of the beam. Compare these gradients to what you would expect from the shear stress equation for beams in bending and from the shear diagram.