ME 354, MECHANICS OF MATERIALS LABORATORY

COMPRESSION AND BUCKLING

01 January 2000 / mgj

PURPOSE

The purpose of this exercise is to study the effects of end conditions, column length, and material properties on compressive behaviour and buckling in columns.

EQUIPMENT

- Solid rods of various lengths of aluminum and steel
- Universal test machine with grips, controller, and data acquisition system

PROCEDURE

Repeat the following steps for each specimen.

- Measure the diameter and lengths of each specimen to 0.02 mm.
- Zero the force output (balance).
- Activate force protect (~50 N) on the test machine to prevent overloading the specimen during installation.
- Install the top end of the test specimen in the top grip of the test machine while the test machine is in displacement control.
- Install the bottom end of the test specimen in the lower grip of the test machine.
- In displacement control adjust the actuator position of the test machine to achieve nearly zero force on the specimen.
- Deactivate force protect.
- Initiate the data acquisition and control program.
- Enter the correct file name and specimen information as required.
- Initiate the test sequence via the computer program.
- Continue the test until buckling or compressive failure of the test specimen occurs
- Examine the force versus displacement trace for each test. Note the force at the onset of buckling or compressive failure (i.e., significant deviation from linearity)

RESULTS

Structures and machines may fail in many ways depending on the materials, kinds of loads, and conditions of support. Many machine elements can be modeled as uniform members under uniaxial tension or compression. For tensile loading, these members tend to self-align and fail either by ductile deformation or brittle fracture depending on the material. In compression, the failure mode is complicated by the possibility of a geometric instability, called buckling, in addition to ductile deformation.

Columns are structural members which support compressive forces. Buckling occurs when the column has a tendency to deflect laterally, out of the line of action of the force. Once buckling initiates, the instability can lead to failure of the column because the eccentric force acts as a moment causing greater stresses and deflections due to the combination of the bending and axial forces.

The possibility of buckling increases for the following column conditions: 1) longer, "thinner" columns, 2) pinned, free, or non-fixed end conditions, 3) initial eccentricity of the force (e.g., bent columns) and/or 4) lower elastic modulus of the column material.

In this exercise, two materials and two column lengths will be studied. Anticipated buckling or compressive failure forces will first be calculated for various length specimens and materials.

For compressive failure, $P_o = \sigma_o A_o$

and (1).

For buckling, $P_{cr} = \frac{\pi^2 EI}{L_e^2}$

where P_o is the compressive failure force (yield), σ_o is proportional limit stress (or yield strength), A_o is the initial area of the gage section, P_{cr} is the Euler critical buckling force, I is the least moment of inertia of the cross section, and L_e is the effective, unsupported length of the column.

The anticipated buckling or compressive failure forces will then be compared to the actual measured forces at the onset of instability. Observations will be made on the effects of end conditions, material type, and column length.

Show all work and answers on the Worksheet, turning this in as the In-class Laboratory report.

References:

"Mechanics of Materials," J.M. Gere and S.P. Timoshenko
"Mechanics of Materials," R.C. Hibbeler

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WORK SHEET

NAME____**DATE**_____________

EQUIPMENT IDENTIFICATION____

1) Determine (look up) the following mechanical properties.

Table 1 Selected Properties for Test Materials

6061-T6 Aluminum
Selected Mechanical Properties (R.T.)

E (GPa)	
σ_o (MPa)	
S_{UTS} (MPa)	
% elongation	

1018 Steel (CD)
Selected Mechanical Properties (R.T.)

E (GPa)	
σ_o (MPa)	
S_{UTS} (MPa)	
% elongation	

2) Measure and record the following dimensions.

Table 2 Pertinent column dimensions

Column Dimensions for Aluminum

Diameter, d (mm)	
Length 1, L1 (mm)	
Length 2, L2 (mm)	

Column Dimensions for Steel

Diameter, d (mm)	
Length 1, L1 (mm)	
Length 2, L2 (mm)	

3) For each column, determine the following geometric quantities.

Aluminum

Moment of Inertia: $I = \frac{\pi d^4}{64}$ _________ mm^4

Cross sectional area: $A = \frac{\pi d^2}{4}$ _________ mm^2

Radius of gyration squared: $k^2 = \frac{I}{A}$ _______ mm^2

Radius of gyration: $k = \sqrt{k^2} = \sqrt{\frac{I}{A}}$ _______ mm

Steel

Moment of Inertia: $I = \frac{\pi d^4}{64}$ _________ mm^4

Cross sectional area: $A = \frac{\pi d^2}{4}$ _________ mm^2

Radius of gyration squared: $k^2 = \frac{I}{A}$ _______ mm^2

Radius of gyration: $k = \sqrt{k^2} = \sqrt{\frac{I}{A}}$ _______ mm

4) Buckling of columns with pinned ends is often called the fundamental case of buckling. However, many other conditions such as fixed ends, elastic supports, and free ends are encountered in practice. The critical forces for buckling for each of these end conditions can be determined by applying the appropriate boundary conditions and solving the differential equations. These solutions lead to the concept of an "effective length," L_e, appropriate for each end condition which is a multiple of the actual length, L, of the column as shown in Table 3 and Figure 1.

Table 3 Effective column length for various end conditions

Pinned/Pinned	Fixed/Free	Fixed/Fixed	Pinned/Fixed
$L_e = L$	$L_e = 2L$	$L_e = L/2$	$L_e = 0.7\ L$

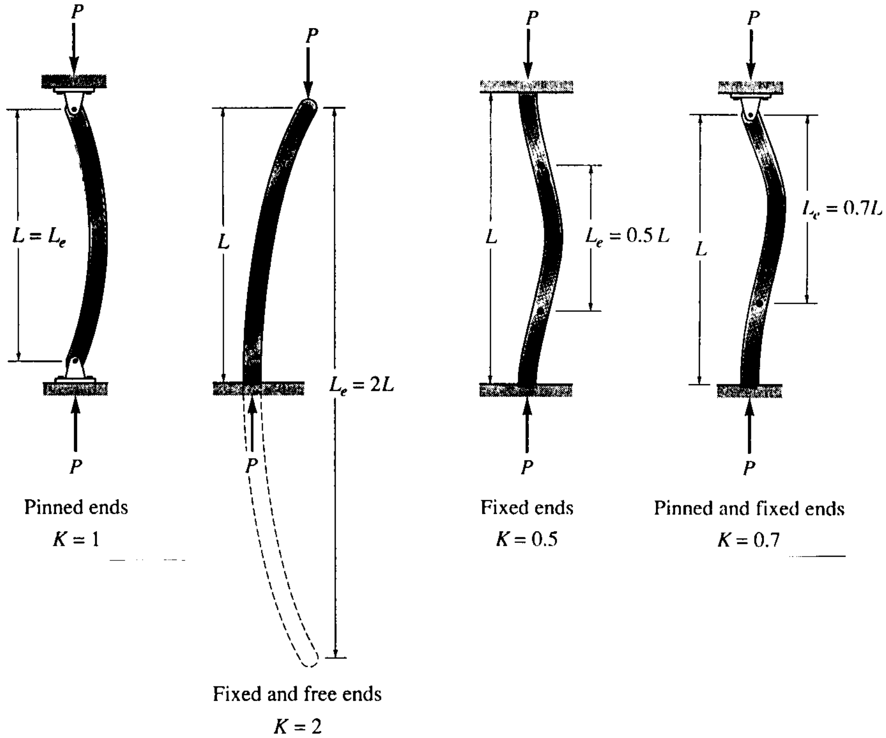


Figure 1 Illustration of end conditions for columns

5) In general, axially-loaded compression members may fail by one of three modes: crushing; a combination of crushing or buckling; or buckling alone. Columns can be placed into three groups:

1) Short columns - the failure mode is by crushing (simple compressive failure)
2) Intermediate columns - the failure mode depends on simple compressive and/or bending stress
3) Long columns - the failure mode is primarily a function of the bending stress (buckling).

A paramter which is employed to group these columns is the slenderness ratio, L_e/k. The minimum slenderness ratio $\left.\frac{L_e}{k}\right|_{min}$ marks the nominal transition from crushing to buckling. If the axial stress, σ , is plotted as a function of slenderness ratio, then the minimum slenderness ratio is the nominal transition from the constant stress for crushing, $\sigma = \sigma_o$, to the stress as function of L_e/k for buckling, $\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$.

Aluminum

Elastic modulus: E=_________MPa

Proportional limit stress: σ_o=_________MPa

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =______

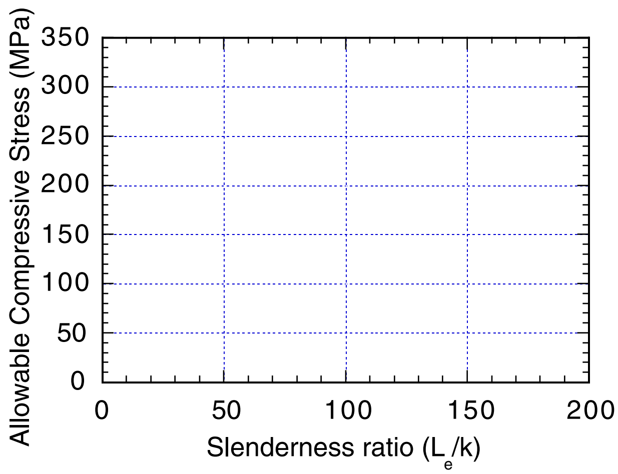
Steel

Elastic modulus: E=_________MPa

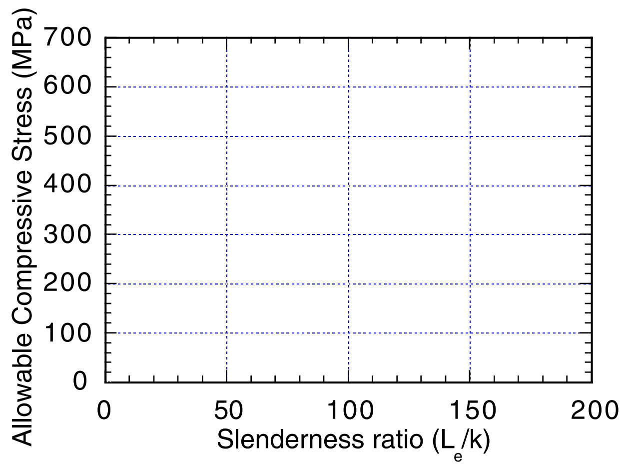
Proportional limit stress: σ_o=_________MPa

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

On the following graphs, plot $\sigma = \sigma_o$ for $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$ **and** $\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ for $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$.



a) Allowable compressive stress for aluminum



b) Allowable compressive stress for steel

Figure 1 Allowable compressive stress for aluminum and steel

6) Determine the following critical compressive forces for the experimental columns

Aluminum

For column length L1, the unsupported length if each grip end is ℓ = _____ mm long such that L=L1-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L1, slenderness ratio, L_e/k =________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$. _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$. _________MPa

Cross sectional area, A=___________mm^2

Use the smaller of the stresses calculated above.
For L1, critical force, $P_{cr}^{L1} = \sigma A$ =___________N

Steel

For column length L1, the unsupported length if each grip end is ℓ = _____ mm long such that L=L1-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L1, slenderness ratio, L_e/k =________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$ _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$. _________MPa

Cross sectional area, A=___________mm^2

Use the smaller of the stresses calculated above.
For L1, critical force, $P_{cr}^{L1} = \sigma A$ =___________N

For column length L2, the unsupported length if each grip end is ℓ= _____ mm long such that L=L2-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L2 slenderness ratio, L_e/k=________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{\min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$=_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{\min}$. _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{\min}$. _________MPa

Cross sectional area, A_=__________mm^2

Use the smaller of the stresses calculated above.
For L2, critical force, $P_{cr}^{L2} = \sigma A$=___________N

For column length L2, the unsupported length if each grip end is ℓ= _____ mm long such that L=L2-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L2, slenderness ratio, L_e/k=________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{\min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$=_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{\min}$ _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{\min}$. _________MPa

Cross sectional area, A=___________mm^2

Use the smaller of the stresses calculated above.
For L2, critical force, $P_{cr}^{L2} = \sigma A$=___________N

7) Measure the actual critical compressive forces for the experimental columns.

For L1, Aluminum

Measured critical compressive force, P_{L1}=________N

For L1, critical force, $P_{cr}^{L1} = \sigma A$=___________N

% diff __________

For L1, Steel

Measured critical compressive force, P_{L1}=_______N

For L1, critical force, $P_{cr}^{L1} = \sigma A$=___________N

% diff __________

For L2, Aluminum

Measured critical compressive force, P_{L2}=________N

For L2, critical force, $P_{cr}^{L2} = \sigma A$=___________N

% diff __________

For L2, Steel

Measured critical compressive force, P_{L2}=_______N

For L2, critical force, $P_{cr}^{L2} = \sigma A$_=__________N

% diff __________

8) Comment on how well the equations predicted the actual critical compression force. Were discrepancies reasonable? If not, what could possible sources of error be attributed to? (Recall that the assumptions for the buckling forces assume no initial eccentricity, perfectly straight columns, and no off-axis loading).

9) As a designer, what steps can be taken to reduce the tendency to buckle, geometrically? material-wise?