ME 354, MECHANICS OF MATERIALS LABORATORY
STRESS CONCENTRATIONS

25 october 2000 / mgj

PURPOSE

The purpose of this exercise is to the study the effects of geometric discontinuities on the stress states in structures and to use photo elasticity to determine the stress concentration factor in a simple structure.

EQUIPMENT

- Un-notched beam of birefringent material (an epoxy).
- Notched bend beam of the same birefringent material as the un-notched beam.
- Four-point flexure loading fixture with load pan and suitable masses for loading
- Circular polariscope with monochromatic light source

PROCEDURE

Part 1. Beam under Pure Bending to Determine the Stress-Optical Coefficient of the Material

i) Install the un-notched beam (see Fig. 1) in the four-point flexure loading fixture
ii) Attach the load pan (Note: The combined pan/fixture mass is ~0.980 kg)
iii) Apply two10-kg masses one at a time to the load pan.
iv) With the polarizer and analyzer crossed (dark field), focus the camera, and record the image using the thermal printer
v) Determine the maximum fringe orders at the top and bottom of the beam including estimates of fractional fringes orders by counting the fringes.
vi) The stress-optical coefficient can be calculated using the following relation:

$$f = \frac{t}{\overline{N}}(\sigma_1 - \sigma_2) \qquad (1)$$

where f is the stress-optical coefficient, $\overline{N}$ is the fringe order, t is the model thickness, and σ_1 and σ_2 are the plane-stress principal stresses.

Part 2.Notched Beam under Pure Bending to Determine the Stress Concentration Factor

i) Install the notched beam (see Fig. 2) in the four-point flexure loading fixture
ii) Attach the load pan (Note: The combined pan/fixture mass is ~0.980 kg)
iii) Apply one 5-kg mass to the load pan. (Note: Do not apply more than 5 kg at one time).
iv) With the polarizer and analyzer crossed (dark field), focus the camera, and record the image using the thermal printer.
v) Determine the maximum fringe orders at the top and bottom of the beam and at the edge of the notch including estimates of fractional fringes orders.
vi) The stress distributions within the beam can be calculated using the relation:

$$(\sigma_1 - \sigma_2) = f\frac{\overline{N}}{t} \qquad (2)$$

where f is the stress-optical coefficient determined previously, $\overline{N}$ is the fringe order, t is the model thickness, and σ_1 and σ_2 are the plane-stress principal stresses.

* REFERENCES

Manual on Experimental Stress Analysis, J.F. Doyle & J.W. Philips, eds, Society for Exper. Mechanics, 1989
Experimental Stress Analysis, J.W. Dally and W.F. Riley, McGraw-Hill, Inc., 1990
Handbook on Experimental Mechanics, A.S. Kobayashi, ed., Prentice Hall, Inc.,1992
Formulas for Stress and Strains, R.J. Roark and W.C. Young, McGraw-Hill, Inc., 1975
Stress Concentration Factors, R.E. Peterson, John Wiley and Sons, Inc., 1974

RESULTS

When loads are applied to a solid body, such as part of a structure or a machine component, stresses which vary from point to point, are set up in the body. At certain points, stress concentrations (sometimes called stress raisers) occur and are potential weak points in the body. Frequently, an alteration in the shape of the body will lead to a reduction in the stresses at such points and to a more even distribution over the whole body. An optimum body is that of uniform load-carrying capability.

The mathematical theory of elasticity provides many valuable solutions involving the stress distributions in bodies of simple geometries and loadings. A common use of these solutions is the determination of stress concentration factors ($k_t = \frac{\sigma_{local}}{\sigma_{remote}}$) resulting from discontinuities or other localized disturbances in the stress field of the body. In more complicated problems, commercially available two- and three-dimensional computer programs for finite element and boundary element analyses (FEA and BEM, respectively) can be used to locate and quantify the stress concentrations.

These theoretical and numerical results are exact solutions to problems which may or may not model the actual situations (usually due to assumptions about loads, load applications and boundary conditions). This uncertainty in modeling often requires experimental verification by spot checking the analytical or numerical results. A frequently cited example involves a threaded joint which seldom produces uniform contact at the threads. Contact analyses based on the idealized boundary condition of uniform contact will grossly underestimate the actual maximum stress concentration at the root of the overloaded thread. The uncertainty in the contact condition requires a stress analysis of the actual threaded joint experimentally despite the proliferation of FEA and BEM programs. Experimental stress analysis is also necessary to study nonlinear structure problems involving dynamic loading and/or plastic/viscoplastic deformations. Available FEA programs cannot provide detailed stress analysis of three-dimensional dynamic structures. Constitutive relations for plastic/viscoplastic materials are still being developed

One such experimental procedure often applied to empirically determine stress states is photoelasticity. Photoelasticity is a relatively simple, whole-field method of elastic stress analysis which is well suited for visually identifying locations of stress concentrations. In comparison with other methods of experimental stress analysis, such as a strain gage technique which is a point measurement method, photoelasticity is inexpensive to operate and provides results with minimum effort.

Photoelasticity consists of examining a model similar to the structure of interest using polarized light. The model is fabricated from transparent polymers possessing special optical properties. When the model is viewed under the type (but not necessarily magnitude) of loading similar to the structure of interest, the model exhibits patterns of fringes from which the magnitudes and directions of stresses at all points in the model can be calculated. The principle of similitude can be used to deduce the stresses which exist in the actual structure.

A disadvantage of photoelasticity is the necessity to test a polymer model which may not be able to withstand extreme loading conditions such as high temperature and/or high strain rates. Although photoelasticity is generally applied to elastic analysis, limited studies on photo plasticity and photo viscoelasticty indicate the potential of extending the technique to nonlinear structural analysis. Further details of photoelasticity can be found in listed references.

Show all work and answers on the Worksheet / In-class Laboratory report.

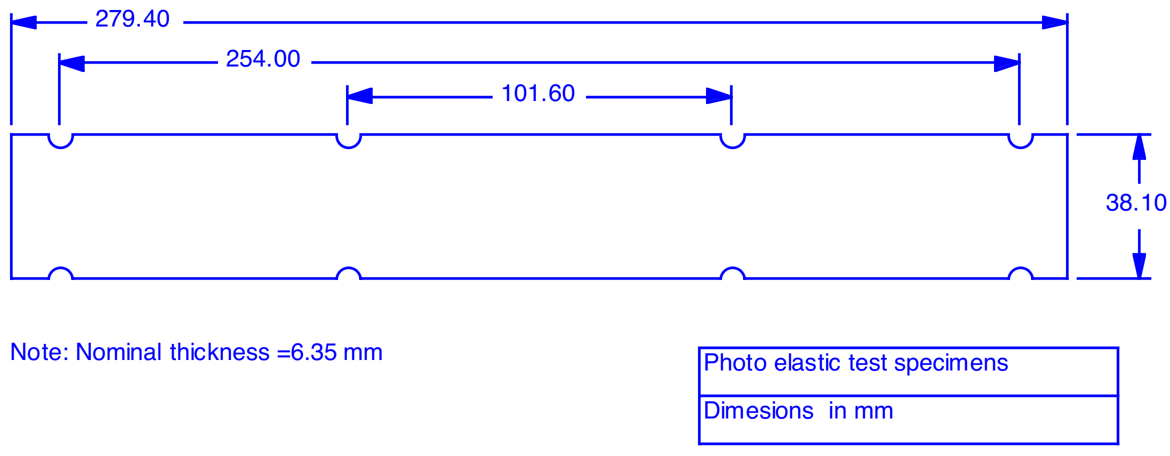

Figure 1 Un-notched Beam

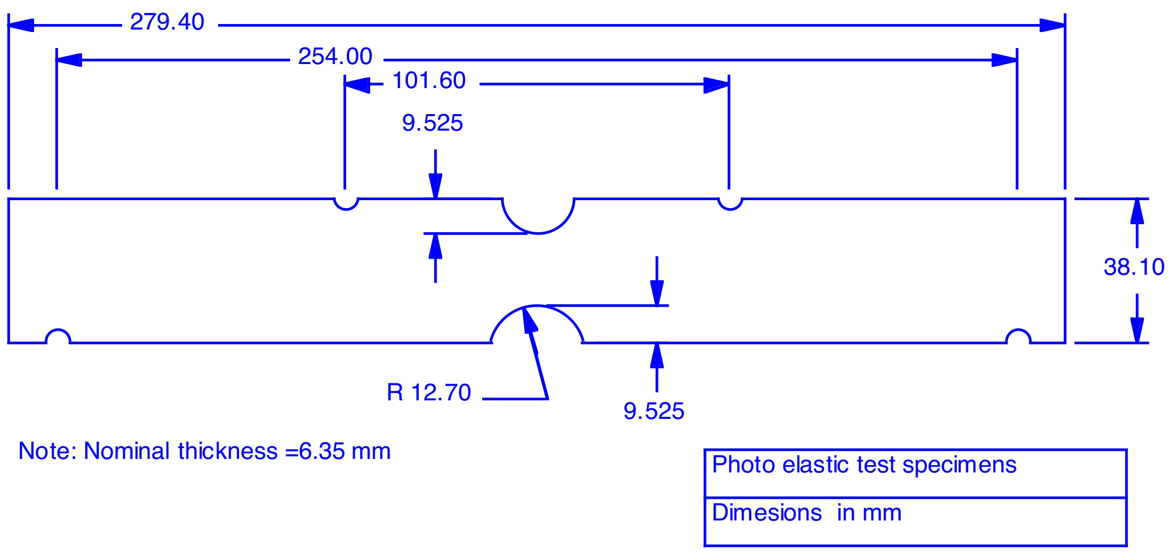

Figure 2 Notched Beam

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WORK SHEET

NAME______________________________________**DATE**_____________

EQUIPMENT IDENTIFICATION______________________________________

1) The properties of two birefringent polymers often used for photoelastic experiments are in Table 1.

Table 1 Some Properties of Two Birefringent Polymers Used in Photoelastic Experiments

PSM-1 (polycarbonate) Selected Properties (R.T.)

Elastic Modulus, E(GPa)	2.5
Proportional Limit σ_o (MPa)	48
Poisson's ratio, ν	0.38
Stress Optical Coefficient, f (MPa-mm/fringe)*	7
Figure of Merit Q=E / f (1/m)	357,143

PSM-9 (epoxy) Selected Properties (R.T.)

Elastic Modulus, E(GPa)	3.3
Proportional Limit σ_o (MPa)	50
Poisson's ratio, ν	0.37
Stress Optical Coefficient, f (MPa-mm/fringe)*	10.5
Figure of Merit Q=E / f (1/m)	314,286

* in green light with wavelength 546 nm

2) For the two beams and loading fixtures, confirm the following information. See Figs. 3 and 4 for nomenclature.

Table 2 Dimensions and Loading for Un-notched and Notched Photoelastic Beams

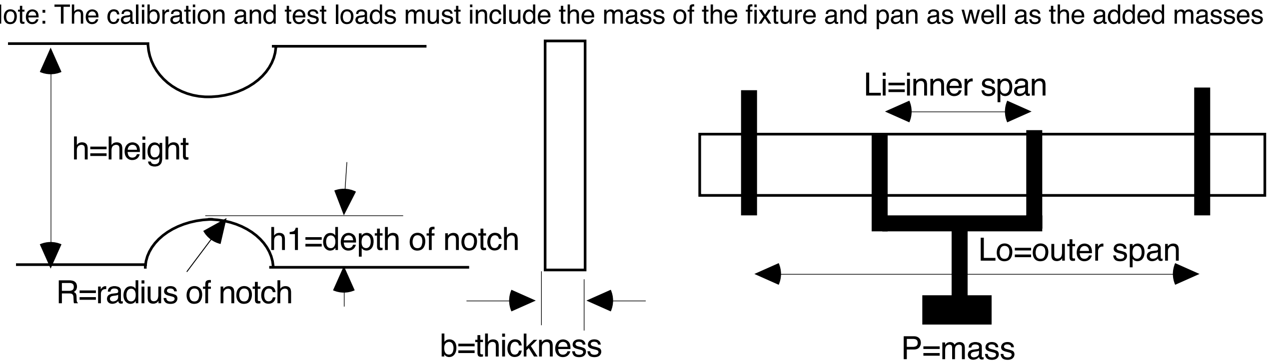
Un-notched beam

Calibration load, P_c $=P_{weight}+P_{fixture}+P_{pan}$ (N)	
Outer Span, L_o (mm)	
Inner Span, L_i (mm)	
Height, h (mm)	
Thickness, b (mm)	
Radius of Notch, R (mm)	----
Depth of notch, h_1 (mm)	----

Notched Beam

Test load, P_c $=P_{weight}+P_{fixture}+P_{pan}$ (N)	
Outer Span, L_o (mm)	
Inner Span, L_i (mm)	
Height, h (mm)	
Thickness, b (mm)	
Radius of Notch, R (mm)	
Depth of notch, h_1 (mm)	

Note: The calibration and test loads must include the mass of the fixture and pan as well as the added masses



a) Notch Detail b) Overall Specimen Detail

Figure 3 Nomenclature for the Beams

3) A unique aspect of the four-point flexure loading arrangement is that the region of interest (the section of the beam within the inner loading span) experiences a pure bending moment as shown in Fig. 4.

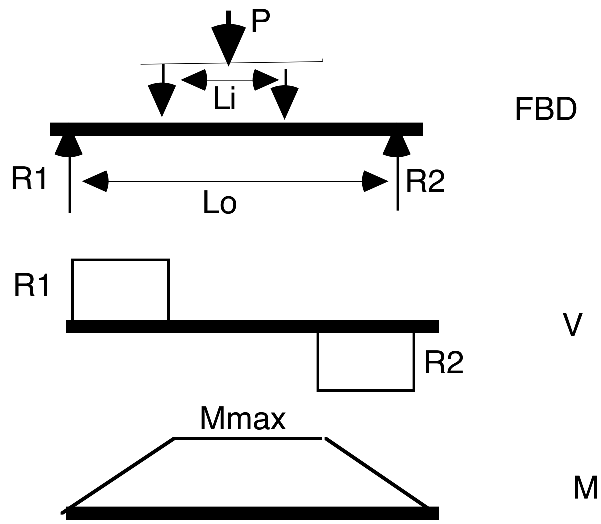


Figure 4 Free Body, Shear and Moment Diagrams for Four-Point Flexure Loading

For the un-notched beam, determine the following:

Moment of Inertia for the rectangular cross section beam, $I = \frac{bh^3}{12}$ =__________________mm^4

Maximum moment when the calibration load, P_C, was applied,

$M_c = \frac{P_c(Lo - Li)}{4}$=______________________N•mm

4) At the outer free edge of the beam (y=c=h/2) the stress state is uniaxial and the photo elastic relation can be used to determine the stress optical coefficient directly from the beam bending relation.

The average fringe value at the upper and lower outer edges of the beam determined at the calibration load, $\overline{N}$=_________________.

Maximum distance to an outer edge of the beam from the neutral axis, c=h/2=___________mm

Maximum uniaxial bending stress at the outer free edge of the beam

$\sigma_1 = \frac{M_c\, c}{I}$=__________MPa.

Calculated stress optical coefficient for the material, $f = \frac{b}{\overline{N}}(\sigma_1)$=________MPa-mm/fringe

Compare this value to that shown in the table. How do the values compare? Discuss any discrepancies and possible reasons (Note: Do not panic if the calculated stress optical coefficient differs from the value listed in Table 1.differences in optical test setup, environmental effects in the material, etc. all require the "calibration" of the material).

5) At the free edge of the notch the stress state is uniaxial and the photoelastic relation can be used to calculate the normal stress using the relation between the fringe order at the free edge, the stress optical coefficient for the material, and the specimen thickness.

The average fringe value at the free edge of the notches determined at the test force, $\bar{N}$=______.

Calculated normal stress at the free edge of the notch, $\sigma_1 = \sigma_{w/\ notch} = \frac{f\,\bar{N}}{b}$=_________MPa.

6) One way to define a stress concentration factor, k_t, is the ratio of the stress at the discontinuity in a body to the maximum stress in the net section (i.e., that part of the body remaining after the discontinuity removes a portion of the cross section) such that: $k_t = \frac{\sigma_{w/\ discontinuity}}{\sigma_{net}}$.

The notched beam is symmetric, therefore the neutral axis is the midpoint of the beam as well as the midpoint of the net cross section beam. The width of the net cross section beam is the distance between the notches, $h_{net} = h - 2h_1$=_________mm.

The moment of inertia for the net cross section is $I_{net} = \frac{bh_{net}^3}{12}$= _________$mm^4$.

The distance from the neutral axis to the outermost edge of the net cross section is $c_{net} = h_{net}/2$= _________mm.

The moment in the beam at the test force, P_t, is $M_t = \frac{P_t(L_o - L_i)}{4}$=_____________mm.

Stress in the net cross section of the beam, $\sigma_{net} = \frac{M_t\, c_{net}}{I_{net}}$=______________MPa.

The stress concentration factor is the ratio of the stress at the notch and to the net cross section stress: $k_t^{measured} = \frac{\sigma_{w/notch}}{\sigma_{net}}$=____________.

7) Several authors have compiled stress concentration factors for simple geometries. The most "famous" compilation is Peterson's book of stress concentration factor graphs. From Peterson's book for the double-notched flat specimen in bending, k_t is plotted as a function of $r/d = R/(h-2h_1)$ for various values of $D/d=h/(h-2h_1)$.

In this case, $\frac{r}{d} = \frac{R}{(h-2h_1)}$=______________ and $\frac{D}{d} = \frac{h}{(h-2h_1)}$=________________.

The stress concentration, k_t can be "picked off" a plot such that k_t^{plot} =______________

Alternatively, a curve fit for a double-notched beam in pure bending (Roarke and Young) is described as follows for $0.25 \le \frac{h_1}{R} \le 2.0$. In this case, $\frac{h_1}{R}$=____________ and the stress concentration factor is: $k_t = K_1 + K_2\left(\frac{2h_1}{h}\right) + K_3\left(\frac{2h_1}{h}\right)^2 + K_4\left(\frac{2h_1}{h}\right)^3$

where $K_1 = 0.723 + 2.845\sqrt{\frac{h_1}{R}} - 0.504\frac{h_1}{R}$=___________

$K_2 = -1.836 - 5.746\sqrt{\frac{h_1}{R}} + 1.314\frac{h_1}{R}$=___________

$K_3 = 7.254 - 1.885\sqrt{\frac{h_1}{R}} + 1.646\frac{h_1}{R}$=____________

$K_4 = -5.140 + 4.785\sqrt{\frac{h_1}{R}} - 2.456\frac{h_1}{R}$=____________

such that: $k_t^{\text{curve fit}} = k_t = K_1 + K_2\left(\frac{2h_1}{h}\right) + K_3\left(\frac{2h_1}{h}\right)^2 + K_4\left(\frac{2h_1}{h}\right)^3$ _________________

8) Compare the k_t measured from the photoelastic analysis to that determined from a compiled handbook (e.g., k_t^{plot} or $k_t^{\text{curve fit}}$). Determine the percent differences between the measured k_t and the compiled values. Since many compiled stress concentration factors were determined from photoelastic analyses, discuss possible reasons for differences between the measured k_t and compiled values.

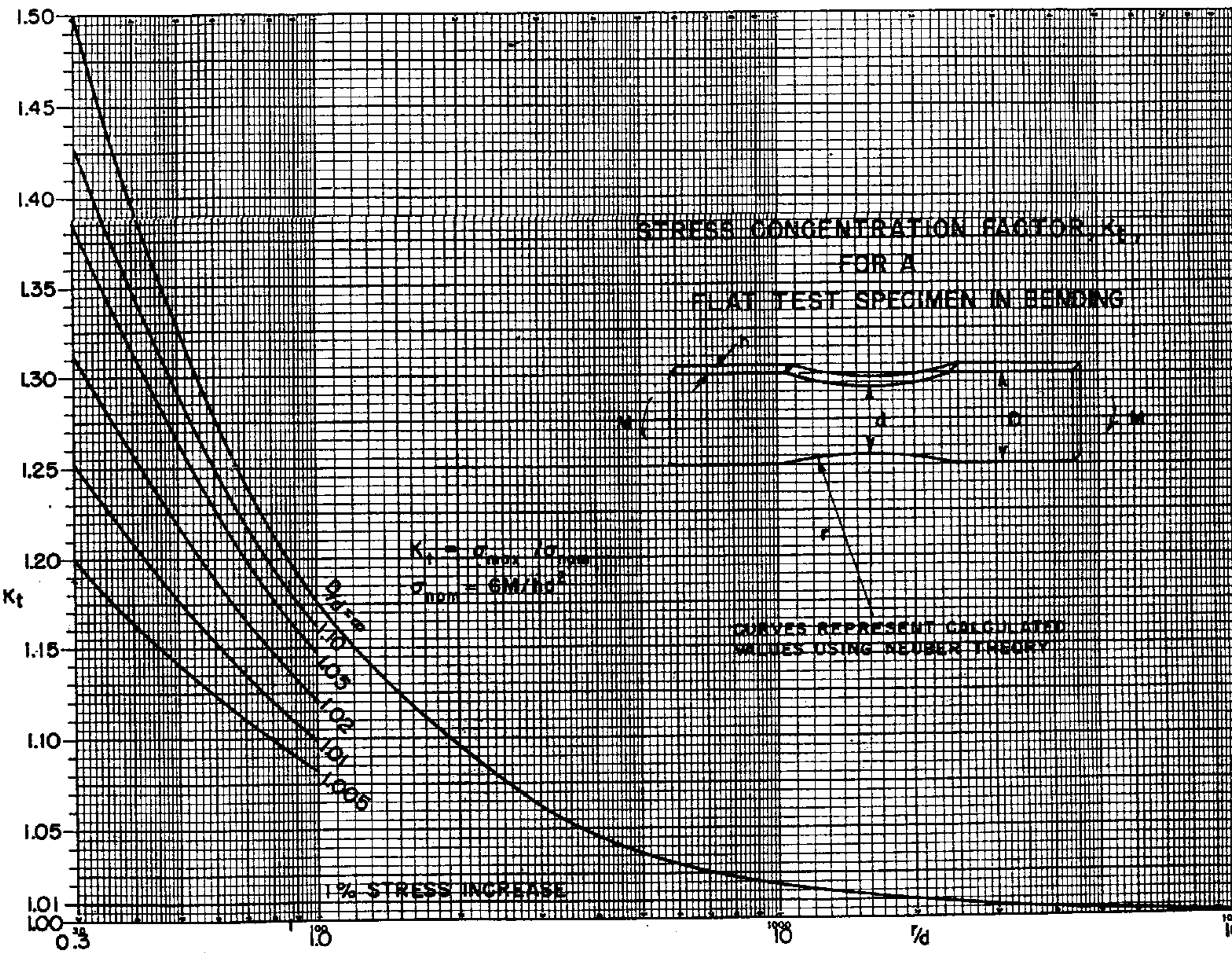
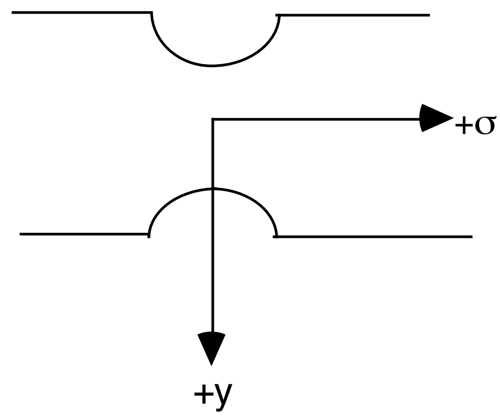

STRESS CONCENTRATION FACTOR, K_t,
FOR A
FLAT TEST SPECIMEN IN BENDING
$K_t = \sigma_{max} / \sigma_{nom}$
$\sigma_{nom} = 6M/hd^2$
CURVES REPRESENT CALCULATED
VALUES USING NEUBER THEORY
D/d = ∞
1.10
1.05
1.02
1.01
1.005
1% STRESS INCREASE
K_t
r/d
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.01
1.00
0.3
1.0
10
100

FIG. 37

Extra effort: Using the fringe orders across the notched beam, assume the stress state is uniaxial, plot the stress across the height of the beam. Compare this stress distribution to that of the un-notched beam at the same force.



Extra effort: Another way to define a stress concentration factor, k_t is the ratio of the stress at the discontinuity in a body to the stress that would have been at the same point in the body without the discontinuity such that $k_t = \frac{\sigma_{\text{w/ discontinuity}}}{\sigma_{\text{w/o discontinuity}}}$.

The notched beam is symmetric, therefore the neutral axis is the midpoint of the beam. The distance from the neutral axis to the edge of the notch is, $y = \frac{h}{2} - h_1$=_________mm

The moment in the beam at the test force, P_t, is $M_t = \frac{P_t(L_o - L_i)}{4}$=_____________mm.

Stress in an un-notched beam at the same point at the edge of the notch in the notched beam is $\sigma_{\text{w/o notch}} = \frac{M_t\, y}{I}$=______________MPa.

The stress concentration factor is the ratio of the stresses at the same location for the notched and un-notched beams, $k_t = \frac{\sigma_{\text{w/ notch}}}{\sigma_{\text{w/o notch}}}$=_____________.

The Peterson stress concentration factor found earlier can be modifed to account for this difference in defintion such that $k_t^{\text{notch}} = \left(k_t^{\text{net}} = \frac{\sigma_{\text{w/ notch}}}{\sigma_{\text{net}}}\right)\left(\frac{I_{\text{beam}}}{I_{\text{net}}}\right)$.

Since, $I_{\text{net}} = \frac{bh_{\text{net}}^3}{12}$=______________ and $I_{\text{beam}} = \frac{bh_{\text{beam}}^3}{12}$=____________, then $\left(\frac{I_{\text{beam}}}{I_{\text{net}}}\right)$=________and $k_t^{\text{notch}} = \left(k_t^{\text{net}} = \frac{\sigma_{\text{w/ notch}}}{\sigma_{\text{net}}}\right)\left(\frac{I_{\text{beam}}}{I_{\text{net}}}\right)$=_____________

Compare the k_t at the notch using this alternative definition and the modified k_t determined from the compiled version in the handbook. Determine the percent differences between the two values. Which definition of k_t seems more "reasonable?" Why?