ME 354, MECHANICS OF MATERIALS LABORATORY

# STRUCTURES

01 January 2000 / mgj

## PURPOSE

The purpose of this exercise is to the study the effects of various assumptions in analyzing the stresses and forces in an engineering structure using engineering mechanics, experimental mechanics, and numerical modeling.

## EQUIPMENT

- Strain-gaged bicycle.
- Strain gage conditioning equipment and data acquisition system.
- "Dial indicators", holders and magnetic bases.

## PROCEDURE

- Re-read the reference document "NOTES on Strain Gages."

- Carefully examine attached Figs. 1-3. Note that a total of 10 stacked rectangular rosettes have been applied at various locations on the bicycle frame. Each rosette has three strain gages such that 30 possible strain gage circuits are involved. Identify all strain gage circuits and strain gage channel numbers on both the figures as well as on the bicycle frame itself.

- Note which strain gage locations will be used in the analysis.
- Note the location of the dial indicator measurement.
- Note the type of input forces and reactions (axle connections) for the bicycle frame.

- If not already done so, set the gage factor to 2.08 and balance each strain gage circuit to zero or a reasonable minimum offset strain.

- Record this offset strain, if any, (starting value with no force applied) for each channel on the data sheet.
- Zero the "dial indicators". Note the location of the deflection measurements on the bicycle frame.

- Apply a modest concentrated force (approximately the weight of a bicyclist with equipment) to the bicycle frame.

- Record the reading for each strain gage channel on the data sheet.

- Record the reading of the dial indicator

- Remove the force from the bicycle frame.

## BACKGROUND

Engineering structures may take many forms, from the simple shapes of square cross section beams to the complex and intricate shapes of trusses. Trusses are one of the major types of engineering structures, providing practical and economical solutions to many engineering situations. Trusses consist of straight members connected at joints (for example, see Figure 1). Note that truss members are connected at their extremities only: thus no truss members are continuous through a joint.

In general, truss members are slender and can support little lateral force. Therefore, major forces must be applied to the various joints and not the members themselves. Often the weights of truss members are assumed to be applied only at the joints (half the weight at each joint). In addition, even though the joints are actually rivets or welds, it is customary to assume that the truss members are pinned together (i.e., the force acting at the end of each truss member is a single force with no couple). Each truss member may then be treated as a two force member and the entire truss is treated as a group of pins and two force members.

A bicycle frame, on first inspection, appears to be an example of a truss. Each tube (truss member) is connected to the other at a joint, the principal forces are applied at joints (e.g., seat, steering head, and bottom bracket), and the reaction forces are carried at joints as well (e.g., front and rear axles). Although the joints are not pinned, a reasonable first approximation for analyzing forces, deflections, and stresses in the various tubes of the bicycle frame might be made using a simple truss analysis.

Forces in various truss members can be found using such analysis techniques as the method of joints or the method of sections. Deflections at any given joint may be found by using such analysis techniques as the unit force method of virtual work.

## REFERENCES

ME354 NOTES on Strain Gages

ANALYSIS

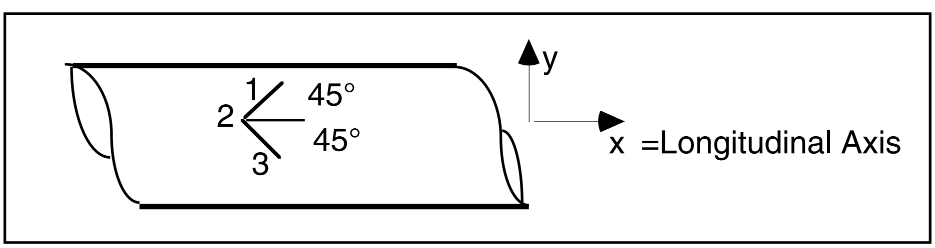
**1)** Draw a free body diagram of the truss, showing your assumptions for the reactions at the front and rear axles.

**2)** For the applied force P, use the assumed dimensions and angles of Fig. 3 along with a simple truss analysis to complete the table. Note: for truss forces, use "+" to indicate tensile force and "-" to indicate compressive force.

| | |
|---|---|
| Applied Force, P (N) | |
| Reaction at Front Axle, Rf (N) | |
| Reaction at Rear Axle, Rr (N) | |
| Force in Top Tube Ftt (N) | |
| Force in Down Tube Fdt (N) | |
| Force in Seat Tube Fst (N) | |
| Force in Seat Stay Fss (N) | |
| Force in Chain Stay Fcs (N) | |
| Force in Head Tube/Fork Ff (N) | |

**3)** Use the strain gage information to find the x-y-z coordinate strains in each member of interest, noting that the orientation of the individual strain gages in each rosette is as shown. Note: this may require using the three strains from the rosette to find the principal strains and then using the complete strain state to find the strains acting in the directions of interest of each member. Use local coordinates on each member to define the coordinate strains. In all cases define x as being along the longitudinal axis of the member, y being in the plane of the member's surface and z being normal to the surface.

| Member | x- Strain (microstrain) | y - Strain (microstrain) | z-Strain (microstrain) |
|---|---|---|---|
| Top Tube (top SG) | | | |
| Top Tube (bottom SG) | | | |
| Down Tube | | | |
| Seat Tube | | | |
| Seat Stay | | | |
| Chain Stay | | | |
| Head Tube/Fork | | | |



**4)** Use 3-D (i.e., Generalized) Hooke's law to find the stress acting in the x-direction (longitudinal direction). Note: $\sigma_x = \frac{E}{(1+\nu)}\varepsilon_x + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right)$ and E for the steel tubes is 200 GPa.

| Member | X- Stress (MPa) |
|---|---|
| Top Tube (top SG) | |
| Top Tube (bottom SG) | |
| Down Tube | |
| Seat Tube | |
| Seat Stay | |
| Chain Stay | |
| Head Tube/Fork | |

**5)** Use the longitudinal stress calculated in Part **4)** to estimate the longitudinal force in each member. Note: Note that this analysis requires the assumption that the stress is uniform across the cross section. The cross sectional dimensions for the members are as follows and the cross secitonal area is $A=(\pi/4(OD^2-ID^2)$:

Top Tube:OD=28.8 mm, ID=26.5 mm; Down Tube: OD=32.2 mm, ID=29.9 mm,
Seat Tube: OD=28.8 mm, ID=26.5 mm; Head Tube/Fork:OD=34 mm, ID=31.7 mm,
Seat and Chain Stays:OD=16.1 mm; ID=13.8 mm.

| Member | X- Stress (MPa) | Cross Sectional Area ($mm^2$) | Longitudinal Force =stress *area (N) |
|---|---|---|---|
| Top Tube (top SG) | | | |
| Top Tube (bottom SG) | | | |
| Down Tube | | | |
| Seat Tube | | | |
| Seat Stay | | | |
| Chain Stay | | | |
| Head Tube/Fork | | | |

**6)** Compare the measured longitudinal forces to the longitudinal forces calculated using the simple truss analysis. Explain any differences by the answering the questions:

i) What assumptions were made in the truss analysis?

ii) What assumptions were made in analyzing the strain gage results to find the forces?

iii) From the strain gage results for the top tube, is the stress distribution uniform across the cross section of the tube? If not, is the truss analysis of uniform axial forces valid?

| Member | Longitudinal Force from Truss Analysis (N) | Longitudinal Force from Strain Gage Analysis (N) | % difference |
|---|---|---|---|
| Top Tube (top SG) | | | |
| Top Tube (bottom SG) | | | |
| Down Tube | | | |
| Seat Tube | | | |
| Seat Stay | | | |
| Chain Stay | | | |
| Head Tube | | | |

**7)** Note that because of the choice of the locations (i.e., A and J) for obtaining strain information, it is possible to separate axial stresses (P/A) from uniaxial bending (Mc/I) at the center of the top tube. By taking the average of the total X-stress at A and total stress at J, the bending component cancels and the axial stress acting in the top tube is obtained.

$$(\text{X-stress}^{A} + \text{X stress}^{J})/2 = \text{axial stress}$$

The axial force can then be obtained by multiplying the axial stress times the cross sectional area.

$$\text{axial force} = \text{axial stress x cross sectional area}$$

The principle of superposition allows the addition of the axial and bending stresses because they are the same type of stress (i.e., normal) acting in the same direction. (i.e., X-stress=axial stress + bending stress). Therefore, once the axial stress is found, the bending stress can be obtained by subtracting the axial stress from the total X-stress

$$\text{bending stress} = \text{X-stress} - \text{axial stress}$$

**8)** As it turns out, due to the variability of the loading scenarios, the stress state in a bicycle frame is more complex than can be analyzed using a simple truss analysis or the simple assumption of uniformly stressed tubes. Finite element analysis (FEA) lends itself to solving this complex stress state. Using the results of an FEA of a model of the bicycle frame for the applied force of this test, quantitatively and qualitatively compare the stresses at the various locations and tubes.

i) Are the stresses uniform across the cross sections?

ii) What are the effects of bending and torsion on the stress state?

iii) Are the axial, bending, and total stresses constant over the lengths of the tubes?

iv) Are there any stress concentrations (e.g., are the maximum stresses greater at the joints than in the middle?

v) Compare the axial (longitudinal) forces determined from the truss analysis to that determined from the strain gage analysis (from the axial stress after substracting the bending stress) to the that determined from the FEA for the top tube. Does bending significantly affect the results?

| Member | Longitudinal Force from Truss Analysis (N) | Longitudinal (Axial) Force from Strain Gage Analysis (N) | Axial Force from FEA (N) |
|---|---|---|---|
| Top Tube | | | |
| | Axial Stress from Truss Analysis (MPa) | Axial Stress (no bending) from Strain Gage Analysis (MPa) | Axial Stress (no bending) from FEA (MPa) |
| Top Tube | | | |
| | Total X-stress from Truss Analysis (MPa) | Total X-stress from Strain Gage Analysis (MPa) | Total X-stress from FEA (MPa) |
| Top Tube (top) | | | |
| Top Tube (bottom) | | | |

**9)** Deflections in trusses can often be found using energy methods. Again, to simplify the analysis it is assumed that the axial force in each tube only acts at the joints and therefore the axial force is constant throughout the length of each member. The unit force method is used as follows in which the deflection at the point of interest is:

$$\Delta = \sum \frac{N_U N_L L}{EA}$$

where $N_U$ and $N_L$ are the forces in each member due to unit and actual forces (in this case use the forces found from the truss analysis, not the experimental measurement), respectively, L is the length of each member, E is the elastic modulus of each member and A is the cross sectional area of each member. In this case, the deflection of interest at the bottom bracket is in the same direction and at the same location as the applied force. Nonetheless, the unit force method can still be used by filling in the appropriate sections of the table where E=200,000 MPa for all members.

| Member | L (mm) | A $(mm^2)$ | $N_L$ [due to P] (N) | $N_U$ [due to unit P] (N) | $\frac{N_U N_L L}{EA}$ |
|---|---|---|---|---|---|
| Top Tube | | | | | |
| Down Tube | | | | | |
| Seat Tube | | | | | |
| Seat Stay | | | | | |
| Chain Stay | | | | | |
| Head Tube/fork | | | | | |

$$\Delta = \sum \frac{N_U N_L L}{EA} =$$ ________________

**10)** Compare the measured deflection at the bottom bracket to the deflection predicted from the unit force method due to axial forces only and the FEA model. Comment on any difference and the reasons (for example, assumptions of the unit force method for deflection or truss analysis for the axial forces). Suggest a other ways to predict the deflections at joints.

| Measured Deflection (mm) | Deflection for Unit Force Analysis (mm) | % difference | Measured Deflection (mm) | Deflection from FEA Model (mm) | % difference |
|---|---|---|---|---|---|
| | | | | | |

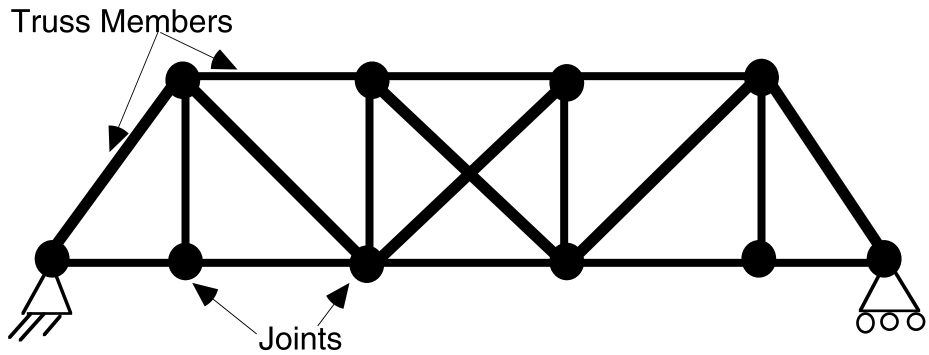



Figure 1 Example of a Simple Truss

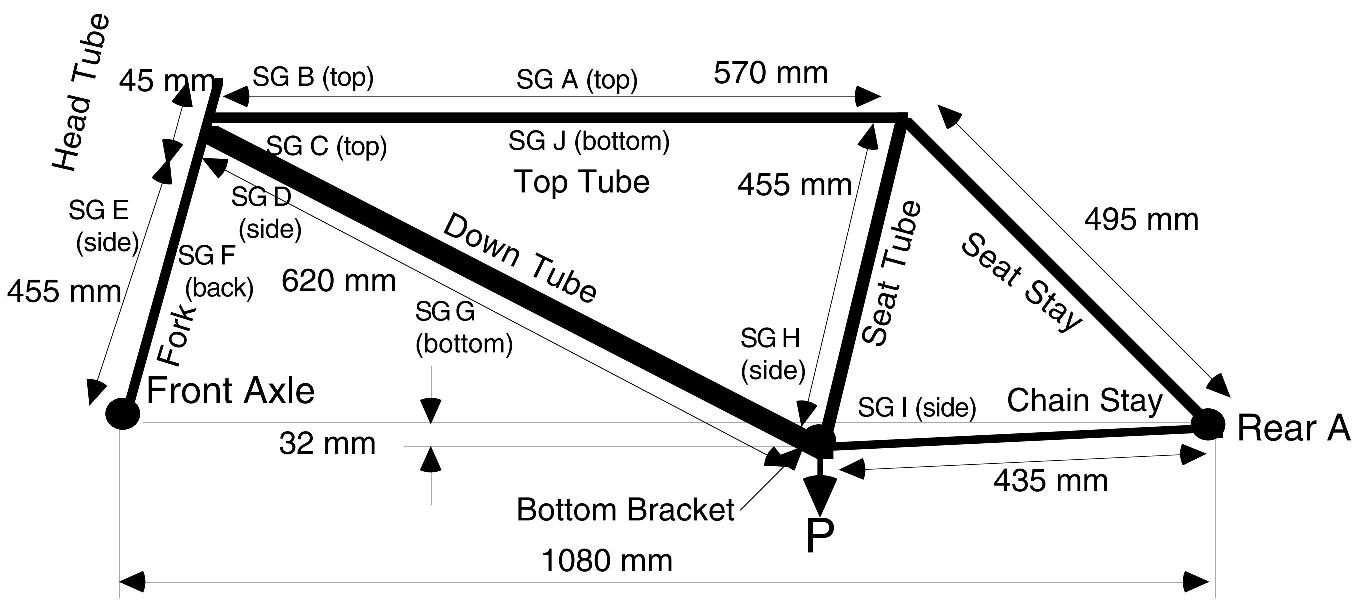
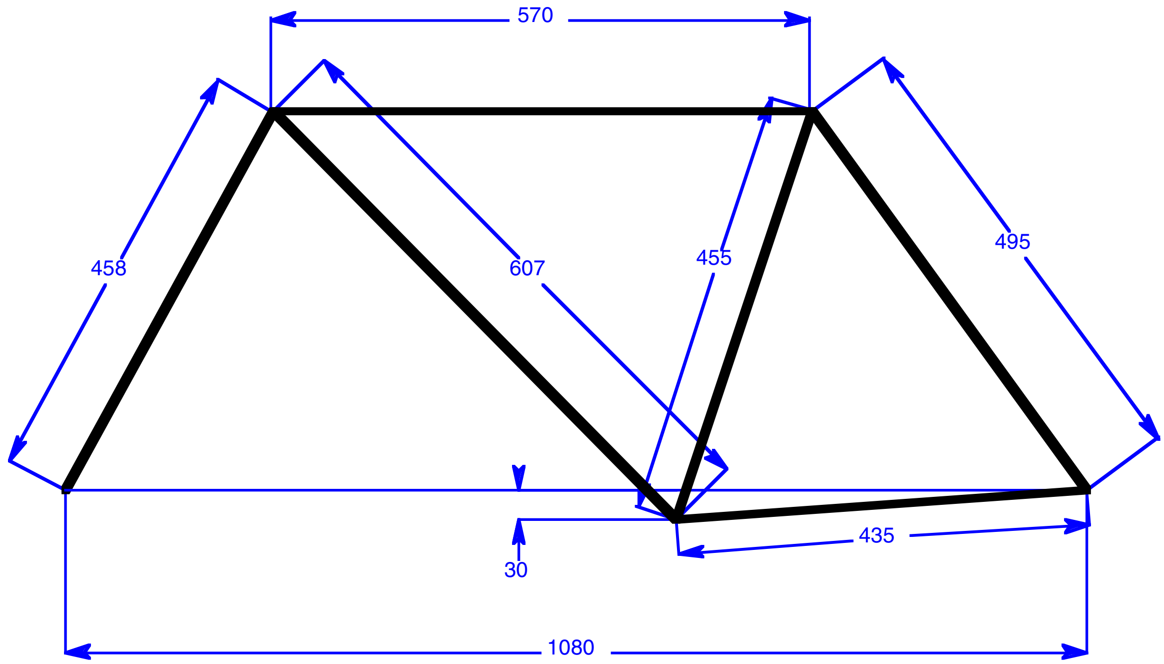
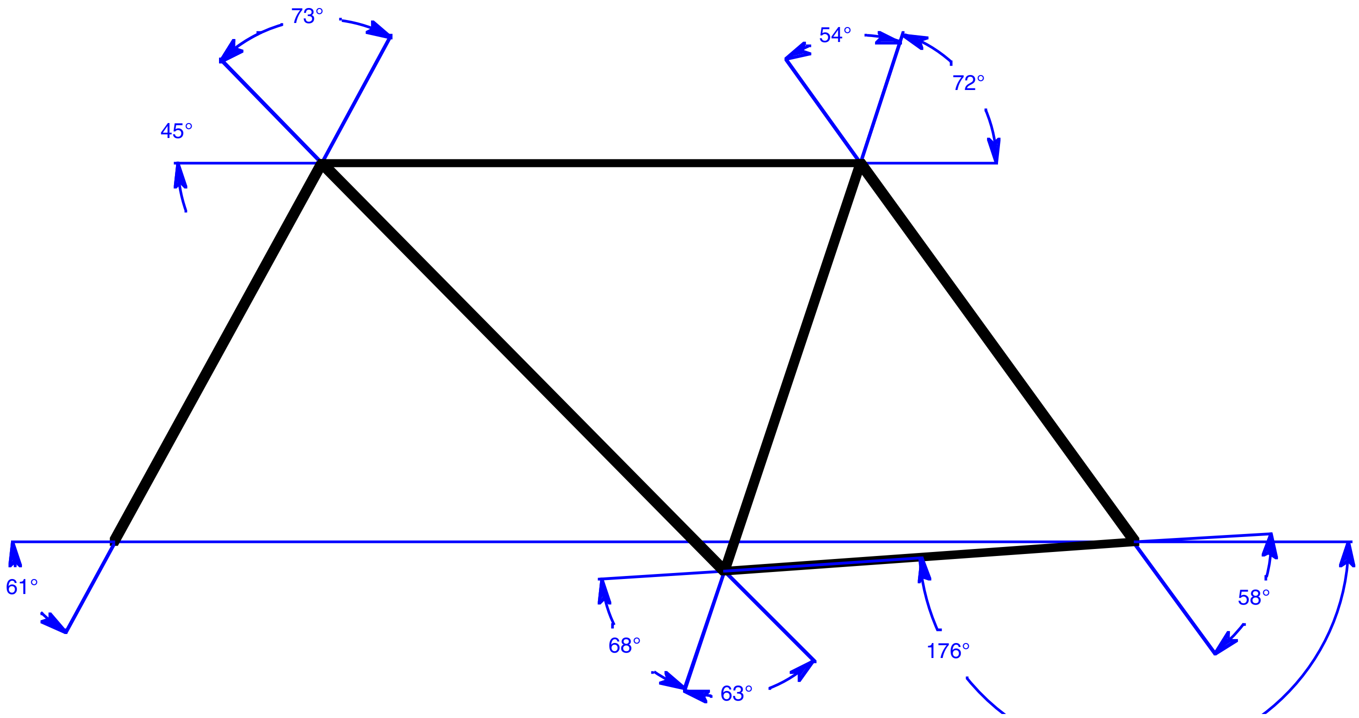



Figure 2 As Measured Dimensions, Nomenclature, and Strain Gage Locations on Bicycle Frame




a) assumed linear dimensions (mm)




b) assumed angular dimensions (degrees)

Figure 3  Assumed Dimensions and Angles for Simplified Truss Analysis

LABORATORY REPORT

1. As a minimum include the following information in the laboratory report.
   a) Free body diagram of the truss, showing your assumptions for the reactions at the front and rear axles.
   b) For forces in each member found from a simple truss analysis. Note: for truss forces, use "+" to indicate tensile force and "-" to indicate compressive force.

| | |
|---|---|
| Applied Force, P (N) | |
| Reaction at Front Axle, Rf (N) | |
| Reaction at Rear Axle, Rr (N) | |
| Force in Top Tube Ftt (N) | |
| Force in Down Tube Fdt (N) | |
| Force in Seat Tube Fst (N) | |
| Force in Seat Stay Fss (N) | |
| Force in Chain Stay Fcs (N) | |
| Force in Head Tube/Fork Ff (N) | |

   c) Comparison of the longitudinal forces in each member from the truss analysis and the experimental measurements.

| Member | Longitudinal Force from Truss Analysis (N) | Longitudinal Force from Strain Gage Analysis (N) | % difference |
|---|---|---|---|
| Top Tube (top SG) | | | |
| Top Tube (bottom SG) | | | |
| Down Tube | | | |
| Seat Tube | | | |
| Seat Stay | | | |
| Chain Stay | | | |
| Head Tube | | | |

   d) Comparison of the deflection at the bottom bracket found from the experimental measurements, energy methods, and FEA model.

| Measured Deflection (mm) | Deflection for Unit Force Analysis (mm) | % difference | Measured Deflection (mm) | Deflection from FEA Model (mm) | % difference |
|---|---|---|---|---|---|
| | | | | | |

2. As a minimum, discuss the following in the laboratory report.

a) Answers to these questions (DO NOT simply answer the questions, but instead use the questions are starting points for explanations about the results:

i) What assumptions were made in the truss analysis?

ii) What assumptions were made in analyzing the strain gage results to find the forces?

iii) From the strain gage results for the top tube, is the stress distribution uniform across the cross section of the tube? Can the axial stress be separated from any bending stress, if any? If the stress distribution is not uniform, is the truss analysis assuming uniform axial forces valid?

iv) From the FEA model, are the stresses uniform across the cross sections?

v) From the FEA model, what are the effects of bending and torsion on the stress state?

vi) From the FEA model, are the stresses constant over the lengths of the tubes?

vii) From the FEA model, are there any stress concentrations?

viii) From the FEA model, how do the deflections compare?

b) Error analysis in the measurements.

3. At a minimum, include the following information in the appendix of the laboratory report. THIS MAY NOT BE ALL THAT IS NECESSARY (i.e., don't limit yourself to this list.)

a. Original data sheets and/or printouts
b. All supporting calculations. Include sample calculations if using a spread sheet program.

ME 354, MECHANICS OF MATERIALS LABORATORY

# STRUCTURES

01 January 2000 / mgj

## DATA SHEET

**NAME________________________________________DATE____________**

**EQUIPMENT IDENTIFICATION________________________________________**

| | |
|---|---|
| Applied Force, P (kg) | |
| Total Deflection at Bottom Bracket (mm) | |
| "Machine" Deflection at Reaction Point (mm) | |

| | Gage 1 (microstrain) | | Gage 2 (microstrain) | | Gage 3 (microstrain) | |
|---|---|---|---|---|---|---|
| | Initial | Final | Initial | Final | Initial | Final |
| Location A | | | | | | |
| Location B | | | | | | |
| Location C | | | | | | |
| Location D | | | | | | |
| Location E | | | | | | |
| Location F | | | | | | |
| Location G | | | | | | |
| Location H | | | | | | |
| Location I | | | | | | |
| Location J | | | | | | |