Appendix B: Laboratory Exercise Handouts

This appendix contains handouts for the various laboratory exercises as follows:

NOTES ON STRAIN GAGES	(used for various laboratory exercises involving strain gages)
STRAINS, DEFLECTIONS AND BEAM BENDING	(laboratory handout for preformatted report)
STRESSES IN STRAIGHT AND CURVED BEAMS	(laboratory handout for preformatted report)
MECHANICAL PROPERTIES & PERFORMANCE: OF MATERIALS: tension, hardness, torsion, impact	(laboratory handout for formal, written report)
STRESS CONCENTRATIONS	(laboratory handout for preformatted report)
FRACTURE	(laboratory handout for preformatted report)
COMPRESSION AND BUCKLING	(laboratory handout for preformatted report)
TIME-DEPENDENT FAILURE: FATIGUE	(laboratory handout for preformatted report)
TIME-DEPENDENT DEFORMATION: CREEP	(laboratory handout for preformatted report)
STRUCTURES	(laboratory handout for formal, written report)

NOTES ON STRAIN GAGES

ME 354, MECHANICS OF MATERIALS LABORATORY

NOTES on Strain Gages

01 January 2000 / mgj

RESISTANCE FOIL STRAIN GAGES

In 1856 Lord Kelvin reported that the electrical resistance of copper and iron wires increased when subjected to tensile stresses. This observation ultimately led to the development of the modern "strain gage" independently at California Institute of Technology and Massachusetts Institute of Technology in 1939. The underlying concept of the strain gage is very simple. In essence, an electrically-conductive wire or foil (i.e. the strain gage) is bonded to the structure of interest and the resistance of the wire or foil is measured before and after the structure is loaded. Since the strain gage is firmly bonded to the structure, any strain induced in the structure by the loading is also induced in the strain gage. This causes a change in the strain gage resistance thus serving as an indirect measure of the strain induced in the structure.

Originally, strain gages were made of wire and, in fact, wire strain gages are still in use under special circumstances. However, today foil strain gages are most widely used. A typical strain gage is shown in the sketch below. The strain sensing region of the strain gage is called the "gage grid." The grid is etched from a thin metallic foil. The orientation of the grid defines the strain sensing axis of the strain gage. Electrical connections are made by soldering lead wires to the strain gage "solder tabs." The entire strain gage is bonded to a thin polymeric backing which helps protect and support the delicate metal foil.

Foil strain gages are available in literally hundreds of shapes and sizes. The strain gage shown is called a "uniaxial strain gage." Other common strain gage configurations are:

Biaxial strain gages which consist of two individual strain gage elements oriented precisely 90° apart, allowing strain measurements in two orthogonal directions.

Rectangular, three-element strain gage rosettes which consist of three individual strain gage elements oriented precisely 45° apart, allowing the resolution of principal strains and principal directions regardless of the orientation of the rosette or the applied stress/strain.

Delta, three-element strain gage rosettes which consist of three individual strain gage elements oriented precisely 60° apart, allowing the resolution of principal strains and principal directions regardless of the orientation of the rosette or the applied stress/strain.

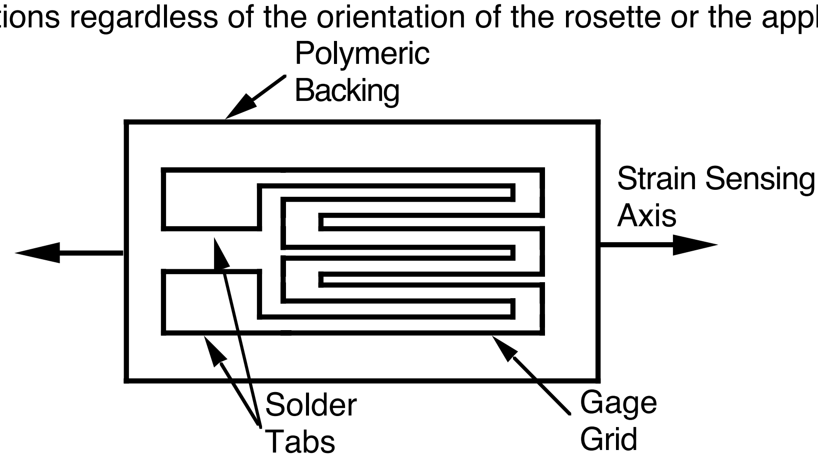


FIGURE 1 - Illustration of a typical, uniaxial resistance foil strain gage

STRAIN GAGE RESISTANCE

Strain gage manufacturers produce strain gages with three standard resistances: 120 Ω, 350 Ω, and 1000 Ω. The user specifies the desired resistance when ordering the strain gage. The 120 Ω resistance is the most commonly used, although 350 Ω and 1000 Ω strain gages are widely available.

As discussed previously, strains are sensed by bonding a strain gage to a structure of interest and subsequently measuring the strain gage resistance before and after the structure is loaded. Consider the magnitude of the resistance change which must typically be measured. Assume a measurement resolution of 10×10^{-6} m/m = 10 μm/m is required (a typical measurement). The change in resistance which corresponds to a strain of 10 μm/m can be calculated using Eq. 1:

$$\Delta R_g = (F_g)(R_g)(\varepsilon_m) = (2.00)(120\Omega)(10 \times 10^{-6} m/m) = 0.0024\Omega \qquad (1)$$

where ΔR_g is the change in resistance, F_g is the gage factor, R_g is the initial gage resistance, and ε_m is the strain in the strain gage. Thus, a resistance change from 120 Ω to 120.0024 Ω must measured...a very small change indeed!!! In fact, it is very difficult to measure such small changes in resistance using "normal" ohmmeters. Instead, special "strain gage circuits" are used to measure these small resistance changes accurately and precisely. The most widely used strain gage circuit is the "Wheatstone bridge" and is described in the following section.

THE WHEATSTONE BRIDGE

As show in Fig. 2, the Wheatstone bridge circuit consists of four "arms." Each arm contains a resistance (i.e. resistances, R_1, R_2, R_3, and R_4). An excitation voltage V_{ex} (typically 2 to 10 volts) is applied across junction A-C and a voltmeter is used to measure the resulting potential across junctions B-D (voltage ΔE). If all the resistances are equal (i.e. $R_1 = R_2 = R_3 = R_4$) then $\Delta E = 0$ and the bridge is said to be balanced.

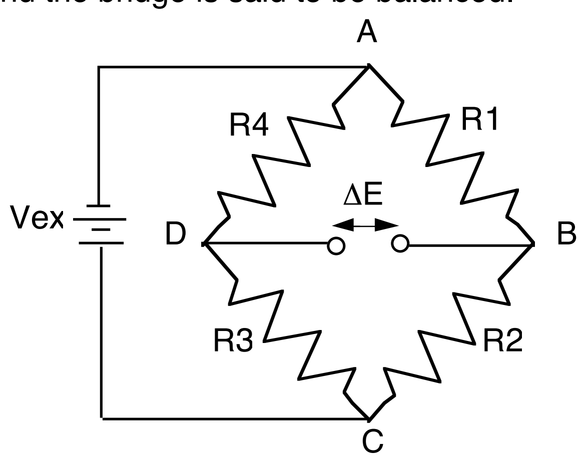


FIGURE 2 - Schematic Diagram of a Wheatstone Bridge

Typically, a 1/4 (quarter) arm Wheatstone bridge circuit is used for individual strain gages where resistance R_1 is the strain gage (i.e., $R_1 = R_g$) and the other three resistances are precision resistors equal to the nominal resistance of R_g (e.g. $R_2 = R_3 = R_4 = 120\ \Omega$)[1] . If the strain gage experiences a strain, the strain gage resistance changes, causing the bridge to become unbalanced. The resulting voltage, ΔE is given by:

$$\Delta E = \frac{V_{ex}}{4}\left[\frac{\Delta R_g}{R_g}\right] \qquad (2).$$

Combining Eqs. 1 and 2 yields:

$$\varepsilon_m = \frac{4}{F_g}\left[\frac{\Delta E}{V_{ex}}\right] \qquad (3)$$

Equation 3 is an important result. It shows that the strain in the strain gage, ε_m, is related to the quantities, F_g, ΔE, and V_{ex}. Generally though, Eq. 3 is not applied directly, Instead, strain gage amplifiers which have been calibrated according to Eq. 3 are used to provide a direct readout of strain.

THE GAGE FACTOR

Strain gage manufacturers perform standard calibration measurements for each lot of strain gages they produce. When a user purchases a strain gage, the manufacturer provides the results of these measurements in the form of several calibration constants. One of these constants is the "gage factor." The gage factor allows the user the convert the change in gage resistance to the corresponding strain level. Specifically, the strain measured in an individual strain gage is related to the change in the strain gage resistance such that:

$$\varepsilon_m = \frac{1}{F_g}\left[\frac{\Delta R_g}{R_g}\right] \qquad (4)$$

The value of the gage factor depends on the metallic alloy used and varies slightly from lot to lot, typically being in the range of 2.0 to 2.1.

THREE-ELEMENT STRAIN GAGE ROSETTES

Three-element strain gage rosettes are used when it is desired to measure ε_x, ε_y, and γ_{xy}, induced at a point (or equivalently, when the principal strains and principal directions are unknown). Referring to the x-y coordinate system in Fig. 3, recall that the normal strain induced in an arbitrary direction from the x-axis defined by angle θ (strain ε_θ) is related to the strains in the x-y coordinate system according to:

---

[1] Actually, the resistances need not be identical. Instead, all that is required is that $(R_1/R_2) = (R_3/R_4)$. However, for the present purposes, it is sufficient to assume $(R_1=R_2= R_3=R_4)$

$$\varepsilon_\theta = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\cos\theta\sin\theta \quad (5)$$

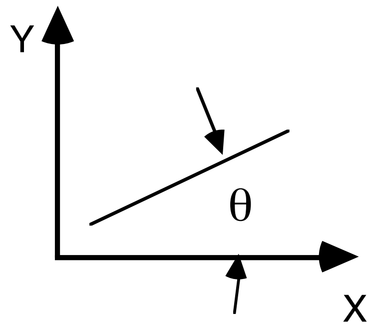


FIGURE 3 - Single Strain Arbitrarily Oriented to the X-Y Coordinate System.

The strain, ε_θ, can be measured by simply mounting a strain gage in the direction defined by angle θ. In solving Eq. 5, ε_θ and θ are known but there are still three unknowns, ε_x, ε_y, and γ_{xy}. Thus, to solve for the unknowns, two more equations are required for a total of three equations (i.e. three equations, three unknowns). If a total of three strain gages are mounted in three distinct but arbitrary directions (θ_1, $\theta 2$, and θ_3) as shown in Fig. 4, then Eq. 5 can be applied three times such that:

$$\varepsilon_{\theta 1} = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\cos\theta_1\sin\theta_1 \quad (6a)$$

$$\varepsilon_{\theta 2} = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\cos\theta_2\sin\theta_2 \quad (6b)$$

$$\varepsilon_{\theta 3} = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\cos\theta_3\sin\theta_3 \quad (6c)$$

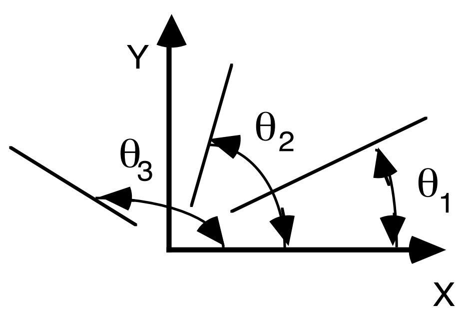


FIGURE 4 - Three Strains Arbitrarily Oriented to the X-Y Coordinate System.

Equations 6a-6c are called the general rosette equations. If θ_1 is set equal to 0°, θ_2 to 45° and θ_3 to 90° the resulting strain gage configuration is called a rectangular rosette as shown in Fig. 5a. The resulting equations for a rectangular rosette are:

$$\varepsilon_{0°} = \varepsilon_x \cos^2(0°) + \varepsilon_y \sin^2(0°) + \gamma_{xy} \cos(0°)\sin(0°) \quad (7a)$$

$$\varepsilon_{45°} = \varepsilon_x \cos^2(45°) + \varepsilon_y \sin^2(45°) + \gamma_{xy} \cos(45°)\sin(45°) \quad (7b)$$

$$\varepsilon_{90°} = \varepsilon_x \cos^2(90°) + \varepsilon_y \sin^2(90°) + \gamma_{xy} \cos(90°)\sin(90°) \quad (7c)$$

which can be reduced to:

$$\varepsilon_x = \varepsilon_{0°} \quad (8a)$$

$$\varepsilon_y = \varepsilon_{90°} \quad (8b)$$

$$\gamma_{xy} = 2\varepsilon_{45°} - (\varepsilon_{0°} + \varepsilon_{90°}) \quad (8c)$$

Similarly, if θ_1 is set equal to 0°, θ_2 to 60° and θ_3 to 120°, the resulting strain gage configuration is called a delta rosette as shown in Fig. 5b. The resulting equations for a delta rosette are:

$$\varepsilon_{0°} = \varepsilon_x \cos^2(0°) + \varepsilon_y \sin^2(0°) + \gamma_{xy} \cos(0°)\sin(0°) \quad (9a)$$

$$\varepsilon_{60°} = \varepsilon_x \cos^2(60°) + \varepsilon_y \sin^2(60°) + \gamma_{xy} \cos(60°)\sin(60°) \quad (9b)$$

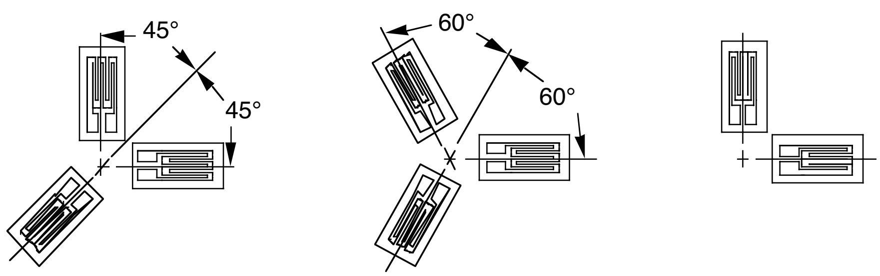
$$\varepsilon_{120°} = \varepsilon_x \cos^2(120°) + \varepsilon_y \sin^2(120°) + \gamma_{xy} \cos(120°)\sin(120°) \quad (9c)$$

which can be reduced to:

$$\varepsilon_x = \varepsilon_{0°} \quad (10a)$$

$$\varepsilon_y = \frac{1}{3}\left[2(\varepsilon_{60°} + \varepsilon_{120°}) - \varepsilon_{0°}\right] \quad (10b)$$

$$\gamma_{xy} = \frac{2\sqrt{3}}{3}\left[\varepsilon_{60°} - \varepsilon_{120°}\right] \quad (10c)$$



a) Rectangular Rosette b) Delta Rosette c) Biaxiial Strain Gage

FIGURE 5 - Rectangular and Delta Rosettes as well as a Biaxial Strain Gage.

STRAINS, DEFLECTIONS AND BEAM BENDING

ME 354, MECHANICS OF MATERIALS LABORATORY

STRAINS, DEFLECTIONS AND BEAM BENDING LABORATORY*

09 october 2000 / mgj

PURPOSE

The purposes of this exercise are a) to familiarize the user with strain gages and associated instrumentation, b) to measure deflections and strains and to compare these results to predicted values, and c) to verify certain aspects of stress-strain relations and simple beam theory.

EQUIPMENT

- Simply-supported 6061-T6 aluminum channel beam instrumented with uniaxial and rosette strain gages.
- Strain gage conditioning equipment and readout unit for an analog strain conditioning system
- A deflectometer (dial indicator) and a load cell ring.

PROCEDURE

- Read the reference document "NOTES on Strain Gages."
- Carefully examine attached Figs. 1 to 3. Note that a total of 10 Wheatstone bridge circuits are involved. Identify all strain gage circuits and strain gage channel numbers on the Figs 1 to 3 as well as on the aluminum beam itself.
- Record the location along the beam length of the turnbuckle loading device.
- Loosen the turnbuckle and prepare the strain gage conditioning equipment according to the manufacturer's instructions. (Note: Use the F_g appropriate for the strain gages and circuit).
- If not already done so, balance each strain gage circuit to zero or a reasonable minimum value. Record this offset strain (starting value with no force applied) on the data sheet for each channel.
- Record the starting reading of the dial indicator on the data sheet and note its position along the beam.
- Apply a modest concentrated force to the beam by tightening the turnbuckle to achieve a reading of ~400 μm/m for the load cell ring (channel 10).
- Record the reading for each channel on the data sheet.
- Record the reading of the dial indicator
- Repeat the force application and data recording for a total of two other forces. (Note: use absolute (after correcting for any initial offset) values for the load cell ring of ~400, and ~800 μm/m).
- Loosen the turnbuckle after completing all force cases.

BACKGROUND AND ANALYSIS OF RESULTS

Experimental mechanics is that branch of engineering mechanics involving the measurement of strains, displacements, stresses and forces acting on or within models, components and/or structures. One of the most useful and widespread measurement devices used in experimental mechanics is the resistance strain gage.

Properly configured multiple strain gages can be arranged in such a way that the complete strain state acting at a point can be determined. Then, by using the correct constitutive relations (e.g., Hooke's law), the complete stress state at that point can be calculated. The principal stresses and strains along with principal directions can be determined from the resulting information. Thus, without even knowing the magnitude or direction of applied loads (e.g., forces, temperature, pressure, etc), the complete stress and strain states along with their maximum values and their orientations can be determined.

For simple structures such as simple beams in bending, mechanics of materials solutions which describe the strain and stress state anywhere in the beam exist. Therefore, it is often not necessary to use experimental mechanics to determine the strain and stress states because the values are readily calculated from the applied force, the loading configuration and the beam dimensions.

However, it is still useful to apply strain gages to a simple beam in bending to verify analytical models or to confirm the response of a particular measurement system. In this laboratory exercise, the force (i.e., reaction force) at the load cell support is calculated as:

$$P_{RLC}\,(N) = C \times \varepsilon_{10}\,(\mu m/m). \tag{1}$$

where C is the calibration constant for the load cell ring in units of N / μm/m. Note that the readout for channel number 10 (ε_{10}) is in micro strain (μ m/m=10^{-6} m/m) for a full strain gage bridge (i.e., four strain gages) adhered to the load cell ring. Note also that P_{RLC} is the reaction force at the load cell ring and not the applied force, P, from tightening the turnbuckle.

After calculating the reaction force, the applied force can be calculated using simple statics relations. Shear (V) and moment (M) diagrams for the test setup and test specimen can then be determined. Using the beam dimensions, the moment of inertia and position of the neutral axis for the aluminum channel beam can be determined.

According to beam theory, a bending moment, M, causes a uniaxial normal stress, σ_x, given by Eq. 2. Since the stress is uniaxial,(although not uniform across the height), Eq. 3 can be used to predict the uniaxial normal strain, ε_x, using the stress calculated from the applied force, test setup dimensions, test specimen dimensions (y for distance from the neutral axis, and I for the moment of inertia) and the elastic modulus, E, of the material.

$$\sigma_x = \frac{My}{I} \tag{2}$$

$$\varepsilon_x = \frac{\sigma_x}{E} \tag{3}$$

Strains predicted using Eqs 2 and 3 can then be compared to those measured with the strain gages at cross section A-A (See Figs. 1 and 3). Principal strains occuring at the sites of the rectangular and delta strain gage rosettes can also be determined from the measurements and compared to strains calculated using Eqs. 2 and 3 at each force level. The orientation of the principal strain coordinate system with respect to the longitudinal axis of the beam, X (see Figs. 1 and 2) can also be determined.

It is well known that the strain and stress distributions across the height of a beam in bending is non uniform. For example, the stress distribution may be maximum at the bottom surface of the beam and a minimum at the top surface. The location in the beam where the stress is zero is called the neutral axis (N/A) and coincides with the centroid of the beam's cross section in straight beams. The location of the centroid can be calculated from the dimensions of the beam's cross section. The measured strains (e.g., strains from strain gages 7, 8 and 9 as shown in Fig. 3) across this cross section can be plotted as a function of distance from a fixed reference such as the bottom of the beam. It is then possible to compare the distance at which the measured strain is zero to the theoretical calculation of the distance for the centroid (neutral axis).

According to beam theory, the vertical deflection of the beam at any longitudinal location can be related to the applied force, the moment of inertia, the beam's dimensions, and the elastic modulus of the beam's material. The relation for this deflection also depends on the type of reaction supports. Knowledge of mechanics of materials can be used to determine the deflection relation (i.e., elastic curve) for this setup. The predicted deflection and measured deflection can be compared for each force.

Analytical and numerical models have been demonstrated and compared to experimental results. Although for simple loading geometries there may be little advantage for measuring the beam response, there may be advantages and disadvantages of using each method for predicting (or measuring) bending response.

* REFERENCES

ME354 NOTES on Strain Gages

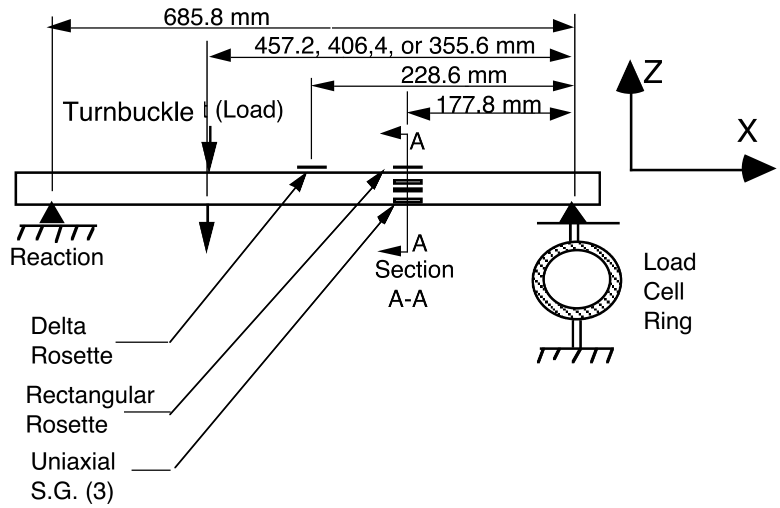


FIGURE 1 - Overall view of Test Specimen Geometry and Setup

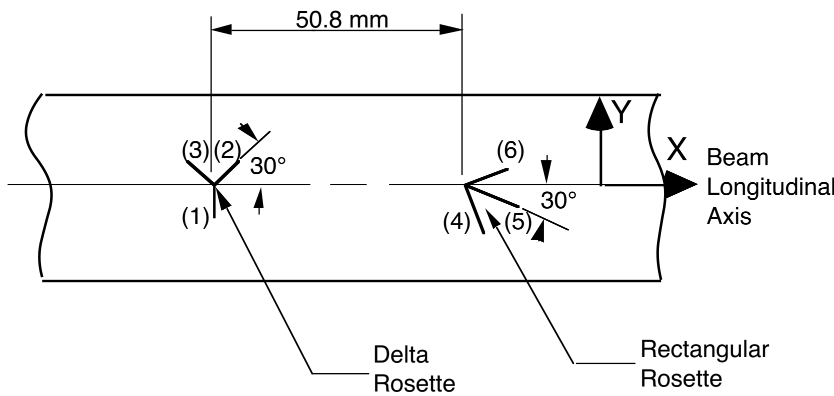
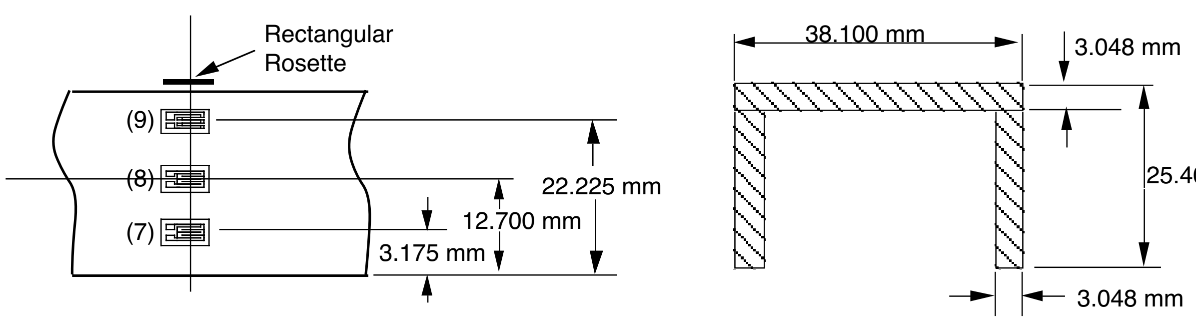


FIGURE 2 - Top View of Specimen Geometry Showing Orientations of 3-Element Strain Gage Rosettes. Note: Strain Gage Channel Numbers are Shown in Parentheses.



a) Side View of Uniaxial Strain Gage Locations.

b) Cross Section of Beam

FIGURE 3 - Strain Gage Locations and Cross Sectional Dimensions of the Beam. Note: Strain Gage Channel Numbers are Shown in Parentheses.

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STRAIN, DEFLECTIONS, AND BEAM BENDING LABORATORY

09 october 2000 / mgj

DATA SHEET

NAME______________________________________**DATE**___________

LABORATORY PARTNER
NAMES____

____

EQUIPMENT IDENTIFICATION_______________________________________

____

____

	STRAIN CONDITIONER CHANNEL NUMBER*										
Test No.	**SG 1** (μm/m)	**SG 2** (μm/m)	**SG 3** (μm/m)	**SG 4** (μm/m)	**SG 5** (μm/m)	**SG 6** (μm/m)	**SG 7** (μm/m)	**SG 8** (μm/m)	**SG 9** (μm/m)	**SG 10** Proving Ring (μm/m)	**Deflection (mm)**	**Reaction Force (N)**
Initial Offset												
1												
2												
3												

* The load cell is connected to channel 10. Strain gages are connected to the remaining nine channels as shown in Figs. 2 and 3.

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STRAINS, DEFLECTIONS AND BEAM BENDING LABORATORY

04 october 2000 / mgj

WORK SHEET

NAME_______________________________________**DATE**_____________

1) Some properties of 6061-T6 aluminum alloy are contained in Table 1.

Table 1 Some Properties of 6061-T6 Aluminum Alloy

Elastic Modulus, E (MPa)	68,000-70,000
Yield Strength (0.2% offset), S_{YP} (MPa)	210-339
Poisson's ratio, ν	0.340-0.350
Ultimate Tensile Strength, S_{UTS} (MPa)	270-345
Percent Elongation, %el (50.8 mm gage length)	12-16

Note: Ranges represent ± 3 standard deviations about a mean

2) Analytical approach to the stress analysis of a beam in bending requires determination of the applied force (i.e. force at the turnbuckle) using the reaction force measured at one support.

For Force #1

Uncorrected strain from load cell ring, SG_{10}=_____ μm/m
Corrected strain from load cell ring, (SG_{10}-Offset) =ε_{10} =_____μm/m
Calibration Constant, C=_______ N/(μm/m)

Reaction force at load cell ring = P_{RLC} = C x ε_{10} = ________ N.

For Force #2

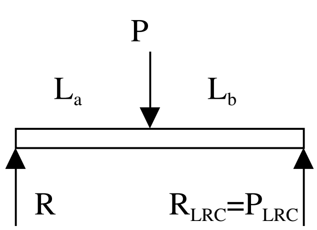
Uncorrected strain from load cell ring, SG_{10}=_____ μm/m
Corrected strain from load cell ring, (SG_{10}-Offset) =ε_{10} =_____μm/m
Calibration Constant, C=_______ N/(μm/m)

Reaction force at load cell ring = P_{RLC} = C x ε_{10} = ________ N.

3) Applied force at turn buckle determined using statics.

For Force #1

Total length of beam = L=____________mm
Distance from Load Cell Ring to Turnbuckle = L_b=_________mm
Distance from Opposite Support to Turnbuckle = L_a=L- L_b =_________mm
Applied Force at turnbuckle = P = $\frac{P_{RLC} L}{L_a}$ =__________N



For Force #2

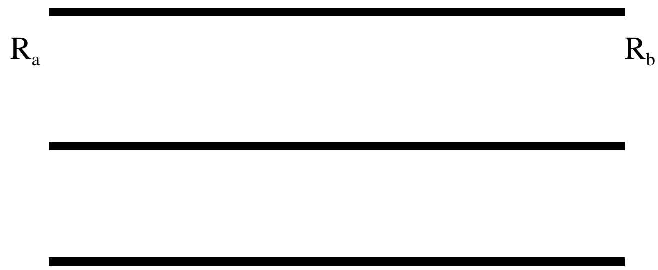
Total length of beam = L=____________mm
Distance from Load Cell Ring to Turnbuckle = L_b=_________mm
Distance from Opposite Support to Turnbuckle = L_a=L- L_b =_________mm
Applied Force at turnbuckle = P = $\frac{P_{RLC} L}{L_a}$ =__________N

4) A free body diagram (FBD) along with Shear and Moment diagrams are used to determined the shear and moments at the strain gage locations. Draw the generalized free body, shear and moment diagrams for the beam. Calculate the shear and moment values at each of the strain gage locations for each of the applied forces using the table.

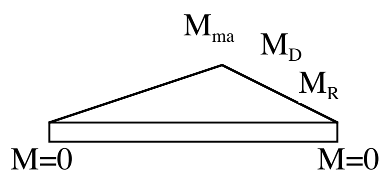
FBD

Shear V

Moment M

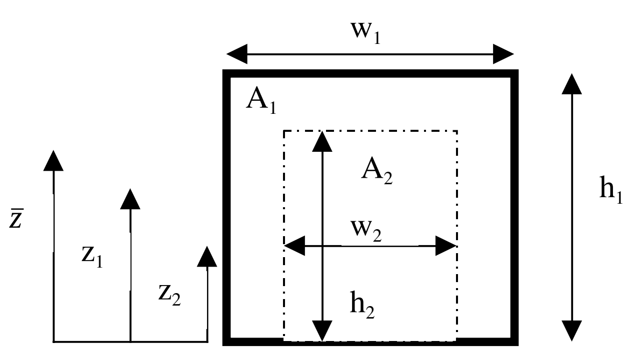


For Force #1
Shear force at Delta Rosette $=V_D=P_{RLC}=$________N
Shear force at Rectangular Rosette $=V_R=P_{RLC}=$_______N
Distance from the Load Cell Ring to the Delta Rosette$=X_D=$__________mm
Distance from the Load Cell Ring to Rectangular Rosette$=X_R=$__________mm
Moment at Delta Rosette= $M_D=\frac{PL_aX_D}{L}=$________N-mm
Moment at Rectangular Rosette = $M_R=\frac{PL_aX_R}{L}=$________N-mm



For Force #2
Shear force at Delta Rosette $=V_D=P_{RLC}=$________N
Shear force at Rectangular Rosette $=V_R=P_{RLC}=$_______N
Distance from the Load Cell Ring to the Delta Rosette$=X_D=$__________mm
Distance from the Load Cell Ring to Rectangular Rosette$=X_R=$__________mm
Moment at Delta Rosette= $M_D=\frac{PL_aX_D}{L}=$________N-mm
Moment at Rectangular Rosette = $M_R=\frac{PL_aX_R}{L}=$________N-mm

5) The dimensions of the beam can be used to determine the moment of inertia and centroid of the beams cross section. The cross section of the beam can be divided into two areas, A_1 (total area) and A_2 (removed, inner area). The centroid can be determined relative to the bottom (i.e. open side) of the beam.



Area / Section	Height (mm)	Width (mm)	Area (mm^2)	Distance from beam bottom (mm)	Area times distance from beam bottom (mm^3)
i=1	h_1=	w_1=	$A_1=h_1*w_1$=	$z_1=h_1/2$=	A_1*z_1=
i=2	h_2=	w_2=	$A_2=h_2*w_2$=	$z_2=h_2/2$=	A_2*z_2=

The vertical distance of the centroid from the bottom of the beam is calculated as $\bar{z} = \frac{\sum A_i z_i}{\sum z_i} = \frac{A_1 z_1 - A_2 z_2}{A_1 - A_2}$=______________mm. (Note that a negative sign is used with A_2 to take into account the "removed" area of A_2). Because of symmetry in the width direction the horizontal distance of the centroid from the front of the beam is the half the distance of the outer width of the beam=$\bar{y}$ =$w_1/2$=__________mm.

The moment of inertia can be calculated from the parallel axis theorem $I=\Sigma(I+A\ dz^2)$

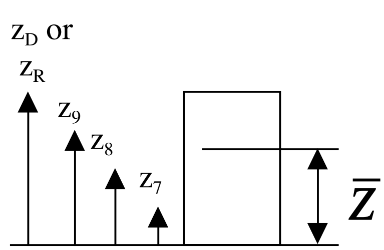
Area / Section	Centroid of Area (mm)	Difference between centroid of area and centroild of section (mm)	Area (mm^2)	Area times difference squared (mm^4)	Moment of inertia of Area (mm^4)
i=1	$z_1=h_1/2$=	$dz_1=\bar{z}-z_1$=	A_1 =	$A_1*(dz_1)^2$=	$I_1=(w_1*(h_1)^3)/12$=
i=2	$z_2=h_2/2$=	$dz_2=\bar{z}-z_2$=	A_2=	$A_2*(dz_2)^2$=	$I_2=(w_2*(h_2)^3)/12$=

The parallel axis theorem is now used to calculate the moment of inertia
$I = \sum\left(I_i + A_i\ dz_i^{\,2}\right) = \left(I_1 + A_1\ dz_1^2\right) - \left(I_2 + A_2\ dz_2^2\right)$=______________$mm^4$.

6) The uniaxial stresses and strains are calculated at each strain gage location. Note that z in each case is the distance from the specific point to the bottom (i.e. open side) of the beam.

For Force #1

Distance from N/A to Delta Rosette=$z_D-\bar{z}$=_________mm
Distance from N/A to Rectangular Rosette= $z_R-\bar{z}$ - =_________mm
Distance from N/A to Strain Gage 9 =$z_9-\bar{z}$=_________mm
Distance from N/A to Strain Gage 8 =$z_8-\bar{z}$=_________mm
Distance from N/A to Strain Gage 7 =$z_7-\bar{z}$=_________mm



For Force #2

Distance from N/A to Delta Rosette=$z_D-\bar{z}$=_________mm
Distance from N/A to Rectangular Rosette= $z_R-\bar{z}$ - =_________mm
Distance from N/A to Strain Gage 9 =$z_9-\bar{z}$=_________mm
Distance from N/A to Strain Gage 8 =$z_8-\bar{z}$=_________mm
Distance from N/A to Strain Gage 7 =$z_7-\bar{z}$=_________mm

Use Table 1 in this worksheet for any mechanical properties

For Force #1

Uniaxial Normal X-Stress at Delta Rosette = $\sigma_{XD} = -\frac{M_D(z_D - \bar{z})}{I}$ =___MPa
Uniaxial Normal X-Stress at Rectangular Rosette = $\sigma_{XR} = -\frac{M_R(z_R - \bar{z})}{I}$ =____MPa
Uniaxial Normal X-Stress at Strain Gage 9 = $\sigma_{X9} = -\frac{M_R(z_9 - \bar{z})}{I}$ =____MPa
Uniaxial Normal X-Stress at Strain Gage 8 = $\sigma_{X8} = -\frac{M_R(z_8 - \bar{z})}{I}$ =____MPa
Uniaxial Normal X-Stress at Strain Gage 7 = $\sigma_{X7} = -\frac{M_R(z_7 - \bar{z})}{I}$ =____MPa

Uniaxial Normal X-Strain at Delta Rosette = $\varepsilon_{XD} = \frac{\sigma_{XD}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Rectangulaur Rosette = $\varepsilon_{XR} = \frac{\sigma_{XR}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Strain Gage 9 = $\varepsilon_{X9} = \frac{\sigma_{X9}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Strain Gage 8 = $\varepsilon_{X8} = \frac{\sigma_{X8}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Strain Gage 7 = $\varepsilon_{X7} = \frac{\sigma_{X7}}{E} \times 10^6$ =______μm/m

For Force #2

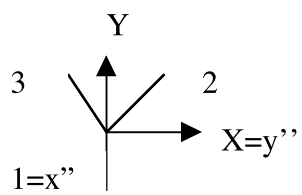
Uniaxial Normal X-Stress at Delta Rosette = $\sigma_{XD} = -\frac{M_D(z_D - \bar{z})}{I}$ =___MPa
Uniaxial Normal X-Stress at Rectangular Rosette = $\sigma_{XR} = -\frac{M_R(z_R - \bar{z})}{I}$ =____MPa
Uniaxial Normal X-Stress at Strain Gage 9 = $\sigma_{X9} = -\frac{M_R(z_9 - \bar{z})}{I}$ =____MPa
Uniaxial Normal X-Stress at Strain Gage 8 = $\sigma_{X8} = -\frac{M_R(z_8 - \bar{z})}{I}$ =____MPa
Uniaxial Normal X-Stress at Strain Gage 7 = $\sigma_{X7} = -\frac{M_R(z_7 - \bar{z})}{I}$ =____MPa

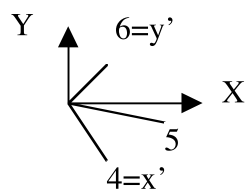
Uniaxial Normal X-Strain at Delta Rosette = $\varepsilon_{XD} = \frac{\sigma_{XD}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Rectangulaur Rosette = $\varepsilon_{XR} = \frac{\sigma_{XR}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Strain Gage 9 = $\varepsilon_{X9} = \frac{\sigma_{X9}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Strain Gage 8 = $\varepsilon_{X8} = \frac{\sigma_{X8}}{E} \times 10^6$ =______μm/m
Uniaxial Normal X-Strain at Strain Gage 7 = $\varepsilon_{X7} = \frac{\sigma_{X7}}{E} \times 10^6$ =______μm/m

7) The deflection of the beam at the location of the rectangular rosette can be calculated from the applied force, the geometry of the setup and the beam dimensions

For Force #1	For Force #2
Deflection at the Rectangular Rosette= $\delta_R = -\left(\frac{PL_aX_R}{6EIL}\right)\left(L^2 - L_a^2 - X_R^2\right)=$ ________mm	Deflection at the Rectangular Rosette= $\delta_R = -\left(\frac{PL_aX_R}{6EIL}\right)\left(L^2 - L_a^2 - X_R^2\right)=$ ________mm

8) The strain gage rosettes can be used to find the values and directions of the principal strains and stresses.





First the offset needs to be removed from each strain gage reading:

For Force #1	For Force #2
Corrected strain from SG1, (SG_1-Offset) =ε_1 =_____μm/m	Corrected strain from SG1, (SG_1-Offset) =ε_1 =_____μm/m
Corrected strain from SG2, (SG_2-Offset) =ε_2 =_____μm/m	Corrected strain from SG2, (SG_2-Offset) =ε_2 =_____μm/m
Corrected strain from SG3, (SG_3-Offset) =ε_3 =_____μm/m	Corrected strain from SG3, (SG_3-Offset) =ε_3 =_____μm/m
Corrected strain from SG4, (SG_4-Offset) =ε_4 =_____μm/m	Corrected strain from SG4, (SG_4-Offset) =ε_4 =_____μm/m
Corrected strain from SG5, (SG_5-Offset) =ε_5 =_____μm/m	Corrected strain from SG5, (SG_5-Offset) =ε_5 =_____μm/m
Corrected strain from SG6, (SG_6-Offset) =ε_6 =_____μm/m	Corrected strain from SG6, (SG_6-Offset) =ε_6 =_____μm/m

Next the coordinate strains are calculated

For Force #1

For the Delta Rosette, strain gage 1 is used as the reference for axis, x" such that $\varepsilon_{x''} = \varepsilon_1$=________$\mu$m/m

Now the strain in the y" direction is

$$\varepsilon_{y''} = \frac{1}{3}\left(2(\varepsilon_2 + \varepsilon_3) - \varepsilon_1\right) =$$

________μm/m

Now the shear strain in the x"y" direction is

$$\gamma_{x''y''} = \frac{2\sqrt{3}}{3}(\varepsilon_2 - \varepsilon_3) =$$

________μm/m

Principal normal strain is

$$\varepsilon_{1D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} + \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2} =$$

________μm/m

Principal normal strain is

$$\varepsilon_{3D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} - \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2} =$$

________μm/m

Principal normal strain is

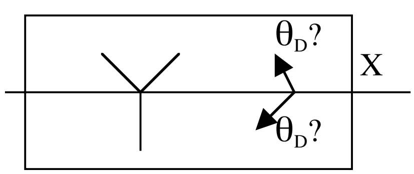
$$\varepsilon_{2D} = \left(\frac{-\nu}{1-\nu}\right)\left(\varepsilon_{1D} + \varepsilon_{3D}\right) =$$

________μm/m

Principal angle of ε_{1D} relative to the X-axis is

$$\theta_D = -90° + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x''y''}}{\left(\varepsilon_{x''} - \varepsilon_{y''}\right)}\right)$$

=________°



For Force #2

For the Delta Rosette, strain gage 1 is used as the reference for axis, x" such that $\varepsilon_{x''} = \varepsilon_1$=________$\mu$m/m

Now the strain in the y" direction is

$$\varepsilon_{y''} = \frac{1}{3}\left(2(\varepsilon_2 + \varepsilon_3) - \varepsilon_1\right) =$$

________μm/m

Now the shear strain in the x"y" direction is

$$\gamma_{x''y''} = \frac{2\sqrt{3}}{3}(\varepsilon_2 - \varepsilon_3) =$$

________μm/m

Principal normal strain is

$$\varepsilon_{1D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} + \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2} =$$

________μm/m

Principal normal strain is

$$\varepsilon_{3D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} - \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2} =$$

________μm/m

Principal normal strain is

$$\varepsilon_{2D} = \left(\frac{-\nu}{1-\nu}\right)\left(\varepsilon_{1D} + \varepsilon_{3D}\right) =$$

________μm/m

Principal angle of ε_{1D} relative to the X-axis is

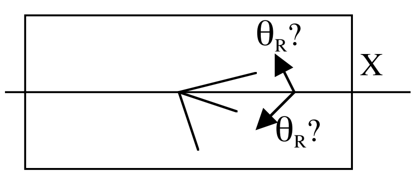
$$\theta_D = -90° + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x''y''}}{\left(\varepsilon_{x''} - \varepsilon_{y''}\right)}\right)$$

=________°

For Force #1

For the Rectangular Rosette, strain gage 1 is used as the reference for axis, x' such that $\varepsilon_{x'} = \varepsilon_4$=________$\mu$m/m
Now the strain in the y' direction is $\varepsilon_{y'} = \varepsilon_6$=________$\mu$m/m
Now the shear strain in the x'y' direction is $\gamma_{x'y'} = 2(\varepsilon_5) - (\varepsilon_4 + \varepsilon_6)$= ________$\mu$m/m

Principal normal strain is $\varepsilon_{1R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} + \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$= ________$\mu$m/m
Principal normal strain is $\varepsilon_{3R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} - \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$= ________$\mu$m/m
Principal normal strain is $\varepsilon_{2R} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1R} + \varepsilon_{3R})$= ________$\mu$m/m
Principal angle of ε_{1R} relative to the X-axis is $\theta_R = -75° + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x'y'}}{(\varepsilon_{x'} - \varepsilon_{y'})}\right)$ =________°



For Force #2

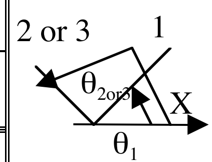
For the Rectangular Rosette, strain gage 1 is used as the reference for axis, x' such that $\varepsilon_{x'} = \varepsilon_4$=________$\mu$m/m
Now the strain in the y' direction is $\varepsilon_{y'} = \varepsilon_6$=________$\mu$m/m
Now the shear strain in the x'y' direction is $\gamma_{x'y'} = 2(\varepsilon_5) - (\varepsilon_4 + \varepsilon_6)$= ________$\mu$m/m

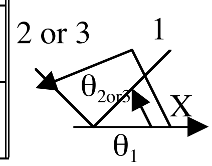
Principal normal strain is $\varepsilon_{1R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} + \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$= ________$\mu$m/m
Principal normal strain is $\varepsilon_{3R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} - \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$= ________$\mu$m/m
Principal normal strain is $\varepsilon_{2R} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1R} + \varepsilon_{3R})$= ________$\mu$m/m
Principal angle of ε_{1R} relative to the X-axis is $\theta_R = -75° + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x'y'}}{(\varepsilon_{x'} - \varepsilon_{y'})}\right)$ =________°

9) The analytical and experimental results can be tabulated and compared. Note that for θ_1, θ_2, and θ_3, the angle θ_1 denotes the angular position of the 1 principal direction relative to the X axis (i.e. longitudinal axis of the beam) and the angle θ_2 (or θ_3) denotes the denotes the angular position of the 2 (or 3) principal direction relative to the X axis.

Rosette strains at each force

		Force #1	Force #2
	Applied Force (N)		
Rectangular rosette	$\varepsilon_1 = \varepsilon_{1R}$ (μ m/m)		
	$\varepsilon_2 = \varepsilon_{2R}$ (μ m/m)		
	$\varepsilon_3 = \varepsilon_{3R}$ (μ m/m)		
Analytical	$\varepsilon_{XR} = \frac{\sigma_{XR}}{E}$		
Relative to X	$\theta_1 = \theta_R$ (°) (CW or CCW?)		
Relative to X	θ_2 (3?) (°) (CW or CCW?)		
Delta rosette	$\varepsilon_1 = \varepsilon_{1D}$ (μ m/m)		
	$\varepsilon_2 = \varepsilon_{2D}$ (μ m/m)		
	$\varepsilon_3 = \varepsilon_{3D}$ (μ m/m)		
Analytical	$\varepsilon_{XD} = \frac{\sigma_{XD}}{E}$		
Relative to X	$\theta_1 = \theta_D$ (°) (CW or CCW?)		
Relative to X	θ_2 (3?) (°) (CW or CCW?)		





Comment on the results. Are the results what you expected? How do the analytical results for the X-strain compare to the experimental results? Are the principal directions what you expected? Explain any difference and similarities…

10) The analytical and experimental results can be tabulated and compared.

The percent difference between calculated and measured values of strain and deflection can be computed as $\%diff = 100\dfrac{measured - calculated}{measured}$ because the measured is taken as the "correct" parameter.

First the offset needs to be removed from each strain gage reading:

For Force #1

Corrected strain from SG9, (SG_9-Offset) $=\varepsilon_9$ =_____μm/m
Corrected strain from SG8, (SG_8-Offset) $=\varepsilon_8$ =_____μm/m
Corrected strain from SG7, (SG_7-Offset) $=\varepsilon_7$ =_____μm/m

For Force #2

Corrected strain from SG9, (SG_9-Offset) $=\varepsilon_9$ =_____μm/m
Corrected strain from SG8, (SG_8-Offset) $=\varepsilon_8$ =_____μm/m
Corrected strain from SG7, (SG_7-Offset) $=\varepsilon_7$ =_____μm/m

Next, the uniaxial strains and deflections at each force are compared

Force #1	Measured value	Calculated value	% Difference
Strain Gage #9 (μ m/m)	ε_9=	ε_{x9}=	
Strain Gage #8 (μ m/m)	ε_8=	ε_{x8}=	
Strain Gage #7 (μ m/m)	ε_7=	ε_{x7}=	
Deflection (mm)	δ=	δ_R=	

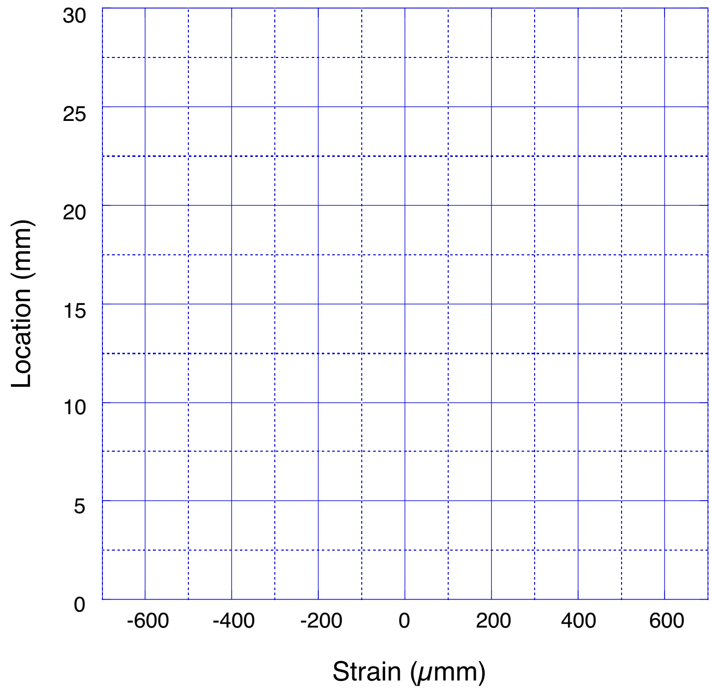
Force #2	Measured value	Calculated value	% Difference
Strain Gage #9 (μ m/m)	ε_9=	ε_{x9}=	
Strain Gage #8 (μ m/m)	ε_8=	ε_{x8}=	
Strain Gage #7 (μ m/m)	ε_7=	ε_{x7}=	
Deflection (mm)	δ=	δ_R=	

Comment on the results. Are the results what you expected? How do the analytical results for the X-strain compare to the experimental results? Are the principal directions what you expected? Explain any difference and similarities…

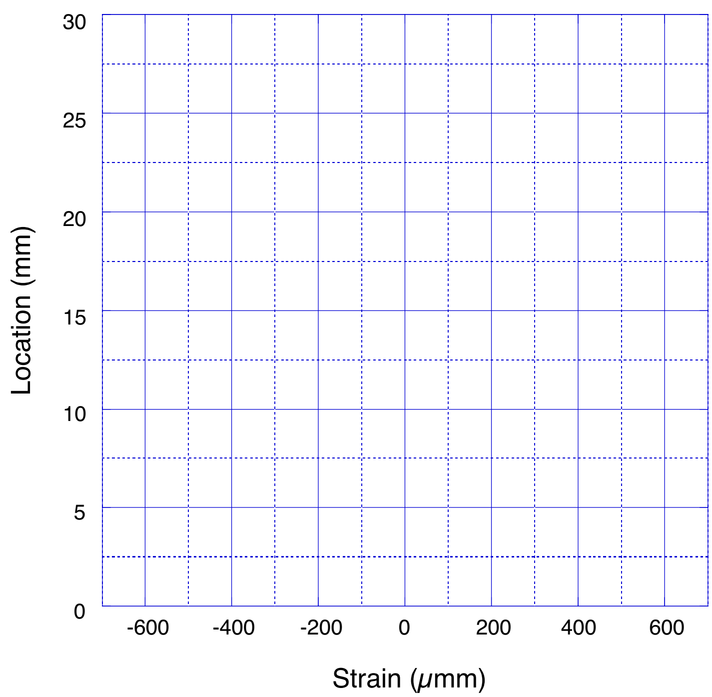
11) Plot the locations of strain gages 7, 8 and 9 as functions of the measured and calculated strains for each force case. Assuming a linear relation of location vs strain, the y-intercept will be the location at which the strain is zero (i.e., the neutral axis). Compare the location of the neutral axis determined using three methods: the measured strains, the calculated strains and the calculated centroid. Explain any differences and similarities.

	For Force #1		For Force #2	
Location	Measured Strain (μm/m)	Calculated Strain (μm/m)	Measured Strain (μm/m)	Calculated Strain (μm/m)
At Rectangular Rosette, Principal strain in X direction, z_R=___mm	$\sim\varepsilon_{3R}=$	$\varepsilon_{XR}=$	$\sim\varepsilon_{3R}=$	$\varepsilon_{XR}=$
At SG9, z_9=_____mm	$\varepsilon_9=$	$\varepsilon_{x9}=$	$\varepsilon_9=$	$\varepsilon_{x9}=$
At SG8, z_8=_____mm	$\varepsilon_8=$	$\varepsilon_{x8}=$	$\varepsilon_8=$	$\varepsilon_{x8}=$
At SG7, z_7=_____mm	$\varepsilon_7=$	$\varepsilon_{x7}=$	$\varepsilon_7=$	$\varepsilon_{x7}=$

For Force #1



For Force #2



12) Compare stress and strain distributions and values from the experimental, analytical and numerical results. Explain and comment on any similarities and differences

STRESSES IN STRAIGHT AND CURVED BEAMS

ME 354, MECHANICS OF MATERIALS LABORATORY

STRESSES IN STRAIGHT AND CURVED BEAMS

09 october 2000/mgj

PURPOSE

The purpose of this exercise is to the study the limitations of conventional beam bending relations applied to curved beams and to use photo elasticity to determine the actual stresses in a curved beam for comparison to analytical and numerical solutions.

The exercise has two main efforts: 1) Experimental Procedures to determine the fringe values for the straight beam of "calibrating" the birefringent test material and 2) Work Sheet calculations of stresses for comparison of analytically-determined stresses with experimental (photoelastic) and numerical (FEA) results.

EQUIPMENT

- Straight beam of a birefringent material.
- Curved beam of the same birefringent material as the straight beam.
- Four-point flexure loading fixture with load pan and suitable masses (straight beam)
- Line-loading fixture with load pan and suitable masses (curved beam)
- Circular polariscope with monochromatic light source

EXPERIMENTAL PROCEDURES

Procedure 1. Straight Beam in Pure Bending to Determine ("calibrate") the Stress-Optical Coefficient of the Material

i) Install the straight beam (see Fig. 1) in the four-point flexure loading fixture
ii) Attach the load pan (Note: The combined pan/fixture mass is ~0.980 kg)
iii) Apply two 10-kg masses one at a time to the load pan.
iv) With polarizer and analyzer crossed (dark field), focus the camera, record the image
v) Determine the maximum fringe orders at the top and bottom of the beam including estimates of fractional fringes orders by counting the fringes.
vi) The stress-optical coefficient can be calculated using the following relation:

$$f = \frac{t}{\overline{N}}(\sigma_1 - \sigma_2) \qquad (1)$$

where f is the stress-optical coefficient, $\overline{N}$ is the average fringe order, t is the model thickness, and σ_1 and σ_2 are the plane-stress principal stresses.

Procedure 2 Curved Beam in Tension and Bending

i) Install the curved beam (see Fig. 2) in the line loading fixture
ii) Attach the load pan (Note: The combined pan/fixture mass is ~0.454 kg)
iii) Apply one 5-kg mass to the load pan. (**Note:** Do not apply more than 5 kg at any time).
iv) With polarizer and analyzer crossed (dark field), focus the camera, record the image.
v) Determine the maximum fringe orders at point A at the inside of the straight part of the "arms," at point B at the inside of the curve, and at point C at the outside of the curve.
vi) The stress in the beam can be calculated using the relation:

$$(\sigma_1 - \sigma_2) = f\,\frac{\overline{N}}{t} \qquad (2)$$

where f is the stress-optical coefficient determined previously, $\overline{N}$ is the fringe order, t is the model thickness, and σ_1 and σ_2 are the plane-stress principal stresses.

* REFERENCES

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Experimental Stress Analysis, J.W. Dally and W.F. Riley, McGraw-Hill, Inc., 1990
Handbook on Experimental Mechanics, A.S. Kobayashi, ed., Prentice Hall, Inc.,1992
Mechanics of Materials, A. Higdon, E.H. Ohlsen, W.B. Stiles, J.A. Weese, W.F. Riley, ed., Wiley&Sons,1978

BACKGROUND FOR RESULTS

When loads are applied to a solid body, such as part of a structure or a machine component, stresses which vary from point to point, are set up in the body. By combining an understanding of engineering statics and mechanics of materials for planar elements (that is, beams) subjected to lateral loading (that is, bending) the well known beam bending relations can be developed (assuming pure bending, constant cross section, linear elastic material, and initially straight beam):

$$\varepsilon_x = -\frac{y}{\rho}$$
$$\sigma_x = -E\frac{y}{\rho} = -Ey\frac{M}{E\int y^2 dA} = -\frac{My}{I} \qquad (1)$$

where ε_x is the normal strain in the x-direction (longitudinal as shown in Fig. 1), y is the vertical direction and distance from the neutral axis (transverse as shown in Fig. 1), ρ is the radius of curvature of the neutral axis due to bending, σ_x is the normal stress in the x-direction, E is the elastic modulus, M is the applied moment, and $I=\int y^2 dA$ is the moment of inertia with respect to the z-axis.

The relations developed in Eq. 1 assume among other things that all longitudinal elements have the same initial length (for example, a "straight beam"). These assumptions lead to the linear variation of strain across the cross section (that is, $\varepsilon_x = -y/\rho$). A more general case of beam bending relations can be developed for the case of initially bent beam (assuming pure bending, constant cross section, linear elastic material and a constant initial radius of curvature):

$$\varepsilon_x = -\left(\frac{r_o\varepsilon_o}{r}\right)\frac{y}{h_t}$$
$$\sigma_x = -\left(\frac{r_o\sigma_o}{r}\right)\frac{y}{h_t} = -\frac{My}{(y+R)Ae} = \frac{My}{Ae(R-y)} \qquad (2)$$

where ε_x is the normal strain in the x-direction (longitudinal as shown in Fig. 2), r_o is the outer radius of the initially curved beam, r is the variable for the radius of the point in question, h_t is the height of the tensile section of the beam, ε_o is the longitudinal strain at the outer surface of the initially curved beam (that is, r=r_o), y is the vertical direction and distance from the neutral axis (transverse as shown in Fig. 2), σ_x is the normal stress in

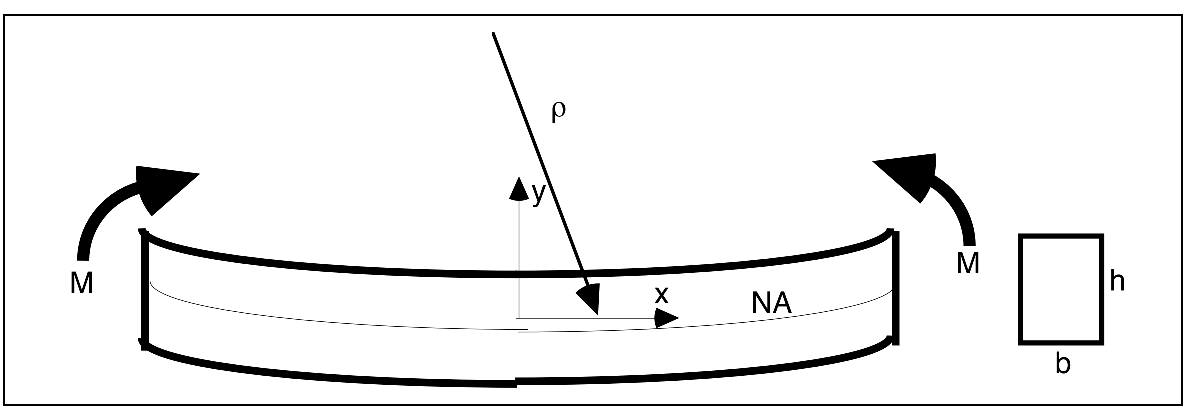


Figure 1 Nomenclature for a straight beam with rectangular cross section in pure bending.

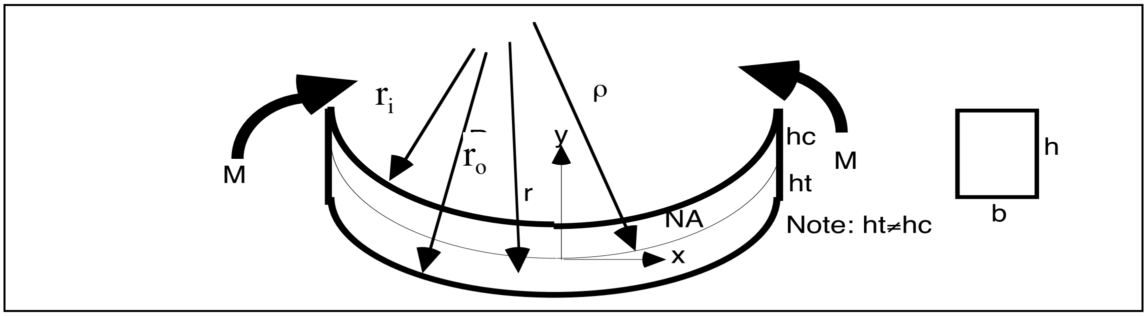


Figure 2 Nomenclature for a curved beam with rectangular cross section in pure bending.

the x-direction, σ_o is the longitudinal stress at the outer surface of the initially curved beam (that is, r=r_o), M is the applied moment, R is the radius of curvature of the neutral axis, Ae is the first moment of the first section (in this case the tensile section) about the neutral axis such that $Ae = \int y dA$. (Note that e can also be thought of as the distance from the centroid of the first section to the neutral axis of the cross section [a.k.a. eccentricity such that $e = \bar{r} - R$ where e=centroid=$(r_o + r_i)/2$]). For a rectangular cross section, $R = \frac{r_o - r_i}{\ln(r_o / r_i)}$ but in general can be found by solving the relation $\int \frac{r - R}{r} dA = 0$.

The mathematical solutions for strains and stress in beams (Eqs 1 and 2) provide valuable information regarding the stress distributions in beam-like components with simple geometries and loadings. In more complicated problems, commercially available two- and three-dimensional computer programs for finite element and boundary element analyses (FEA and BEM, respectively) can be used to determine and visualize stress distributions.

These theoretical and numerical results are exact solutions to problems which may or may not model the actual situations (usually due to assumptions about loads, load applications and boundary conditions). This uncertainty in modeling often requires experimental verification by spot checking the analytical or numerical results. A frequently cited example involves a threaded joint which seldom produces uniform contact at the threads. Contact analyses based on the idealized boundary condition of uniform contact will grossly underestimate the actual maximum stress concentration at the root of the overloaded thread. The uncertainty in the contact condition requires a stress analysis of the actual threaded joint experimentally despite the proliferation of FEA and BEM programs. Experimental stress analysis is also necessary to study nonlinear structure problems involving dynamic loading and/or plastic/viscoplastic deformations. Available FEA programs cannot provide detailed stress analysis of three-dimensional dynamic structures. Constitutive relations for plastic/viscoplastic materials are still being developed

One such experimental procedure often applied to empirically determine stress states is photoelasticity. Photoelasticity is a relatively simple, **whole-field method** of elastic stress analysis which is well suited for visually identifying locations of stress concentrations. In comparison with other methods of experimental stress analysis, such as a strain gage technique which is a **point measurement method**, photoelasticity is inexpensive to operate and provides results with minimum effort.

Photoelasticity consists of examining a model similar to the structure of interest using polarized light. The model is fabricated from transparent polymers possessing special optical properties. When the model is viewed under the type (but not necessarily magnitude) of loading similar to the structure of interest, the model exhibits patterns of fringes from which the magnitudes and directions of stresses at all points in the model can be calculated. The principle of similitude can be used to deduce the stresses which exist in the actual structure.

A disadvantage of photoelasticity is the necessity to test a polymer model which may not be able to withstand extreme loading conditions such as high temperature and/or high strain rates. Although photoelasticity is generally applied to elastic analysis, limited studies on photo plasticity and photo viscoelasticty indicate the potential of extending the technique to nonlinear structural analysis. Further details of photoelasticity can be found in listed references.

In this exercise, show all work and answers on the Worksheet, turning this in as the In-class Lab report.

ME 354, MECHANICS OF MATERIALS LABORATORY

STRESSES IN STRAIGHT AND CURVED BEAMS

09 october 2000 / mgj

WORKSHEET

NAME_______________________________________**DATE**_____________

EQUIPMENT IDENTIFICATION_______________________________________

1) Confirmation of Birefringent Test Material: The properties of two birefringent polymers often used for photoelasticity are shown in **Table 1**. Note which material is used for these laboratory exercises.

Table 1 Selected Properties of Two Birefringent Polymers for Photoelastic Experiments

Homolite 911 a.k.a.. CR-39 (allyl diglycol)
Selected Properties (R.T.)

Elastic Modulus, E(GPa)	1.7
Proportional Limit σ_o (MPa)	21
Poisson's ratio, ν	0.40
Stress Optical Coeff, f (kN/m)*	16
Figure of Merit Q=E / f (1/m)	106,250

Epoxy (Araldite, Epon)
Selected Properties (R.T.)

Elastic Modulus, E(GPa)	3.3
Proportional Limit σ_o (MPa)	55
Poisson's ratio, ν	0.37
Stress Optical Coeff, f (kN/m)*	11
Figure of MeritQ=E / f (1/m)	300,000

* in green light with wavelength 546 nm

2) Confirmation of Dimensions: For the two beams and loading fixtures, confirm the following information. See **Figs. 3, 4, and 5** for nomenclature and values.

Table 2 Dimensions and Loading for Straight and Curved Photoelastic Beams

Straight beam

Calibration Force, F_o $=(M_{weight}+M_{fixture}+M_{pan})*g$ (N)	
Outer Span, L_o (mm)	
Inner Span, L_i (mm)	
Height (straight), h_s (mm)	
Thickness, b (mm)	

Curved Beam

Test Force, F_t $=(M_{weight}+M_{pan}) * g$ (N)	
Outer Radius, r_o (mm)	
Inner Radius, r_i (mm)	
Height (straight), h_s (mm)	
Thickness, b (mm)	
Average Radius, $\bar{r} = (r_o + r_i)/2$ (mm)	
Height (curve), h_c (mm)	

Note: The calibration and test forces must include the masses of the fixture and pan as well as the added masses (a.k.a. weights) in kg. Gravitational constant is g=9.816 kg • m/s^2.

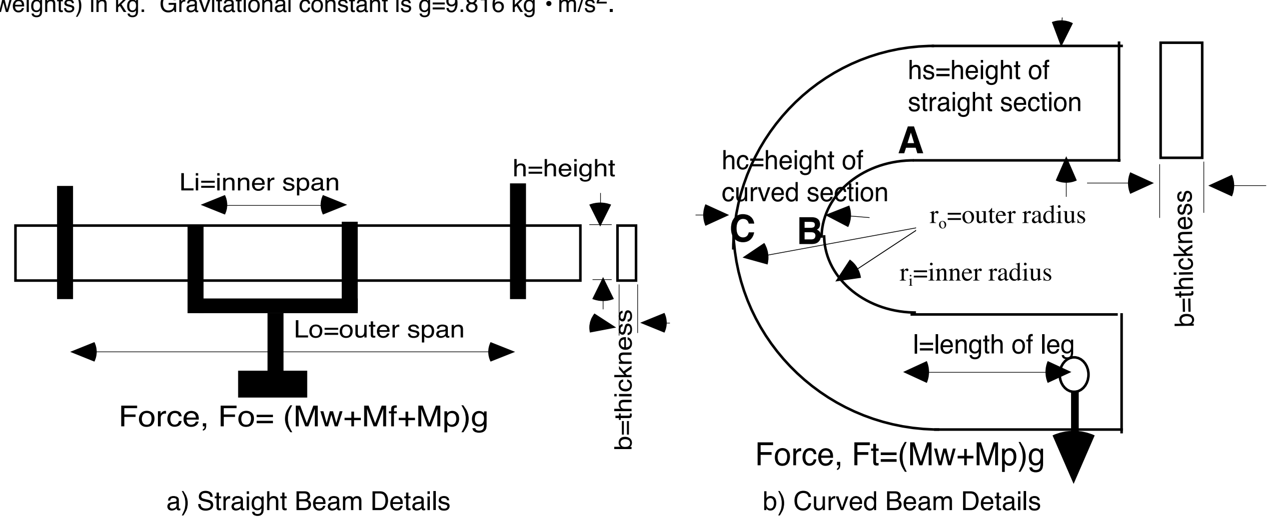


Figure 3 Nomenclature for the Beams

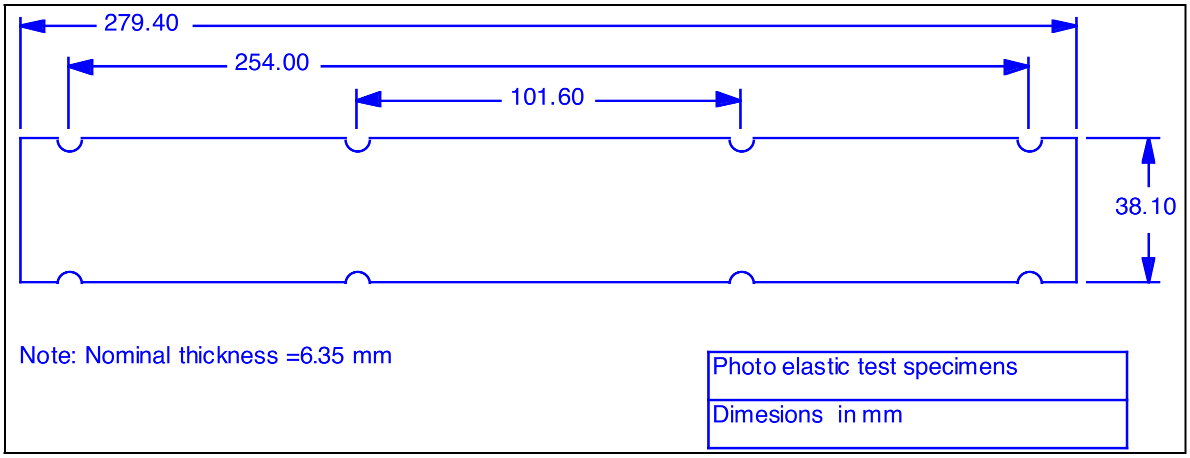


Figure 4 Straight Beam

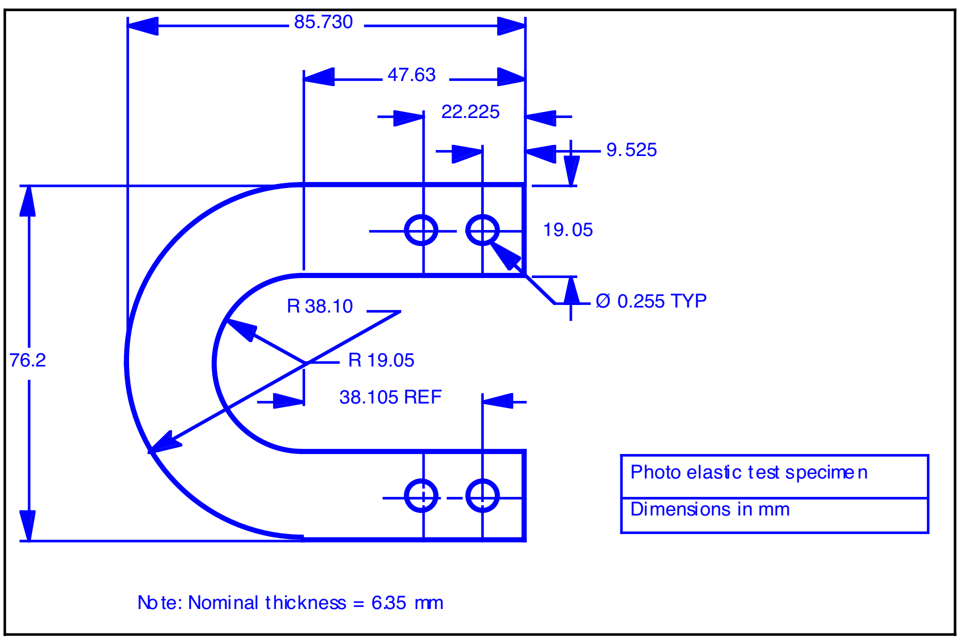


Figure 5 Curved Beam

3) Determination of Stress Optical Coefficient for Material and Setup

A unique aspect of the four-point flexure loading arrangement is that the region of interest (the section of the beam within the inner loading span) undergoes a pure bending moment as shown in Fig. 6.

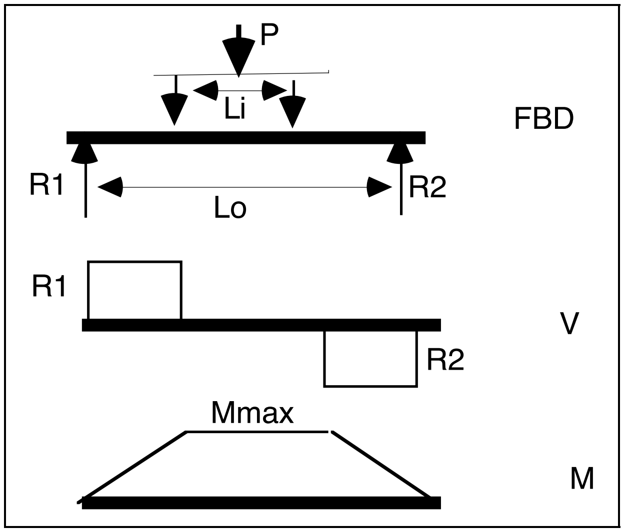


Figure 6 Free Body, Shear and Moment Diagrams for Four-Point Flexure Loading

a) For the straight beam, determine the following if at the outer free edge of the beam (y=c=h/2) the stress state is uniaxial.

Moment of Inertia for the rectangular cross section beam, $I = \frac{bh^3}{12}$ = ___________mm^4

Maximum moment due to calibration force, F_O, $M_o = \frac{F_o(L_o - L_i)}{4}$ = ___________N•mm

Maximum distance to outer edge of the beam from neutral axis, c=h/2 = ________mm

Maximum uniaxial bending stress at the outer free edge of the beam

$\sigma_1 = \sigma_x = \frac{M_o c}{I}$ = __________MPa.

b) The photoelastic relation can be used to determine the stress optical coefficient directly from the beam bending relation.

From the Experimental Procedure, the **average** fringe value at the upper and lower outer edges of the beam determined at the calibration force is $\bar{N}$=__________.

Calculated stress optical coefficient for the material, $f = \frac{b}{\bar{N}}(\sigma_1)$=________MPa-mm/fringe.

4) Compare the calculated value of the "calibrated" stress value to that shown in **Table 1** for the material used in this exercise. How do the values compare? Discuss any discrepancies and possible reasons (Note: Do not panic if the calculated stress optical coefficient differs from the value listed in **Table 1**....differences in optical test setup, environmental effects in the material, etc. all require the "calibration" of the material).

5) Experimentally-Measured Stresses in the Curved Beam Using Photoelasticity

At the free edges of selected locations of the curved beam (**A**, **B** and **C** in **Figure 3b**), the stress states are uniaxial and the photoelastic relation can be used to calculate the normal stresses using the relation between the fringe order at the free edge, the stress optical coefficient for the material, and the specimen thickness.

Fill in the table with values for the loaded test specimen that are used in the following calculations.

Fringe Value Counted at A, N_A	
Fringe Value Counted at B, N_B	
Fringe Value Counted at C, N_C	
Thickness, b (mm)	
Calculated Stress Optical Coefficient, f (MPa-mm/fringe)	

a) At **"A"**, the fringe value at the free edge of the curved beam determined at the test force is N_A=_____.

The photoelastically-determined total normal stress at **"A"** is $\sigma_A = \frac{fN_A}{b}$=_______MPa.

(Note, apply the appropriate sign('+'=tension,' –' =compression based on observation)

b) At **"B"**, the fringe value at the free edge of the curved beam determined at the test force is N_B=_____.

The photoelastically-determined total normal stress at **"B"** is $\sigma_B = \frac{fN_B}{b}$=_______MPa.

(Note, apply the appropriate sign('+'=tension,' –' =compression based on observation)

c) At **"C"**, the fringe value at the free edge of the curved beam determined at the test force is N_C=_____.

The photoelastically-determined total normal stress is $\sigma_C = \frac{fN_C}{b}$=_______MPa.

(Note, apply the appropriate sign('+'=tension,' –' =compression based on observation)

6) Analytically-Determined Stresses Using Straight Beam Relations

One approach to analytically determine stresses in the curved beam is to assume that the relations for straight beams apply (that is, Eq. 1).

Fill in the table with values for the loaded test specimen that are used in the following calculations.

Applied Test Force, F_t (N)	
Length of Straight Leg, ℓ (mm)	
Height of Straight Leg, h_s (mm)	
Thickness of Straight Leg, b (mm)	
Outer Radius, r_o (mm)	
Inner Radius, r_i (mm)	
"Height" of Curve, h_c (mm)	
Thickness of Curve, b (mm)	

a) At **"A"**, the moment is determined as the applied force, F_t= _____N multiplied by the length of the leg, ℓ=_____mm such that $M_A = F_t * \ell$=______N•mm.

The moment of inertia is calculated from the height of the beam, h_s, and the thickness of the beam, b, such that $I_A = \frac{bh_s^3}{12}$=______mm^4.

The distance from the neutral axis to point **"A"** (+y is up) is c=-h_s/2=______mm.

The normal stress at **"A"** for a straight beam assumption is $\sigma_A^{straight} = \frac{-M_A c}{I_A}$=______MPa.

(Confirm that the normal stress at **"A"** should be tension (i.e. **+**σ))

b) At **"B"**, the moment is determined as the applied force, F_t, multiplied by the length of the leg, ℓ, mm plus the average radius, $\bar{r} = (r_o + r_i)/2$=_______mm such that M_B= $F_t*(\ell + \bar{r})$ =______N•mm.

The moment of inertia at **B-C** is calculated from the height of the beam, h_c, and the thickness of the beam, b, such that $I_{BC} = \frac{bh_c^3}{12}$=______mm^4.

The distance from the assumed neutral axis (centroid) to point **"B"** is c = -h_c/2 =______mm (note that c is positive outward from the center of radius).

The assumed normal bending stress at **"B"** for a straight beam assumption is $\sigma_B^{straight} = \frac{-M_B c}{I_{BC}}$=_______MPa. (Confirm that the normal stress at **"B"** is tension (i.e. **+**σ))

c) At **"C"**, the moment is the same as at **"B"** such that $M_C = M_B$ =______N•mm. (see 6b)

The moment of inertia at **B-C** is $I_{BC} = \frac{bh_c^3}{12}$=______mm^4. (**see 6b**)

The distance from the assumed neutral axis to point **"C"** is c= h_c/2 =______mm (note that c is positive outward from the center of radius).

The assumed normal bending stress at **"C"** for a straight beam assumption is $\sigma_c^{straight} = \frac{-M_c c}{I_{BC}}$=______MPa. (Confirm that normal stress at **"C"** is compression (i.e. **-**σ))

7) Analytically-Determined Stresses Using Curved Beam Relations

For the curved part of the beam (in this case points **"B"** and **"C"** the analytical calculation must take into account the initial curvature of the beam (Eq. 2).

Fill in the table with values for the loaded test specimen that are used in the following calculations.

Applied Moment, M_B=M_C (N•mm)	
Outer Radius, r_o (mm)	
Inner Radius, r_i (mm)	
Average radius, $\bar{r} = (r_o + r_i)/2$ (mm)	
"Height" of Curve, h_C (mm)	

a) At the line in the curve connecting **"B"** and **"C"**, the radius of the neutral axis for a rectangular cross section can be calculated from the outer radius, r_o, and the inner radius r_i, such that $R = \frac{(r_o - r_i)}{\ln(r_o / r_i)}$=___________mm.

The eccentricity, e, can be calculated from the average radius of the centroid, $\bar{r}$, and the radius of the neutral axis,R =_____mm such that $e = \bar{r} - R$=______mm.

The cross sectional area is calculated from the thickness, b, and the curved beam height, h_C, such that A=b•h_C= ________mm^2. (Note that the distance from the neutral axis to the point of interest is y=R-r).

b) At **"B"**, r=r_i, therefore y_B=R-r_i=_______mm.

The curved beam, normal bending stress at **"B"** is $\sigma_B^{curved} = \frac{M_B y_B}{Ae(R - y_B)}$=_____MPa.

c) At **"C"**, r=r_o, therefore y_C=R-r_o=_______mm.

The curved beam, normal bending stress at **"C"** is $\sigma_C^{curved} = \frac{M_C y_C}{Ae(R - y_C)}$=_______MPa.

8) Additional Axial Normal Stress Component

Because the bending moment at **"B-C"** is produced by a transverse force (that is, not a pure bending moment), the total normal stress at **"B-C"** has two components: a tensile axial (in the loading direction) stress and a tensile/compressive bending stress.

a) The tensile axial stress is calculated from the applied test force, F_t=_______N and the cross sectional area, A=b h_C = _______mm^2.

The axial tensile stress is $\sigma^{axial} = \frac{F_t}{A}$=___________MPa (Confirm that this axial normal stress is tension (i.e. $+\sigma$))

9) Comparisons of Total Normal Stress (bending and axial) at "B" and "C"

Note that the stress values σ_A, σ_B and σ_C are the stresses determined experimentally as calculated in Section 5.

a) At **"B"**, the total calculated stress using the straight beam assumption is $\sigma_B^{total(straight)} = \sigma^{axial} + \sigma_B^{straight}$ =_______MPa.

Percent difference between the actual photoelastically-measured stress and the calculated stress is $100\dfrac{\sigma_B^{total(straight)} - \sigma_B}{\sigma_B}$ =_________%.

b) At **"B"**, the total calculated stress using the curved beam relation is $\sigma_B^{total(curved)} = \sigma^{axial} + \sigma_B^{curved}$ =_______MPa.

Percent difference between the actual photoelastically-measured stress and the calculated stress is $100\dfrac{\sigma_B^{total(curved)} - \sigma_B}{\sigma_B}$ =_________%.

c) At **"B"**, the numerically-determined (from the finite element analysis (FEA)) normal stress in the y-direction is σ_B^{FEA} =_______MPa.

Percent difference between the actual photoelastically-measured stress and the numerically-determined stress is $100\dfrac{\sigma_B^{FEA} - \sigma_B}{\sigma_B}$ =_________%.

d) At **"C"**, the total calculated stress using the straight beam assumption is $\sigma_C^{total(straight)} = \sigma^{axial} + \sigma_C^{straight}$ =_______MPa.

Percent difference between the actual photoelastically-measured stress and the calculated stress is $100\dfrac{\sigma_C^{total(straight)} - \sigma_C}{\sigma_C}$ =_________%.

e) At **"C"**, the total calculated stress using the curved beam relation is $\sigma_C^{total(curved)} = \sigma^{axial} + \sigma_C^{curved}$ =_______MPa.

Percent difference between the actual photoelastically-measured stress and the calculated stress is $100\dfrac{\sigma_C^{total(curved)} - \sigma_C}{\sigma_C}$ =_________%.

f) At **"C"**, the numerically-determined (from the finite element analysis (FEA)) normal stress in the y-direction is σ_C^{FEA} =_______MPa.

Percent difference between the actual photoelastically-measured stress and the numerically-determined stress is $100\dfrac{\sigma_C^{FEA} - \sigma_C}{\sigma_C}$ =_________%.

10) Comparisons of Normal Stress (bending) at "A"

a) At **"A"**, the total calculated stress using the straight beam relation is $\sigma_A^{(total)straight} = \sigma_A^{straight} + 0$ =_______MPa.

Percent difference between the actual photoelastically-measured stress and the calculated stress is $100\dfrac{\sigma_A^{(total)straight} - \sigma_A}{\sigma_A}$ =_________%.

b) At **"A"**, the numerically-determined (from the finite element analysis (FEA)) is σ_A^{FEA}=________MPa.

Percent difference between the actual photoelastically-measured stress and the numerically-determined stress is $100\dfrac{\sigma_A^{FEA} - \sigma_A}{\sigma_A}$=__________%.

11) Discussion of Analytical, Experimental, and Numerical Results

Comment on similarities and differences between the experimental (photoelasticity), the analytical (straight and curved beam relations) and the numerical (FEA) at points **"A"**, **"B"**, and **"C."** Does the straight-beam assumption give a conservative (i.e., over predict stresses) or non conservative (i.e., under predict stresses) result?

Extra effort: Compare the neutral axis positions in the curved part of the beam for the photoelastic and the FEA results. Are they quantitatively and qualitatively similar?

Extra effort: Using the fringe values from the photoelastic analysis, plot the stresses across the curved cross section from **"B"** to **"C".** Plot the results from the FEA analysis on the same plot. Finally, calculate the stresses from **"B"** ($r=r_i$) to **"C"** ($r=r_o$) for the relation $\sigma = \frac{F}{A} + \frac{M_c y}{Ae(R - y)} = \frac{F}{A} + \frac{M_c(R - \bar{r})}{Ae(R - (R - \bar{r}))}$. Compare the results. Is the stress vs. distance relation linear or non linear? Would you expect the straight beam assumption to give a linear or non linear relation? Would the straight beam assumption over or under predict stresses.

MECHANICAL PROPERTIES & PERFORMANCE OF MATERIALS:
tension, hardness, torsion, impact

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: OVERVIEW

01 Januarly 2000 / mgj

PURPOSE

The purpose of this exercise is to obtain a number of experimental results important for the characterization of the mechanical properties and performance of materials. Four different types of tests will be performed over the course of two weeks A single laboratory report will be written focusing on materials testing in general, but keying on each type of test in specific sub sections.

First Week

Tensile test - the most fundamental test for obtaining information about materials for design

Hardness test - a superficial test for quality control and to determine degree of heat treatments

Second Week

Torsion test - application of pure shear to determine performance of material in plastic range

Impact test - determination of notch + temperature sensitivities of materials under high strain rates

EQUIPMENT and PROCEDURE

Each test is described in detail in the appropriate laboratory hand out.

ANALYSIS

The analysis of the raw data is described in detail in the appropriate hand out.

LABORATORY REPORT

One laboratory report on Mechanical Properties and Performance of Materials should be prepared. This report should contain the results of all four tests. The report should provide descriptions, discussions, presentations of test results, and discussions and conclusions in sufficient detail so as to allow an engineering manager to make comparative decisions about the usefulness and applicability of each type of materials test.

The basic layout of the report follows the required format for the course. The Title and Objectives sections should key on materials testing in general. However, the each of the sections on Test Description, Results, and Discussion/Conclusions should be divided into four subsections each. Each subsection should focus on the particular test.

Total score for the report will be 200 points to reflect the four tests being included in the report.

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TENSILE TESTING*

04 september 2000 / mgj

PURPOSE

The purpose of this exercise is to obtain a number of experimental results important for the characterization of the mechanical properties and performance of materials. The tensile test is a fundamental mechanical test for material properties which are used in engineering design, analysis of structures, and materials development.

EQUIPMENT

- Reduced gage section tensile test specimens of 6061-T6 aluminum
- Reduced gage section tensile test specimens of hot-rolled 1018 or A36 steel
- Reduced gage section tensile test specimens of polymethymethacrylate (PMMA(acrylic))
- Reduced gage section tensile test specimens of polycarbonate (Lexan™ (PC))
- Clip-on extensometer
- Tensile test machine with grips, controller, and data acquisition system
- Calipers

PROCEDURE

per ASTM E8M "Standard Test Methods of Tension Testing of Metallic Materials [Metric]"

For each material, perform the following steps.

- Measure the diameter of the gage section for each test specimen to 0.02 mm.
- Measure the marked gage length of the gage section (in this case 50.8 mm).
- Zero the force output (balance).
- Activate force protect (~500 N) on the test machine to prevent overloading the test specimen during installation.
- Install the one end of the tensile test specimen in the top grip of the test machine while the test machine is in displacement control.
- Install the other end of the tensile test specimen in the lower grip of the test machine.
- In displacement control adjust the actuator position of the test machine to achieve nearly zero force on the test specimen.
- Attach the extensometer to the gage section of the test specimen, centering it in the gage section. Zero the output from the strain conditioner.
- Deactivate force protect.
- Initiate the data acquisition and control program.
- Enter the correct file name and specimen information as required.
- Initiate the test sequence via the computer program.
- At maximum force (i.e. after some amount of necking in the gage section), only if necessary, remove the extensometer to avoid damage to the extensometer at fracture.
- Continue the test until test specimen fracture.
- Measure the smallest diameter of the gage section at the location of failure. Measure the final length between the marks which denoted the original gage length of the test specimen.

ANALYSIS

The analysis is conducted from the raw data [P, force (kN) vs. DL, change in length (mm)] which are available in either computer readable text files or on hard copy text files.

Plot or determine ALL of the following for ALL of the materials.

• Plot engineering stress ($\sigma \approx \frac{P}{A_o}$ MPa) versus engineering strain (use %, m/m or μm/m for $\varepsilon \approx \frac{\Delta L}{L_o}$)

• Determine the following from the engineering stress vs. engineering strain plots.

a) proportional limit stress, $\sigma_p = \sigma_o$

b) 0.2% offset yield stress, S_{yp}

c) ultimate tensile strength, S_{uts}

d) modulus of elasticity (by approximate formula ($E \approx \frac{\sigma_p}{\varepsilon_p}$) and/or numerical method (E=m from linear regression of σ vs ε))

e) modulus of resilience (by approximate formula ($U_r = \int_0^{\varepsilon_o} \sigma d\varepsilon \approx \frac{1}{2}\sigma_o \varepsilon_o$)and/or numerical method ($U_r = \int_0^{\varepsilon_o} \sigma d\varepsilon \approx \sum_{i=1}^{i=n@\varepsilon_o} \left(\frac{\sigma_{i+1} + \sigma_i}{2}\right)\left(\varepsilon_{i+1} - \varepsilon_i\right)$))

f) modulus of toughness. (by approximate formula $U_T = \int_0^{\varepsilon_f} \sigma d\varepsilon \approx \frac{S_{uts} + \sigma_o}{2}\varepsilon_f$ and/or numerical method ($U_T = \int_0^{\varepsilon_f} \sigma d\varepsilon \approx \sum_{i=1}^{i=n@\varepsilon_f} \left(\frac{\sigma_{i+1} + \sigma_i}{2}\right)\left(\varepsilon_{i+1} - \varepsilon_i\right)$)))

• From the diameter and length measurements, determine the following.

a) true fracture stress, $S_f^T = \frac{P_{max}}{A_f}$

b) percent reduction in area, $\%RA = q = 100\frac{A_o - A_f}{A_o}$

c) percent elongation, $\%el = q = 100\frac{L_f - L_o}{L_o}$

Plot or determine ALL of the following for ONLY the aluminum alloy.

• Plot the true stress, s, versus true strain, e, curve along with the engineering stress, σ, versus engineering strain, ε, on the same graph from 0 to maximum force only. Determine the true stress at maximum force and the true uniform strain (i.e., true strain at maximum force, prior to onset of necking). (Note: $s = \sigma(1+\varepsilon)$ and e = $\ln(1+\varepsilon)$) for region of uniform strain.

• Construct a plot of log true stress versus log true strain and determine, using linear regression, the 'best' values of n and K (or H) for the approximate constitutive relation:

$$s = Ke^{n} = He^{n} \qquad (1)$$

where s is the true stress, e is the true plastic strain, K or H is the strength coefficient, n is the strain hardening exponent per ASTM E646 "Standard Test Method for Tensile Strain-Hardening Exponents (n-Values) of Metallic Sheet Materials."

• Add the plot of this constitutive approximation (i.e. calculate the stress using K, n, and measured strain) to the plots of measured true stress versus measured true strain and measured engineering stress versus measured engineering strain. Determine the percent error between the true stress calculated from the approximate constitutive relation (Eq. 1) and the measured true stress at measured true strain values of 0.1%, 1%, and 5%.

* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01
E8 Standard Test Methods of Tension Testing of Metallic Materials
E8M Standard Test Methods of Tension Testing of Metallic Materials [Metric]

E646 Standard Test Method for Tensile Strain-Hardening Exponents (n-Values) of Metallic Sheet Materials

LABORATORY REPORT

1. Include the following information in the laboratory report.

	1018 (HR) or A36 steel	6061-T6 aluminum	PMMA (acrylic)	PC (polycarbonate)
proportional limit stress (MPa).........				
0.2 % offset yield strength (MPa)....			-	
ultimate tensile strength (MPa)........				
modulus of elasticity (GPa)......[AF]				
modulus of elasticity (GPa)......[NM]				
% difference..................................				
modulus of resilience (J/m^3)....[AF]				
modulus of resilience (J/m^3)....[NM]				
% difference..................................				
modulus of toughness (J/m^3)...[AF]				
modulus of toughness (J/m^3)...[NM]				
% difference..................................				
true fracture strength (MPa)...........				
% reduction in area.........................				
% elongation..................................				
true stress @ maximum force(MPa)	- - - - -		- - - - -	- - - - -
true uniform strain..........................	- - - - -		- - - - -	- - - - -
strain hardening exponent, n...........	- - - - -		- - - - -	- - - - -
strength coefficient, K (MPa)..........	- - - - -		- - - - -	- - - - -
true stress at 0.1% true strain(MPa)	- - - - -		- - - - -	- - - - -
$s = Ke^n$ at 0.1% true strain (MPa)....	- - - - -		- - - - -	- - - - -
% difference..................................	- - - - -		- - - - -	- - - - -
true stress at 1% true strain (MPa)..	- - - - -		- - - - -	- - - - -
$s = Ke^n$ at 1% true strain (MPa).......	- - - - -		- - - - -	- - - - -
% difference..................................	- - - - -		- - - - -	- - - - -
true stress at 5% true strain (MPa)..	- - - - -		- - - - -	- - - - -
$s = Ke^n$ at 5% true strain (MPa).......	- - - - -		- - - - -	- - - - -
% difference..................................	- - - - -		- - - - -	- - - - -

Note: AF = approximate formula. NM = numerical method (least squares or numerical integration).

2. Include the following information in the laboratory report.
   a. Engineering stress vs. engineering strain for all materials.
   b. Engineering stress vs. engineering strain and true stress vs. true strain on the same graph for the alumimum alloy.
   c. Log-log plot of true stress vs. true strain along with the curve fit on the same graph for the aluminum alloy
   e. Eng. stress vs. eng. strain and true stress vs. true strain with calculated stress vs. strain from Eq. 1 for the aluminum alloy on the same graph
   f. Compare results of these tests for each material to 'book' values from a source such as the ASM Metals Handbook. Comment on any differences.
   g. Compare fracture surface appearances and mechanical properties for each material (metals & polymers). Comment on the brittle/ductile behaviour of each.
3. Include the following information in the appendix of the laboratory report. THIS MAY NOT BE ALL THAT IS NECESSARY (i.e., don't limit yourself to this list.)
   a. Original data sheets and/or printouts
   b. All supporting calculations. Include sample calculations if using a spread sheet program. DO NOT INCLUDE ALL TABULATED RAW OR CALCULATED DATA.

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TENSILE TESTING*

DATA SHEET

01 January 2000 / mgj

NAME______________________________________**DATE**____________

LABORATORY PARTNER NAMES____

____

EQUIPMENT IDENTIFICATION______________________________________

____

Aluminium	**metal**
Initial (units)	
D_o ( )	
L_o ( )	
Final (units)	
D_f ( )	
L_f ( )	
Observed Maximum Force ( )	
Observed Fracture Force ( )	

Steel	**metal**
Initial (units)	
D_o ( )	
L_o ( )	
Final (units)	
D_f ( )	
L_f ( )	
Observed Maximum Force ( )	
Observed Fracture Force ( )	

PMMA (acrylic)	**polymer**
Initial (units)	
D_o ( )	
L_o ( )	
Final (units)	
D_f ( )	
L_f ( )	
Observed Maximum Force ( )	
Observed Fracture Force ( )	

Polycarbonate	**polymer**
Initial (units)	
D_o ( )	
L_o ( )	
Final (units)	
D_f ( )	
L_f ( )	
Observed Maximum Force ( )	
Observed Fracture Force ( )	

01 January 2000 / mgj

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: HARDNESS TESTING*

PURPOSE

The purpose of this exercise is to obtain a number of experimental results important for the characterization of the mechanical properties and performance of materials. The hardness test is a mechanical test for material properties which are used in engineering design, analysis of structures, and materials development.

EQUIPMENT

- Fractured "halves" of reduced gage section tensile specimen of 6061-T6 aluminum.
- Fractured "halves" of reduced gage section tensile specimen of 1018 (hot rolled).
- Flat coupons of 6061 T6 aluminum.
- Flat coupons of 1018 (hot rolled).
- Rockwell hardness tester with 1/16 inch ball indenter tip and 100 kg of mass weights.
- Tensile test machines with compression platen, 10-mm diameter Brinell indenter ball fixture and controller
- Reticular eye piece microscope

PROCEDURE

Brinell Hardness Test

per ASTM E10 "Standard Test Method for Brinell Hardness of Metallic Materials"

- Place the flat coupon of one of the materials on the compression platen of the test machine, ensuring that the specimen is centered and resting flat on the platen.
- In displacement control, with force protect ON and set to 5 kg, adjust the actuator position of the test machine such that the Brinell indenter ball just contacts the surface of the flat coupon with a NEGATIVE force.
- Turn off force protect, switch to force control, use waveform to ramp the force to -500 kg.
- Maintain the maximum compressive force for not more than 15 s.
- Ramp the force back to ~-10 kg.
- Switch to displacement control and adjust the actuator position of the test machine such that the Brinell indenter ball is no longer in contact with the surface of the flat coupon.
- Remove the flat coupon from the compression platen.
- Use the Micro Mike to measure the diameter of the indentation of the surface
- Repeat these steps for the other material.

Rockwell Hardness Test

per ASTM E18 "Standard Test Method for Rockwell Hardness and Rockwell Superficial Hardness of Metallic Materials"

- Place the cylindrical gripped end of one half of a fractured tensile specimen of one of the materials in the V-notched platen of the Rockwell hardness tester.
- With the load handle pulled forward, raise the specimen and load fixture until the indenter contacts the specimen
- Continue raising the specimen until the small dial hand is pointing at the small black dot (this applies a 10 kg preload).
- Rotate the Rockwell dial until the large dial hand is pointing at "0".
- Depress the loading bar, allowing the machine to apply the maximum load of 100 kg.
- Wait until the large dial hand stops moving, holding the load for not more than 25 s.
- Pull the load handle forward again
- Read the number on the B-scale indicated by the large dial hand
- Repeat this hardness test for the flat sections of the gripped end of one half of the tensile specimen and the flat coupon of the same material using the flat platen

- Repeat these steps for the other material.

ANALYSIS

The analysis is conducted from recorded data.

The Brinell hardness number is obtained by dividing the applied force (in kilograms) by the curved surface of the indentation which is a segment of sphere such that:

$$BHN = HB = \frac{2P}{\pi D\left[D - \sqrt{\left(D^2 - d^2\right)}\right]} \qquad (1)$$

where P is the applied load in kg, D is the diameter of the ball (nominally 10 mm) and is the diameter of the indention. See Figure 1 for a schematic illustration of the Brinell hardness test.

The Rockwell hardness number (HRX or RX) is determined from the differences of the indentation depths at the preload and the maximum load. The Rockwell number is read directly from the dial of the indenter, but the number must be reported along with the Rockwell scale which automatically identifies the type of indenter type and the maximum load (otherwise the number is meaningless). See Figure 2 for a schematic of the Rockwell hardness test.

Use ASTM E 140-88 "Standard Hardness Conversion Tables for Metals (Relationship Between Brinell Hardness, Vickers Hardness, Rockwell Hardness, Rockwell Superficial Hardness, and Knoop Hardness)" to convert the BHN to RB and vice versa. Are the measured and converted values similar? Why or why not? Compare the size of "artifacts" left by both indenters. What conclusions might you draw about the possible effects of indents on the mechanical properties of indented components?

Compare the hardness values obtained from flat coupons / flat sections of the component and those obtained on the curved surfaces of the component (i.e., the tensile specimens). Are the values similar? If not, which value shows a "softer" material? Would you expect this? What type of recommendation might you have about indenting components and curved surfaces, in general.

The deformations caused by a hardness indenter are of similar magnitude to those occurring at the ultimate tensile strength of a tension test. However, an important difference is that the material cannot freely flow outward, so that a complex triaxial stress state exists under the indenter. Nevertheless, empirical correlations can be established between hardness and tensile properties, primarily the engineering ultimate tensile strength, S_{uts}.

Use appropriate empirical relations (e.g., see Mechanical Behaviour of Materials by Dowling or ASM Metals Reference Book with various editors) estimate ultimate tensile strengths for the two materials from the hardness numbers. Compare these estimated strengths to those measured from tensile tests (those of this class or from the literature).

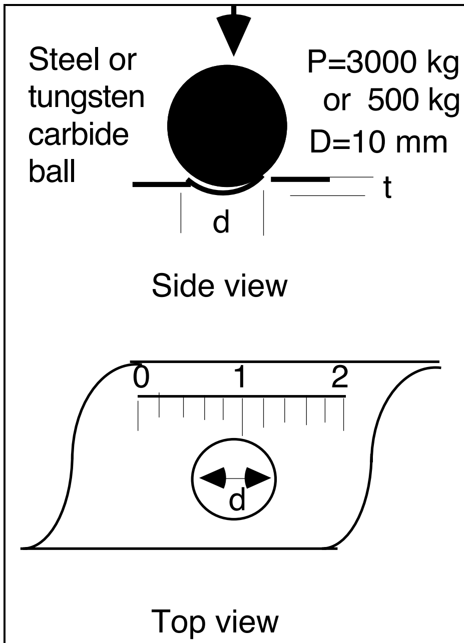
* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01

E10 Standard Test Method for Brinell Hardness of Metallic Materials

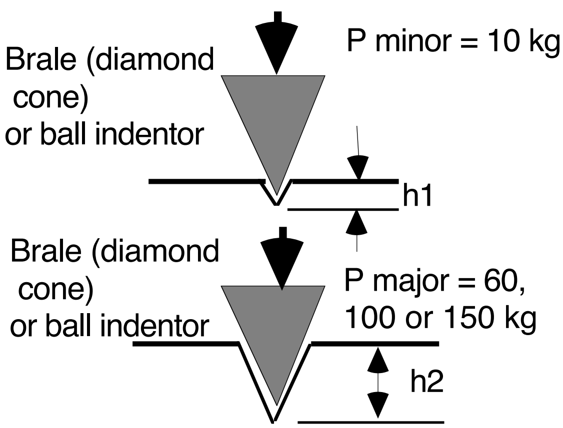
E18 Standard Test Method for Rockwell Hardness and Rockwell Superficial Hardness of Metallic Materials

E 140 Standard Hardness Conversion Tables for Metals (Relationship Between Brinell Hardness, Vickers Hardness, Rockwell Hardness, Rockwell Superficial Hardness, and Knoop Hardness)



$$BHN = HB = \frac{P}{\pi D t} = \frac{2P}{\pi D\left[D - \sqrt{\left(D^2 - d^2\right)}\right]}$$

Figure 1 - Schematic Diagram of Brinell Hardness Test



Rockwell Scale (X =)	Indentor	Pmajor (kg)
A	Brale (diamond)	60
B	1/16" ball	100
C	Brale (diamond)	150
D	Brale (diamond)	100
E	1/8" ball	100
F	1/8" ball	60
M	1/4" ball	100
R	1/2" ball	60

$$HRX = R_X = M - \frac{\left(h_2 - h_1\right)}{0.002}$$

M = 100 for A, C, and D scales

M = 130 for B, E, M, R, etc. scales

Figure 2 - Schematic Diagram of Rockwell Hardness Test

LABORATORY REPORT

1. Include the following information in the laboratory report.

	6061-T6 aluminum	1018 (HR) or A36 steel
BHN (kg/mm^2)......[measured]		
BHN (kg/mm^2). [literature]		
% difference....................................		
S_{uts} (MPa) [estimated from BHN]...		
S_{uts} (MPa) [measured or literature].		
% difference....................................		
RB ....[measured, flat coupon]..........		
RB ....[literature]		
% difference....................................		
S_{uts} (MPa) [estimated from RB].......		
S_{uts} (MPa) [measured or literature]		
% difference....................................		
RB ....[measured, tensile specimen cylindrical grip]		
RB ....[literature]..............................		
% difference....................................		
S_{uts} (MPa) [estimated from RB].......		
S_{uts} (MPa) [measured or literature]		
% difference....................................		
RB ....[measured, tensile specimen flat grip]		
RB ....[literature]..............................		
% difference....................................		
S_{uts} (MPa) [estimated from RB].......		
S_{uts} (MPa) [measured or literature].		
% difference....................................		

2. Include the following information in the laboratory report.
   a. Compare results of the hardness tests for each metal to 'book' values from a source such as the ASM Metals Handbook. Comment on any differences.
   b. Compare the size of the artifact (i.e., indentation) from each type of test. Discuss the possible effect of such "artifacts" on material response if hardness tests are used for quality control of components.
   c. Comment on the empirical relations which allow estimates of ultimate tensile strength of each material. Discuss the merits of using hardness versus tensile tests for determining/estimating mechanical properties of materials for engineering design.
3. Include the following information in the appendix of the laboratory report. THIS MAY NOT BE ALL THAT IS NECESSARY (i.e., don't limit yourself to this list.)
   a. Original data sheets and/or printouts
   b. All supporting calculations. Include sample calculations if using a spread sheet program. DO NOT INCLUDE ALL TABULATED RAW OR CALCULATED DATA.

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: HARDNESS TESTING*

DATA SHEET

01 January 2000 / mgj

NAME____**DATE**______________

LABORATORY PARTNER NAMES____

____

EQUIPMENT IDENTIFICATION____

____

Aluminium

Flat Coupon	
Brinell	
Maximum Load (kg), P	
Brinell Ball Dia (mm), D	
Indentation Dia (mm), d	
Rockwell	
Load (kg)	
Indenter Size/Type	
Rockwell Scale	
Rockwell Number	
Tensile Specimen	
Cylindrical Grip	
Rockwell	
Load (kg)	
Indenter Size/Type	
Rockwell Scale	
Rockwell Number	
Flat Grip	
Rockwell	
Load (kg)	
Indenter Size/Type	
Rockwell Scale	
Rockwell Number	

Steel

Flat Coupon	
Brinell	
Maximum Load (kg), P	
Brinell Ball Dia (mm), D	
Indentation Dia (mm), d	
Rockwell	
Load (kg)	
Indenter Size/Type	
Rockwell Scale	
Rockwell Number	
Tensile Specimen	
Cylindrical Grip	
Rockwell	
Load (kg)	
Indenter Size/Type	
Rockwell Scale	
Rockwell Number	
Flat Grip	
Rockwell	
Load (kg)	
Indenter Size/Type	
Rockwell Scale	
Rockwell Number	

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TORSION TESTING*

MGJ/08 Feb 1999

PURPOSE

The purpose of this exercise is to obtain a number of experimental results important for the characterization of materials. In particular, the results from the torsion test will be compared to the results of the engineering tensile test for a particular alloy using the effective stress-effective strain concept.

EQUIPMENT

- Constant-diameter gage section torsion specimen of 6061-T6 aluminum
- Torsion test machine with grips, troptometer, and force sensor.

PROCEDURE

- Measure the diameter (D=2R) of the gage section for each test specimen to 0.02 mm.
- Install the bottom end of the torsion test specimen in the lower grip of the test machine. Rotate the lever arm as far to the right as possible. (Note: unscrew the horizontal threaded drive rod as much as possible).
- Rotate the top grip as far as possible in the direction necessary to remove the 'slack' from the reaction cables and install the top end of the torsion test specimen in the top grip of the test machine.
- Zero the output of the force sensor.
- Use the threaded drive rod to apply torque to the base of the test specimen and record the applied torque, T, versus angular rotation, θ, at 2° increments until 30° of rotation.
- Remove the horizontal threaded drive rod and find the torque after 90° and 360° of rotation, being careful not to allow elastic unloading.
- After 360° of rotation, unload and remove the specimen. Measure the gage length L (grip to grip length) of the installed specimen to 0.1 mm.

RESULTS

- Plot measured torque, T, versus angular displacement per unit length, $\theta' = \frac{d\theta}{d\ell}$. Using linear regression, fit the curve to 30° of relative rotation. (It is assumed that T is proportional to θ' from θ=0° to θ = 30°). (Note that θ must be in radians, i.e. π radians = 180°).
- Calculate the shear modulus, G, from the linear portion of the T-θ' using linear regression to find dT/dθ' from θ=0° to θ = 30°. Compare this value of G to the shear modulus determined from the tensile test results (i.e. $G = \frac{E}{2(1+\nu)}$) using ν=0.345 for aluminum.
- Using the strength coefficient coefficient, K (or H), and the strain hardening exponent, n, determined from the tensile test for the approximate constitutive relation $\sigma = K\varepsilon^n = H\varepsilon^n$, integrate the predicted shear stress, τ, versus radial distance, r, to obtain the predicted torque, T, after 90° and after 360° of rotation. Compare these values of T to those measured experimentally. (Note that θ must be in radians for the calculations, i.e. π radians = 180°). Use the attached "cook book" method to facilitate your work.
- On the same graph, plot shear stress, τ, and engineering shear strain, γ as functions of radial distance, r, at 30° of rotation. Construct similar plots τ and γ vesus r for 90° and 360° of rotation. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

LABORATORY REPORT

1. As a minimum include the following information in the laboratory report.

a. Raw data (typed in tabular form)

b. Two values for the shear modulus, G (tension and torsion)

c. Two values of the torsional yield stress, τ_o (tension and torsion)

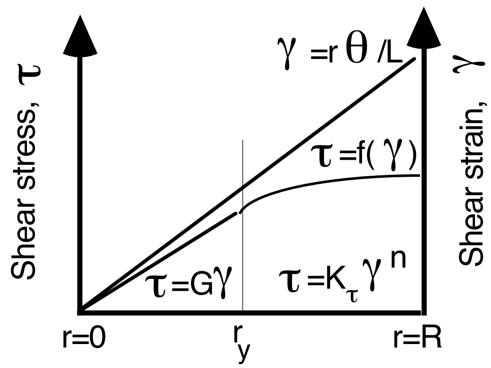
d. "n" and "K" from the tension test (use these in the calculations)

e. Total torque as required in the table:

Angle of Rotation .... 90° 360° Predicted Torque ( )
Measured Torque ( ) . % Difference ....

f. Plot of Torque vs. Angular displacement per unit length (T vs θ')

g. One graph each of τ and γ as functions of radial distance, r, for θ = 30°, 90°, and 360° (2 plots on each graph for a total of 3 graphs)



h. Discuss comparisons of basic mechanical properties as determined from tension and torsion tests. Compare results of these tests for each alloys to 'book' values from such sources as the ASM Metals Handbook. Comment on any differences. Compare the shapes of the stress vs. radial distance curves and the magnitudes of the plastic and elastic torques.

2. Include the following information in the appendix of the laboratory report. THIS MAY NOT BE ALL THAT IS NECESSARY (i.e., don't limit yourself to this list.)

a. Original data sheets and/or printouts

b. All supporting calculations. Include sample calculations if using a spread sheet program.

c. All "cookbook" calculations from the Torsion Test Solution Path.

* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01 E143 Standard Test Methods for Shear Modulus at Room Temperature.

ME354 NOTES on Torsion Testing

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TORSION TESTING*

MGJ/08 Feb 1999

DATA SHEET

NAME____**DATE**_____________

LABORATORY PARTNER NAMES____

____

EQUIPMENT IDENTIFICATION____

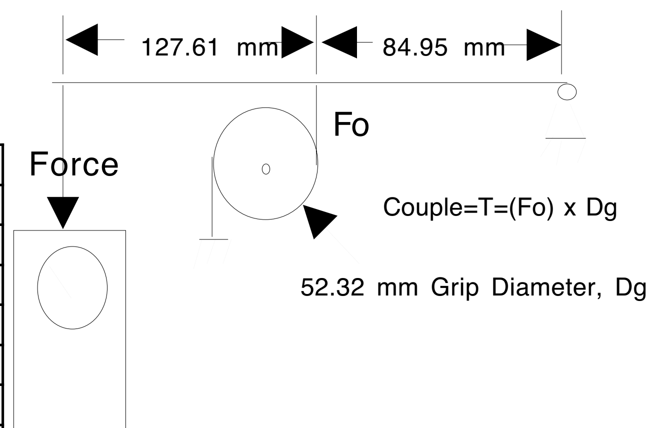
____

Note: Be sure to record units for each quantity.

Specimen measurments

D ( )	
L ( )	
J ( )	

Angle(degrees)	Force ( )
0	
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	
22	
24	
26	
28	
30	
90	
360	



ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TORSION TESTING*

Torsion Test Solution Path

TORSION TEST

The initial set of calculations has input parameters obtained only from the torsion test. The results of these calculations will later be compared to results calculated with information obtained from the tension test.

1. Record the torsion specimen diameter (D=2R) and the length of the gripped section of the torsion specimen, L. Calculate the polar moment of inertia for a solid rod, $J = \frac{\pi D^4}{32}$.

   D = mm

   L = mm

   J = mm^4

2. From the measured torque, T, versus angular rotation, θ, data points, plot T versus relative angular deflection, $\theta' = \frac{d\theta}{d\ell}$ between two cross sections (i.e. $\frac{\theta}{L}$). Obtain the "best fit" of the linear portion of the T versus θ' data using linear regression. (It is assumed that T is proportional to θ' from θ=0° to θ = 30°). Determine the slope, dT/dθ' from θ=0° to θ = 30°. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

   dT/dθ' = (N-mm) / (rad/mm)

3. The shear modulus, G, from the torsion test can now be calculated from the relation:

   $$G = \frac{dT_{(0-30°)}}{d\theta'_{(0-30°)}} \frac{1}{J} =$$ (N/mm^2=MPa)

4. Finally, record the measured torques and calculate $\theta' = \frac{d\theta}{d\ell}$ for θ = 90° and 360°. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

   $T_{90°}$ = N-mm

   $\theta'_{90°}$ = /mm

   $T_{360°}$ = N-mm

   $\theta'_{360°}$ = /mm

TENSION TEST

This set of calculations has input parameters obtained only from the tension test. The results of these calculations will later be compared to results calculated with information obtained from the torsion test.

1. Record the uniaxial elastic modulus, E, uniaxial yield stress, σ_o, the strain hardening coefficient, K, and the strain hardening exponent, n, determined from the tension test.

E = N/mm2

σ_o = N/mm2

K = N/mm2

n =

2. Calculate the value of the shear modulus from the results of the tension test:

$$G = \frac{E}{2(1+\nu)}$$ using ν=0.345 for aluminum.

3. Using the effective stress concept, calculate the shear strength indicated by the tension test data such that:

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{xz}{}^2\right)\right]^{\frac{1}{2}}$$

and setting $\bar{\sigma} = \sigma_o$ and solving for $\tau_{xy} = \tau_y$ (yield stress in shear) with all other stress equal to zero.

τ_y = N/mm2

EVALUATION OF TORSION TEST RESULTS FOR YIELDING

This set of calculations has input parameters obtained only from the torsion and tension tests. The results of these calculations are used to evaluate the shear stresses and shear strains across the radius of the torsion specimen as it yields.

1. Find the radius of the torsion specimen at yielding, r_y for $\theta = 90°$ (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

$$r_y = \frac{\tau_y}{G}\frac{R}{\gamma_{max}} \text{ where } \gamma_{max} = \left(\theta'_{90°}\right)R$$

$r_y^{90°}$ = mm

2. Within the elastic domain, the shear stress is a linear function of radial distance, r, such that

$$\tau(r) = \frac{\tau_y}{r_y} r$$

3. The shear stress as a function of radial distance, r, can now be multiplied by differential area element, $2\pi r\,dr$ and a moment arm, r, and integrated to find the torque over the elastic domain. (i.e. $\sum M = 0$).

$$T_e = \int_o^{r_y} \left(\frac{\tau_y}{r_y} r\right)(r)2\pi r\,dr$$

4. In the plastic domain, only the shear strain remains a linear function of radial distance, r. Therefore, it is advantageous to change the integration variable to γ. In order to accomplish this variable change, the shear stress, τ, moment arm, r, and differential area of integration, $2\pi r\,dr$, must be expressed as function of γ.

In the tension test the uniaxial stress, σ, was expressed as a function of uniaxial strain, e, through the strenght coefficient, K (or H), strain hardening exponent, n, such that:

$$\sigma = K\varepsilon^n = H\varepsilon^n$$

Since the uniaxial stress is identical to the effective stress, and the uniaxial strain is identical to the effective strain, the equation relating effective stress to effective strain would be exactly the same.

$$\bar{\sigma} = K\bar{\varepsilon}^n = H\bar{\varepsilon}^n$$

When the effective stress and effective strain are evaluated for the case of pure torsion, the shear stress can be found as a function of the shear strain.

$$\tau(\gamma) = \left(\frac{1}{\sqrt{3}} K \left(\frac{\gamma}{\sqrt{3}} \right)^n \right)$$

Since $\gamma = \theta' r$ it is also true that $r = \left(\frac{\gamma}{\theta'} \right)$ and, since θ' is a constant .

Substituting these relations into the basic torsion integral yields:

$$T_e = \int_o^{r_y} \left(\frac{\tau_y}{r_y} r \right) (r) 2\pi \, r \, dr$$ for the elastic torque

$$T_p = \int_{\gamma_y}^{\gamma_{max}} \left(\frac{1}{\sqrt{3}} K \left(\frac{\gamma}{\sqrt{3}} \right)^n \right) \left(\frac{\gamma}{\theta'} \right) 2\pi \, \frac{\gamma}{\theta'} \frac{d\gamma}{\theta'}$$ for the plastic torque.

Note that the limits of integration are $\gamma_y = \frac{\tau_y}{G}$ and $\gamma_{max} = \theta' R$. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

The total torque, T, is found as the sum of the elastic and plastic torques such that:

$T = T_e + T_p$ This torque value is then compared to the value measured in the torsion test.

For $\theta = 90°$, calculated torques are:

$T_e =$ N-mm

$T_p =$ N-mm

$T =$ N-mm

For $\theta = 90°$, measured torque is:

$T_{90°} =$ N-mm

5. Steps 1 to 4 are repeated for θ = 360° (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

For θ = 360°, calculated torques are:

T_e = N-mm

T_p = N-mm

T = N-mm

For θ = 360°, measured torque is:

$T_{360°°}$= N-mm

6. Finally, plot τ and γ as functions of r after θ = 30° for relative rotations of θ = 90° and θ = 360°. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TORSION TESTING*

NOTES on Torsion Testing

STRESSES IN THE ELASTIC RANGE

In the elastic range, stresses in the shaft will remain less than the proportional limit and less than the elastic limit as well. For this case Hooke's law will apply and there will be no permanent deformation. Hooke's Law for shear stress is as follows:

$$\tau = G\gamma$$

G = Modulus of rigidity (shear modulus)
τ = Shear Stress
γ = Shear Strain

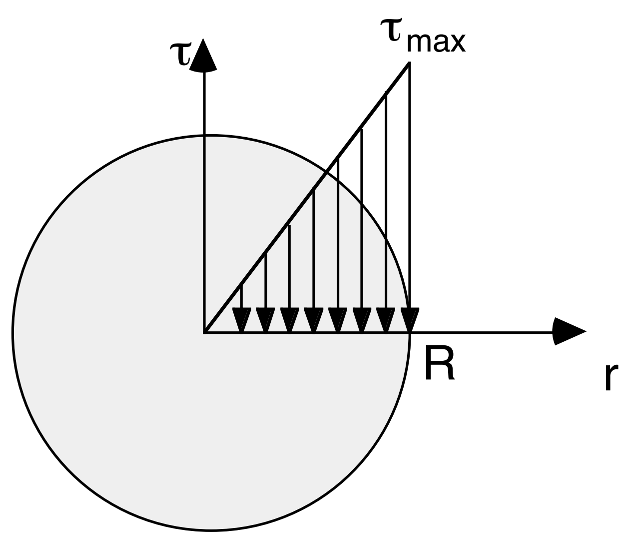


Figure 1. Distribution of Stress in the Elastic Range.

The elementary forces exerted on any cross section of the shaft must be equal to the magnitude T of the torque exerted on the shaft:

$$\int_o^R r\left(\tau dA\right) = T \quad \text{where} \quad dA = 2\pi\, r\, dr$$

$$\gamma = \frac{r}{R}\gamma_{max}$$

$$G\gamma = \frac{r}{R}G\gamma_{max}$$

$$\tau = \frac{r}{R}\tau_{max}$$

$$T = \int r\, \tau dA = \frac{\tau_{max}}{R} \int r^2 dA$$

$$\int r^2 dA = J = \frac{1}{2} \pi R^4$$

$$T = \frac{\tau_{max}}{R} J$$

$$\boxed{\tau_{max} = \frac{TR}{J}}$$

$$\boxed{\tau = \frac{Tr}{J}}$$

The last two equations are known as the elastic torsion formulas.

ANGLE OF TWIST IN THE ELASTIC RANGE

For this section the entire shaft will again be assumed to be in the elastic range. Therefore Hooke's Law applies.

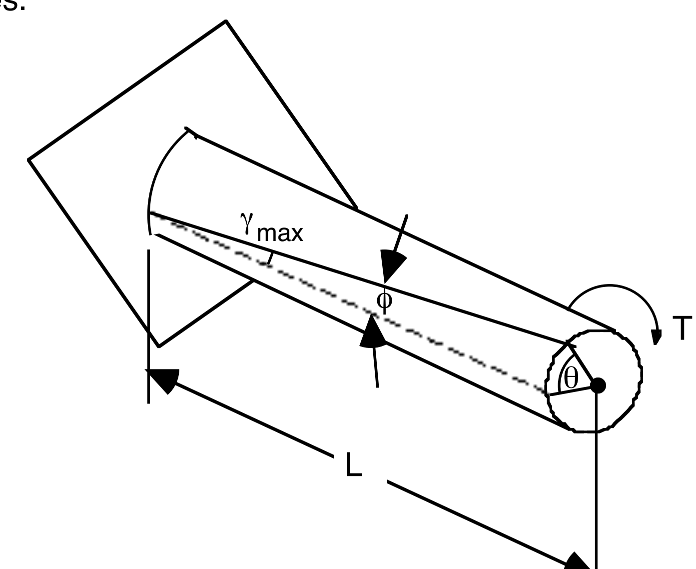


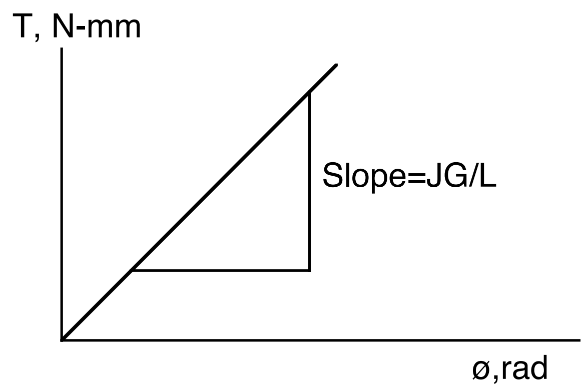
Figure 2. Demonstration of the Angle of Stress and the Shearing Strain.

$$\gamma_{max} = \frac{R\phi}{L}$$

$$\gamma_{max} = \frac{\tau_{max}}{G} = \frac{TR}{JG}$$

$$\boxed{\phi = \frac{TL}{JG}}$$

The angle of twist, Ø, is expressed in radians. The angle of twist is proportional to the torque T applied to the shaft. The above equation provides a convenient method for determining the modulus of rigidity, G. Torques of increasing magnitude T are applied to the specimen, and the corresponding values of the angle of twist in a length L of the specimen are recorded. As long as the yield stress of the material is not exceeded, the points obtained by plotting Ø against T will fall on a straight line. The slope of the line represents the quantity JG/L from which the modulus of rigidity, G, may be computed.



PLASTIC DEFORMATIONS IN CIRCULAR SHAFTS

If the yield strength is exceeded in some portion of the shaft the relations discussed in the earlier sections cease to be valid. The purpose of this section is to develop a more general method for determining the distribution of stresses in the solid circular shaft, and for computing the torque required to produce a given force.

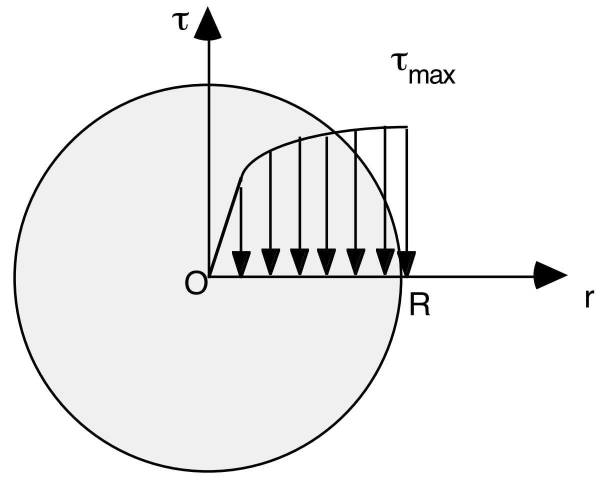


Figure 3. Stress Distribution in a Shaft for Plastic Deformation.

As the torque increases, τ_{max} eventually reaches the shearing yield stress, τ_{y}, of the material. Solving for the corresponding value of T, we obtain the value of T_y at the onset of yield:

$$T_Y = \frac{J}{R}\tau_Y$$

T_y is referred to as the maximum elastic torque, since it is the greatest torque for which deformation remains fully elastic. Recalling that, for a solid circular shaft, J/R=1/2 (πR^3) we have:

$$T_Y = \frac{1}{2}\pi R^3 \tau_Y$$

The τ_Y can be found using the data from the tension test and the idea of effective stress. Using the Distoritonal Energy (von Mises) criterion and the yield stress from the tensile test laboratory τ_Y can be determined.

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{xz}{}^2\right)\right]^{\frac{1}{2}}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[6\tau_{xy}{}^2\right]^{\frac{1}{2}} = \sqrt{3}\tau_{xy}$$

$$\bar{\sigma} = \sigma_0 = \text{yield stress from tension test}$$

$$\tau_{xy} = \tau_Y = \frac{\bar{\sigma}}{\sqrt{3}} = 0.577\,\bar{\sigma}.$$

$$\gamma_y = \frac{\tau_y}{G}, \text{measured G from the elastic part of the torsion test.}$$

$$r_y = \frac{\gamma_y}{d\theta/d\ell}, \text{ where } d\theta/d\ell = \frac{\pi/2}{L} \text{ (at } 90^\circ\text{) and } \frac{2\pi}{L} \text{ (at } 360^\circ\text{) for this lab.}$$

The total torque is a function of the torque in the elastic range and the torque in the plastic range.

$$T_{total} = T_{elastic} + T_{plastic}$$

$$T_{total} = \int_0^{r_y} r\tau(r)2\pi r dr + \int_{r_y}^{R} r\tau(r)2\pi r dr$$

for the elastic range

$$\tau(r) = \frac{\tau_y}{r_y} r \quad \therefore \quad T_{elas} = \int_0^{r_y} \frac{\tau_y}{r_y} 2\pi r^3 dr$$

for the plastic range

$$\sigma = K\varepsilon^n = \sqrt{3}\tau, \quad \varepsilon = \frac{\gamma}{\sqrt{3}}, \quad \sqrt{3}\tau = K\left(\frac{\gamma}{\sqrt{3}}\right)^n$$

$$\tau = \frac{K}{\sqrt{3}}\left(\frac{\gamma}{\sqrt{3}}\right)^n = 0.577K\left(\frac{\gamma}{\sqrt{3}}\right)^n$$

$$\therefore \quad T_{plastic} = \int_{r_y}^{R} \frac{K}{\sqrt{3}}\left(\frac{\gamma}{\sqrt{3}}\right)^n 2\pi r^2 dr$$

$$r = \frac{\gamma}{\frac{d\theta}{d\ell}}, \quad dr = \frac{d\gamma}{\frac{d\theta}{d\ell}}$$

$$T_{plastic} = \int (const)(\gamma)^{n+2} d\gamma$$

$$T_{plastic} = \int_{\tau_y/G}^{\gamma_{max}} \frac{K}{\sqrt{3}}\left(\frac{\gamma}{\sqrt{3}}\right)^n 2\pi \frac{1}{\left(d\theta/d\ell\right)^3} \gamma^2 d\gamma$$

OR

$$T_{plastic} = \int_{r_y}^{R} \frac{K}{\sqrt{3}}\left(\frac{r\frac{d\theta}{d\ell}}{\sqrt{3}}\right)^n 2\pi r^2 dr, \quad \frac{d\theta}{d\ell} = \frac{\theta}{L}$$

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: CHARPY V-NOTCH IMPACT*

01 January 2000 / mgj

PURPOSE

The purpose of this exercise is to obtain a number of experimental results important for the characterization of the mechanical behavior of materials. The Charpy V-notch impact is a mechanical test for determining qualitative results for material properties and performance which are useful in engineering design, analysis of structures, and materials development.

EQUIPMENT

- Charpy V-notch test specimens of 6061-T6 aluminum and 1018 (hot rolled) or A36 steel
- Charpy testing machine with 800-mm long pendulum arm and 22.6-kg impact head
- Type K thermocouple and digital readout unit
- Beakers of room-temperature water, warm water and boiling water
- Beakers of plain iced water
- Cryo-beakers of salted iced water and super cold liquids

PROCEDURE

CAUTION: When using the Charpy testing machine, stand well clear of the swinging area of the pendulum both when the arm is cocked and for some time after the arm is released for a test while it is still swinging. Serious injury will result from a swinging pendulum arm.

For each material repeat the following steps

- Designate a person as the "operator" of the Charpy test machine: all other persons must stand clear during testing
- Designate a person as the "monitor and recorder" of temperatures and impact energies
- Designate a person as the "test specimen loader" who will remove test specimens from the liquid bath, quickly placing them on the test fixture of the Charpy testing machine
- Designate a person as the "test specimen retriever" who will retrieve the broken halves of the test specimens, will bind the halves together and will mark the test temperature on each pair of specimen halves for later examination and inspection.

Use the following procedure to conduct tests in the order shown after exposure to the pre-conditions to give the approximate test temperatures indicated:

Room temperature water (20 to 25°C)
Warm water (50-60 °C)
Boiling water (95-100°C)
Ice water (0 to 4°C)
Salted ice water (-15 to -18°C)
Acetone with some dry ice (-50 to -57°C)
Acetone with much dry ice (-80 to -85°C)

- Place the thermocouple probe in the appropriate liquid being sure to allow both the test specimens and the thermocouple to equilibrate for at least five minutes prior to testing.
- Record the indicated temperature
- "Cock" the pendulum by activating the "raise" mechanism and stand clear while the pendulum is held in the "cocked" position.
- Using the tongs, quickly remove the test specimen from the bath and place it on the test fixture with the notch opening facing away from the direction of the cocked pendulum
- Stand clear
- Release the pendulum
- Secure the pendulum in its rest position (i.e., hanging vertically) and retrieve the fractured specimen halves.
- Record the impact energy (read directly from the dial on the Charpy testing machine)
- Repeat these steps for the each temperature and each material.

BACKGROUND AND ANALYSIS

Static or quasi-static properties and performance of materials are very much a function of the processing of the material (heat treatments, cold working, etc.) in addition to design and service factors such as stress raisers and cracks.

The behaviour of materials is also dependent on the rate at which the force is applied. For example, a polycarbonate tensile specimen which might show a relatively low yield point but up to 200% elongation at a low loading rate may show a much greater yield point but at only 5% elongation at an order of magnitude faster loading rate. Low carbon steels, such as 1018, may show considerable increases in yield strength and work hardening at high strain rates.

In quasi-static tests, the amount of energy required to deform a material is determined from the area under the tensile stress-strain curve and is know as the modulus of toughness. Under dynamic loading, stress-strain response is typically not recorded. Instead, the transfer of energy from a device such as a drop weight or a swinging specimen to the deforming or breaking specimen is equated to the "impact energy."

The Charpy impact test uses a standard Charpy impact machine to evaluate this impact energy. The machine consists of a rigid specimen holder and a swinging pendulum hammer for striking the impact blow to a v-notched specimen as shown in Figs. 1 and 2.

Unfortunately, while the test, including machine and specimen geometry, has been standardized, the test results do not provide definitive information about material properties and thus are not directly applicable to design (as for example might be a yield strength). However, the test is useful for comparing variations in the metallurgical structure of materials and in determining environmental effects, such as temperature on the dynamic response of the material.

One of the most dramatic results of Charpy impact tests is in the form of plots of impact energy versus temperature in which sigmoidally-shaped curves (see Fig. 3) show substantial decreases in some materials' abilities to absorb energy below a certain transition temperature. This ductile to brittle transition is most apparent in materials with BCC and HCP crystalline structures as for example in steels and titanium. A classic and dramatic example of this ductile to brittle behaviour is the low carbon steel Victory ships of WWII cracking in half under even the mild conditions of sitting at anchor in a harbor. Materials with FCC structures (e.g., aluminum and copper) have many slip systems and are more resistant to brittle fracture at low temperatures.

In this laboratory exercise the primary outcome will be plots of impact energy versus temperature for two materials (FCC-606-T6 aluminum and BCC-1018 steel). Note the effects of temperature and material type on the levels and shapes of the curves.

Examine the fracture surfaces of specimens and compare the type and degree of deformation to the impact energy and the corresponding temperature. Consider not only the type of material, but also the effect of notches and temperature in making design decisions.

* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01

E23 Standard Test Methods for Notched Bar Impact Testing of Metallic Materials

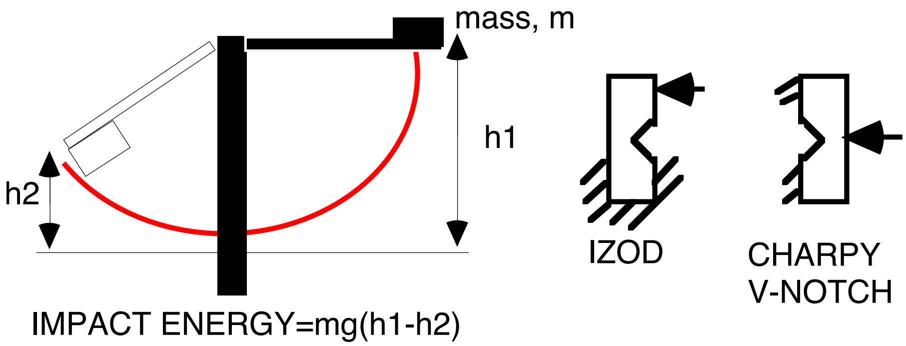
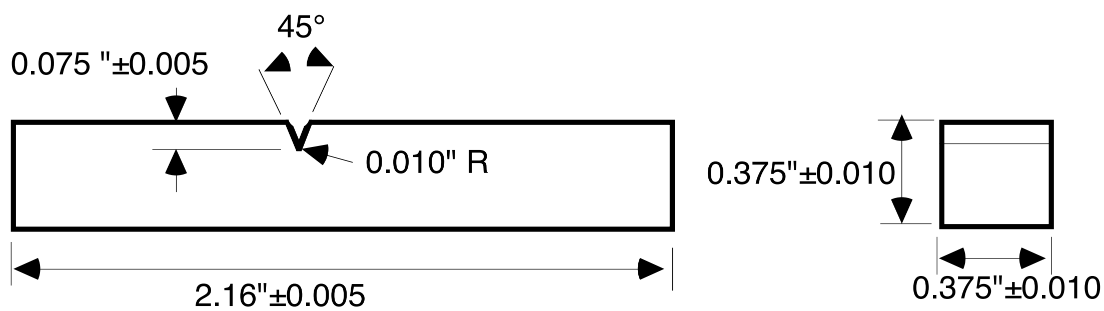
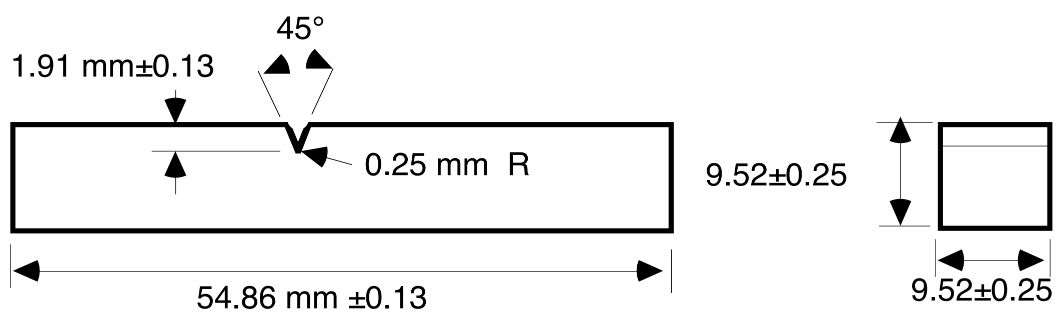


Figure 1 Schematic of Charpy Impact Testing including Izod and Charpy V-notch specimens



a) Dimensions in inches



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b) Dimensions in mm

Figure 2 Charpy V-notch specimen used in these tests

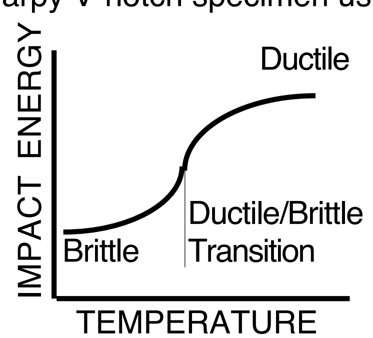


Figure 3 Schematic of plot of impact energy versus temperature showing sigmoidal curve

LABORATORY REPORT

1. Include the following information in the laboratory report.

	Impact Energy (J)	
	6061-T6 aluminum	1018 (HR) or A36 steel
Boiling hot temperature (°C)		
Warm temperature (°C)		
Room temperature (°C)		
Freezing temperature (°C)		
Cold temperature (°C)		
Very cold temperature (°C)		
Extremely cold temperature (°C)		

2. Include the following information in the laboratory report.
   a. Plot the impact energy versus temperature for each material on the same graph.
   b. Compare these impact results for each metal to tabulated values from a source such as the ASM Metals Handbook. Comment on differences and similarities.
   c. Examine the type and degree of deformation of each fracture surface. Correlate this information with the corresponding impact energies. Comment on the correlations.

3. Include the following information in the appendix of the laboratory report. THIS MAY NOT BE ALL THAT IS NECESSARY (i.e., don't limit yourself to this list.)
   a. Original data sheets and/or printouts
   b. All supporting calculations. Include sample calculations if using a spread sheet program. DO NOT INCLUDE ALL TABULATED RAW OR CALCULATED DATA.

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: CHARPY V-NOTCH IMPACT

DATA SHEET

01 Januarly 2000 / mgj

NAME____**DATE**____________

LABORATORY PARTNER NAMES____

____

EQUIPMENT IDENTIFICATION____

____

Aluminium

Pretest Conditioning	**Temperature (°C)**	**Impact Energy (J)**
Boiling water		
Warm water		
Room temperature water		
Ice water		
Salted ice water		
Acetone with some dry ice		
Acetone with much dry ice		

STEEL

Pretest Conditioning	**Temperature (°C)**	**Impact Energy (J)**
Boiling water		
Warm water		
Room temperature water		
Ice water		
Salted ice water		
Acetone with some dry ice		
Acetone with much dry ice		

STRESS CONCENTRATIONS

ME 354, MECHANICS OF MATERIALS LABORATORY
STRESS CONCENTRATIONS

25 october 2000 / mgj

PURPOSE

The purpose of this exercise is to the study the effects of geometric discontinuities on the stress states in structures and to use photo elasticity to determine the stress concentration factor in a simple structure.

EQUIPMENT

- Un-notched beam of birefringent material (an epoxy).
- Notched bend beam of the same birefringent material as the un-notched beam.
- Four-point flexure loading fixture with load pan and suitable masses for loading
- Circular polariscope with monochromatic light source

PROCEDURE

Part 1. Beam under Pure Bending to Determine the Stress-Optical Coefficient of the Material

i) Install the un-notched beam (see Fig. 1) in the four-point flexure loading fixture
ii) Attach the load pan (Note: The combined pan/fixture mass is ~0.980 kg)
iii) Apply two10-kg masses one at a time to the load pan.
iv) With the polarizer and analyzer crossed (dark field), focus the camera, and record the image using the thermal printer
v) Determine the maximum fringe orders at the top and bottom of the beam including estimates of fractional fringes orders by counting the fringes.
vi) The stress-optical coefficient can be calculated using the following relation:

$$f = \frac{t}{\overline{N}}(\sigma_1 - \sigma_2) \qquad (1)$$

where f is the stress-optical coefficient, $\overline{N}$ is the fringe order, t is the model thickness, and σ_1 and σ_2 are the plane-stress principal stresses.

Part 2.Notched Beam under Pure Bending to Determine the Stress Concentration Factor

i) Install the notched beam (see Fig. 2) in the four-point flexure loading fixture
ii) Attach the load pan (Note: The combined pan/fixture mass is ~0.980 kg)
iii) Apply one 5-kg mass to the load pan. (Note: Do not apply more than 5 kg at one time).
iv) With the polarizer and analyzer crossed (dark field), focus the camera, and record the image using the thermal printer.
v) Determine the maximum fringe orders at the top and bottom of the beam and at the edge of the notch including estimates of fractional fringes orders.
vi) The stress distributions within the beam can be calculated using the relation:

$$(\sigma_1 - \sigma_2) = f\frac{\overline{N}}{t} \qquad (2)$$

where f is the stress-optical coefficient determined previously, $\overline{N}$ is the fringe order, t is the model thickness, and σ_1 and σ_2 are the plane-stress principal stresses.

* REFERENCES

Manual on Experimental Stress Analysis, J.F. Doyle & J.W. Philips, eds, Society for Exper. Mechanics, 1989
Experimental Stress Analysis, J.W. Dally and W.F. Riley, McGraw-Hill, Inc., 1990
Handbook on Experimental Mechanics, A.S. Kobayashi, ed., Prentice Hall, Inc.,1992
Formulas for Stress and Strains, R.J. Roark and W.C. Young, McGraw-Hill, Inc., 1975
Stress Concentration Factors, R.E. Peterson, John Wiley and Sons, Inc., 1974

RESULTS

When loads are applied to a solid body, such as part of a structure or a machine component, stresses which vary from point to point, are set up in the body. At certain points, stress concentrations (sometimes called stress raisers) occur and are potential weak points in the body. Frequently, an alteration in the shape of the body will lead to a reduction in the stresses at such points and to a more even distribution over the whole body. An optimum body is that of uniform load-carrying capability.

The mathematical theory of elasticity provides many valuable solutions involving the stress distributions in bodies of simple geometries and loadings. A common use of these solutions is the determination of stress concentration factors ($k_t = \frac{\sigma_{local}}{\sigma_{remote}}$) resulting from discontinuities or other localized disturbances in the stress field of the body. In more complicated problems, commercially available two- and three-dimensional computer programs for finite element and boundary element analyses (FEA and BEM, respectively) can be used to locate and quantify the stress concentrations.

These theoretical and numerical results are exact solutions to problems which may or may not model the actual situations (usually due to assumptions about loads, load applications and boundary conditions). This uncertainty in modeling often requires experimental verification by spot checking the analytical or numerical results. A frequently cited example involves a threaded joint which seldom produces uniform contact at the threads. Contact analyses based on the idealized boundary condition of uniform contact will grossly underestimate the actual maximum stress concentration at the root of the overloaded thread. The uncertainty in the contact condition requires a stress analysis of the actual threaded joint experimentally despite the proliferation of FEA and BEM programs. Experimental stress analysis is also necessary to study nonlinear structure problems involving dynamic loading and/or plastic/viscoplastic deformations. Available FEA programs cannot provide detailed stress analysis of three-dimensional dynamic structures. Constitutive relations for plastic/viscoplastic materials are still being developed

One such experimental procedure often applied to empirically determine stress states is photoelasticity. Photoelasticity is a relatively simple, whole-field method of elastic stress analysis which is well suited for visually identifying locations of stress concentrations. In comparison with other methods of experimental stress analysis, such as a strain gage technique which is a point measurement method, photoelasticity is inexpensive to operate and provides results with minimum effort.

Photoelasticity consists of examining a model similar to the structure of interest using polarized light. The model is fabricated from transparent polymers possessing special optical properties. When the model is viewed under the type (but not necessarily magnitude) of loading similar to the structure of interest, the model exhibits patterns of fringes from which the magnitudes and directions of stresses at all points in the model can be calculated. The principle of similitude can be used to deduce the stresses which exist in the actual structure.

A disadvantage of photoelasticity is the necessity to test a polymer model which may not be able to withstand extreme loading conditions such as high temperature and/or high strain rates. Although photoelasticity is generally applied to elastic analysis, limited studies on photo plasticity and photo viscoelasticty indicate the potential of extending the technique to nonlinear structural analysis. Further details of photoelasticity can be found in listed references.

Show all work and answers on the Worksheet / In-class Laboratory report.

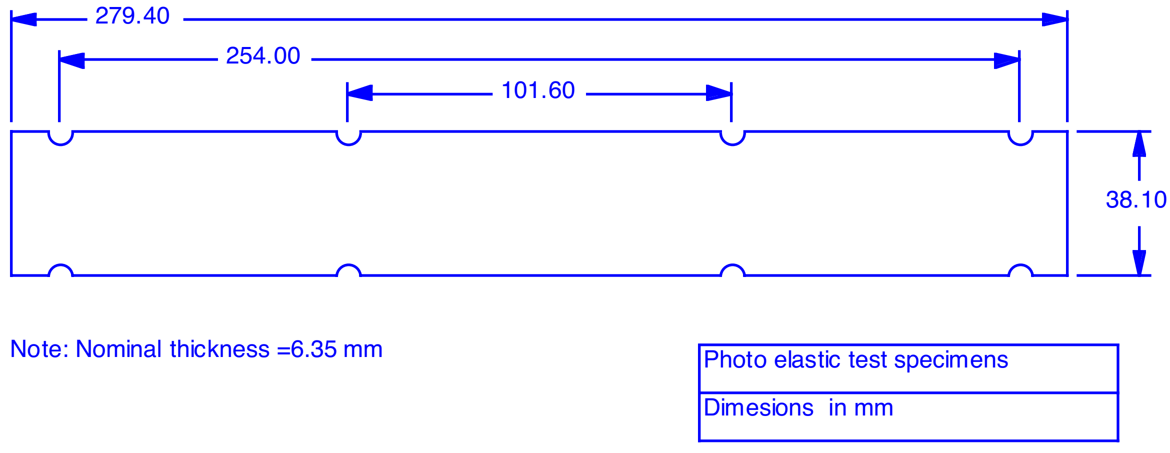


Figure 1 Un-notched Beam

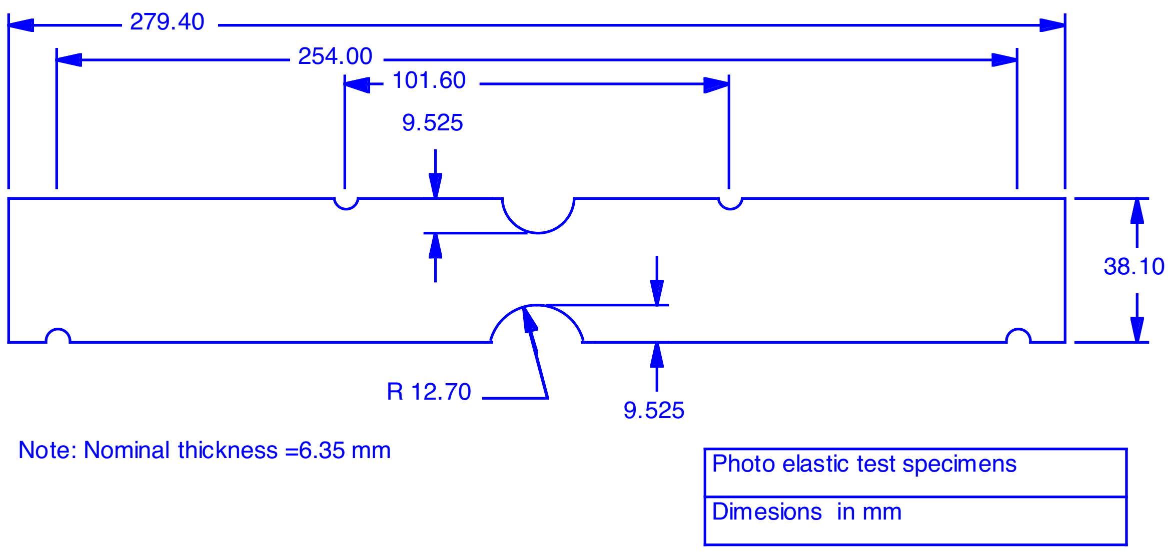


Figure 2 Notched Beam

ME 354, MECHANICS OF MATERIALS LABORATORY

STRESS CONCENTRATIONS

25 october 2000 / mgj

WORK SHEET

NAME______________________________________**DATE**_____________

EQUIPMENT IDENTIFICATION______________________________________

1) The properties of two birefringent polymers often used for photoelastic experiments are in Table 1.

Table 1 Some Properties of Two Birefringent Polymers Used in Photoelastic Experiments

PSM-1 (polycarbonate) Selected Properties (R.T.)

Elastic Modulus, E(GPa)	2.5
Proportional Limit σ_o (MPa)	48
Poisson's ratio, ν	0.38
Stress Optical Coefficient, f (MPa-mm/fringe)*	7
Figure of Merit Q=E / f (1/m)	357,143

PSM-9 (epoxy) Selected Properties (R.T.)

Elastic Modulus, E(GPa)	3.3
Proportional Limit σ_o (MPa)	50
Poisson's ratio, ν	0.37
Stress Optical Coefficient, f (MPa-mm/fringe)*	10.5
Figure of Merit Q=E / f (1/m)	314,286

* in green light with wavelength 546 nm

2) For the two beams and loading fixtures, confirm the following information. See Figs. 3 and 4 for nomenclature.

Table 2 Dimensions and Loading for Un-notched and Notched Photoelastic Beams

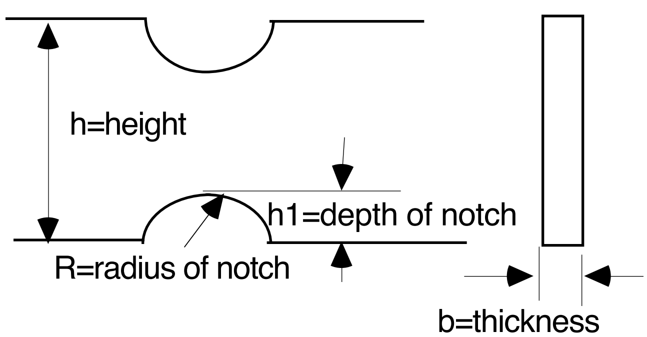
Un-notched beam

Calibration load, P_c $=P_{weight}+P_{fixture}+P_{pan}$ (N)	
Outer Span, L_o (mm)	
Inner Span, L_i (mm)	
Height, h (mm)	
Thickness, b (mm)	
Radius of Notch, R (mm)	----
Depth of notch, h_1 (mm)	----

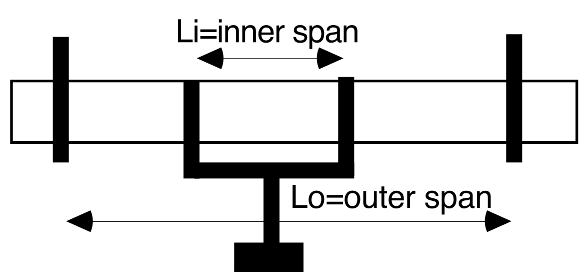
Notched Beam

Test load, P_c $=P_{weight}+P_{fixture}+P_{pan}$ (N)	
Outer Span, L_o (mm)	
Inner Span, L_i (mm)	
Height, h (mm)	
Thickness, b (mm)	
Radius of Notch, R (mm)	
Depth of notch, h_1 (mm)	

Note: The calibration and test loads must include the mass of the fixture and pan as well as the added masses



a) Notch Detail



b) Overall Specimen Detail

Figure 3 Nomenclature for the Beams

3) A unique aspect of the four-point flexure loading arrangement is that the region of interest (the section of the beam within the inner loading span) experiences a pure bending moment as shown in Fig. 4.

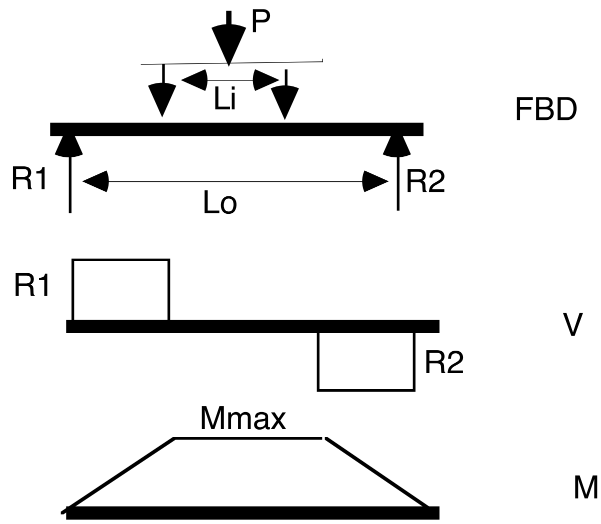


Figure 4 Free Body, Shear and Moment Diagrams for Four-Point Flexure Loading

For the un-notched beam, determine the following:

Moment of Inertia for the rectangular cross section beam, $I = \frac{bh^3}{12}$ =__________________mm^4

Maximum moment when the calibration load, P_C, was applied,

$M_c = \frac{P_c(Lo - Li)}{4}$=______________________N•mm

4) At the outer free edge of the beam (y=c=h/2) the stress state is uniaxial and the photo elastic relation can be used to determine the stress optical coefficient directly from the beam bending relation.

The average fringe value at the upper and lower outer edges of the beam determined at the calibration load, $\overline{N}$=_________________.

Maximum distance to an outer edge of the beam from the neutral axis, c=h/2=___________mm

Maximum uniaxial bending stress at the outer free edge of the beam

$\sigma_1 = \frac{M_c\, c}{I}$=__________MPa.

Calculated stress optical coefficient for the material, $f = \frac{b}{\overline{N}}(\sigma_1)$=________MPa-mm/fringe

Compare this value to that shown in the table. How do the values compare? Discuss any discrepancies and possible reasons (Note: Do not panic if the calculated stress optical coefficient differs from the value listed in Table 1.differences in optical test setup, environmental effects in the material, etc. all require the "calibration" of the material).

5) At the free edge of the notch the stress state is uniaxial and the photoelastic relation can be used to calculate the normal stress using the relation between the fringe order at the free edge, the stress optical coefficient for the material, and the specimen thickness.

The average fringe value at the free edge of the notches determined at the test force, $\bar{N}$=______.

Calculated normal stress at the free edge of the notch, $\sigma_1 = \sigma_{w/\ notch} = \frac{f\,\bar{N}}{b}$=_________MPa.

6) One way to define a stress concentration factor, k_t, is the ratio of the stress at the discontinuity in a body to the maximum stress in the net section (i.e., that part of the body remaining after the discontinuity removes a portion of the cross section) such that: $k_t = \frac{\sigma_{w/\ discontinuity}}{\sigma_{net}}$.

The notched beam is symmetric, therefore the neutral axis is the midpoint of the beam as well as the midpoint of the net cross section beam. The width of the net cross section beam is the distance between the notches, $h_{net} = h - 2h_1$=_________mm.

The moment of inertia for the net cross section is $I_{net} = \frac{bh_{net}^3}{12}$= _________mm^4.

The distance from the neutral axis to the outermost edge of the net cross section is $c_{net} = h_{net}/2$= _________mm.

The moment in the beam at the test force, P_t, is $M_t = \frac{P_t(L_o - L_i)}{4}$=_____________mm.

Stress in the net cross section of the beam, $\sigma_{net} = \frac{M_t\, c_{net}}{I_{net}}$=______________MPa.

The stress concentration factor is the ratio of the stress at the notch and to the net cross section stress: $k_t^{measured} = \frac{\sigma_{w/notch}}{\sigma_{net}}$=____________.

7) Several authors have compiled stress concentration factors for simple geometries. The most "famous" compilation is Peterson's book of stress concentration factor graphs. From Peterson's book for the double-notched flat specimen in bending, k_t is plotted as a function of r/d = R/(h-2h_1) for various values of D/d=h/(h-2h_1).

In this case, $\frac{r}{d} = \frac{R}{(h-2h_1)}$=______________ and $\frac{D}{d} = \frac{h}{(h-2h_1)}$=________________.

The stress concentration, k_t can be "picked off" a plot such that k_t^{plot} =______________

Alternatively, a curve fit for a double-notched beam in pure bending (Roarke and Young) is described as follows for $0.25 \le \frac{h_1}{R} \le 2.0$. In this case, $\frac{h_1}{R}$=____________ and the stress concentration factor is: $k_t = K_1 + K_2\left(\frac{2h_1}{h}\right) + K_3\left(\frac{2h_1}{h}\right)^2 + K_4\left(\frac{2h_1}{h}\right)^3$

where $K_1 = 0.723 + 2.845\sqrt{\frac{h_1}{R}} - 0.504\frac{h_1}{R}$=___________

$K_2 = -1.836 - 5.746\sqrt{\frac{h_1}{R}} + 1.314\frac{h_1}{R}$=___________

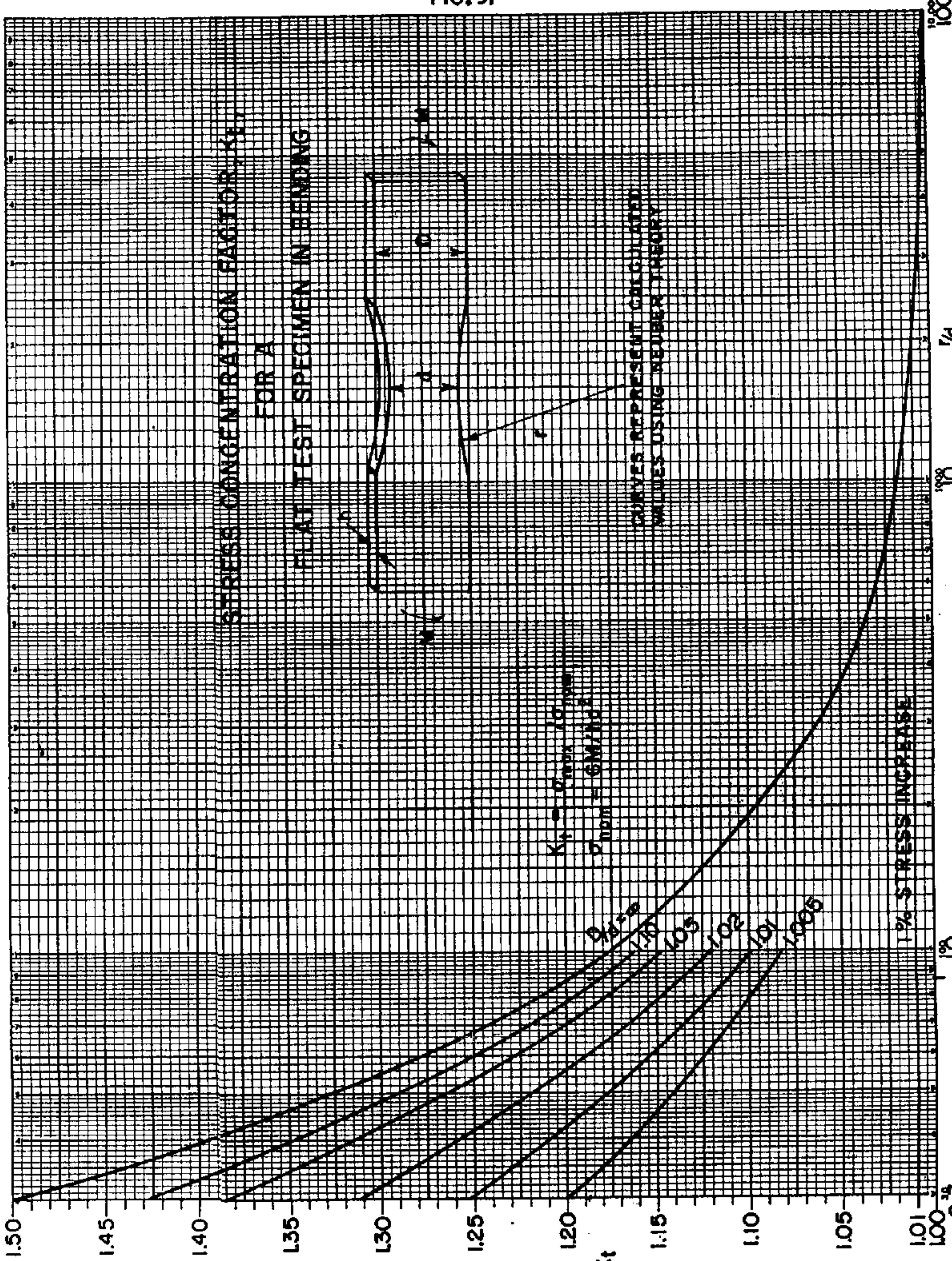
$K_3 = 7.254 - 1.885\sqrt{\frac{h_1}{R}} + 1.646\frac{h_1}{R}$=____________

$K_4 = -5.140 + 4.785\sqrt{\frac{h_1}{R}} - 2.456\frac{h_1}{R}$=____________

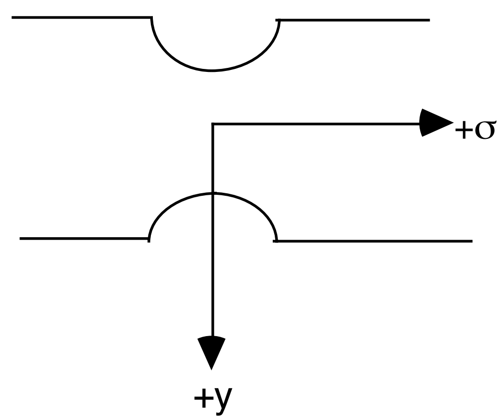
such that: $k_t^{\text{curve fit}} = k_t = K_1 + K_2\left(\frac{2h_1}{h}\right) + K_3\left(\frac{2h_1}{h}\right)^2 + K_4\left(\frac{2h_1}{h}\right)^3$ _________________

8) Compare the k_t measured from the photoelastic analysis to that determined from a compiled handbook (e.g., k_t^{plot} or $k_t^{\text{curve fit}}$). Determine the percent differences between the measured k_t and the compiled values. Since many compiled stress concentration factors were determined from photoelastic analyses, discuss possible reasons for differences between the measured k_t and compiled values.

FIG. 37


STRESS CONCENTRATION FACTOR, Kt, FOR A FLAT TEST SPECIMEN IN BENDING
Kt = σmax / σnom
σnom = 6M/hd²
CURVES REPRESENT CALCULATED VALUES USING NEUBER THEORY
1% STRESS INCREASE
D/d = ∞
1.10
1.05
1.02
1.01
1.005
Kt
r/d
1.50
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.01
1.00
0.3
1.0
10
100

Extra effort: Using the fringe orders across the notched beam, assume the stress state is uniaxial, plot the stress across the height of the beam. Compare this stress distribution to that of the un-notched beam at the same force.



Extra effort: Another way to define a stress concentration factor, k_t is the ratio of the stress at the discontinuity in a body to the stress that would have been at the same point in the body without the discontinuity such that $k_t = \frac{\sigma_{\text{w/ discontinuity}}}{\sigma_{\text{w/o discontinuity}}}$.

The notched beam is symmetric, therefore the neutral axis is the midpoint of the beam. The distance from the neutral axis to the edge of the notch is, $y = \frac{h}{2} - h_1$=________mm

The moment in the beam at the test force, P_t, is $M_t = \frac{P_t(L_o - L_i)}{4}$=____________mm.

Stress in an un-notched beam at the same point at the edge of the notch in the notched beam is $\sigma_{\text{w/o notch}} = \frac{M_t\, y}{I}$=_____________MPa.

The stress concentration factor is the ratio of the stresses at the same location for the notched and un-notched beams, $k_t = \frac{\sigma_{\text{w/ notch}}}{\sigma_{\text{w/o notch}}}$=____________.

The Peterson stress concentration factor found earlier can be modifed to account for this difference in defintion such that $k_t^{\text{notch}} = \left(k_t^{\text{net}} = \frac{\sigma_{\text{w/ notch}}}{\sigma_{\text{net}}}\right)\left(\frac{I_{\text{beam}}}{I_{\text{net}}}\right)$.

Since, $I_{\text{net}} = \frac{bh_{\text{net}}^3}{12}$=_____________ and $I_{\text{beam}} = \frac{bh_{\text{beam}}^3}{12}$=___________, then $\left(\frac{I_{\text{beam}}}{I_{\text{net}}}\right)$=_______and $k_t^{\text{notch}} = \left(k_t^{\text{net}} = \frac{\sigma_{\text{w/ notch}}}{\sigma_{\text{net}}}\right)\left(\frac{I_{\text{beam}}}{I_{\text{net}}}\right)$=____________

Compare the k_t at the notch using this alternative definition and the modified k_t determined from the compiled version in the handbook. Determine the percent differences between the two values. Which definition of k_t seems more "reasonable?" Why?

FRACTURE

ME 354, MECHANICS OF MATERIALS LABORATORY

FRACTURE

01 January 2000 / mgj

PURPOSE

The purpose of this exercise is to study the effects of cracks in decreasing the load-carrying ability of structures and to determine the plane strain critical stress intensity factor, K_{Ic}, for single-edge notched specimens.

EQUIPMENT

- Single-edge notched tensile specimens of polymethyl methacrylate (PMMA) and polycarbonate (PC)
- Tensile test machine with grips, controller, and data acquisition system

PROCEDURE

- Measure the width and thickness of the gage section for each specimen to 0.02 mm.
- Measure the notch length for each specimen to 0.02 mm.
- Zero the force output (balance).
- Activate force protect (~50 N) on the test machine to prevent overloading the specimen during installation.
- Install the top end of the tensile specimen in the top grip of the test machine while the test machine is in displacement control.
- Install the bottom end of the tensile specimen in the lower grip of the test machine.
- In displacement control adjust the actuator position of the test machine to achieve nearly zero force on the specimen.
- Deactivate force protect.
- Initiate the data acquisition and control program.
- Enter the correct file name and specimen information as required.
- Initiate the test sequence via the computer program.
- Continue the test until specimen fracture.
- Confirm the initial notch length for each specimen.
- Examine the fracture surface to note any evidence of subcritical crack growth. Note the appearance of the fracture surfaces.
- Examine the force versus displacement trace each test. Note the force at fracture initiation, P_Q, and maximum force, P_{max}, at fracture.

* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01

E399 Standard Test Method for Plane Strain Fracture Toughness of Metallic Materials

RESULTS

Anticipated fracture forces will first be calculated for un-notched and notched specimens at yield and ultimate tensile strengths. These forces will be compared to anticipated fracture forces assuming single-edge notched tensile specimens such that:

$$K_Q = F(\alpha)\frac{P_Q}{WB}\sqrt{\pi a}$$

$$\text{where } \alpha = a/W \qquad (1).$$

$$F(\alpha) = 0.265(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}} \quad \text{for } (h/W \geq 1)$$

where P_Q is the tentative fracture force, W is the gage section width, B is the gage section thickness, a is the notch/crack length, h is half the gage section length and K_Q is the tentative fracture toughness value.

Compare the relation of K_{Ic} versus yield strength for these alloys to that of other materials and comment on the susceptibility of these materials to fracture or yielding. Use your own sourses of information (e.g. tensile test laboratory results).

Silicon nitride (ceramic) alloys		6061-T6 Aluminum alloy		1018 HR Steel alloy	
E (GPa)	310	E (GPa)		E (GPa)	
σ_o (MPa)	= S_{UTS} =500-1000	σ_o (MPa)		σ_o (MPa)	
S_{UTS} (MPa)	= σ_o =500-1000	S_{UTS} (MPa)		S_{UTS} (MPa)	
% elongation	0.25-0.5	% elongation		% elongation	
K_{Ic} $\left(MPa\sqrt{m}\right)$	5-10	K_{Ic} $\left(MPa\sqrt{m}\right)$		K_{Ic} $\left(MPa\sqrt{m}\right)$	

Design Concerns and Failure Criterion
(Fracture Mechanics, Maximum Normal Stress, or Yield Stress?)

PMMA polymer		PC polymer	
E (GPa)		E (GPa)	
σ_o (MPa)		σ_o (MPa)	
S_{UTS} (MPa)		S_{UTS} (MPa)	
% elongation		% elongation	
K_{Ic} $\left(MPa\sqrt{m}\right)$		K_{Ic} $\left(MPa\sqrt{m}\right)$	

Design Concerns and Failure Criterion
(Fracture Mechanics, Maximum Normal Stress, or Yield Stress?)

Show all work and answers on the Worksheet, turning this in as the In-class Laboratory report.

ME 354, MECHANICS OF MATERIALS LABORATORY

FRACTURE

01 January 2000 / mgj

WORK SHEET

NAME____**DATE**______________

EQUIPMENT IDENTIFICATION____

1) Determine (look up) the following mechanical properties.

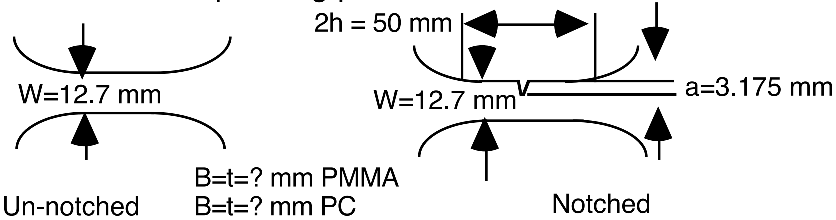
PMMA (acrylic)
Selected Mechanical Properties (R.T.)

E (GPa)	
σ_o (MPa)	estimate as S_{uts}
S_{UTS} (MPa)	
% elongation	
K_{Ic} $\left(MPa\sqrt{m}\right)$	

PC (polycarbonate)
Selected Mechanical Properties (R.T.)

E (GPa)	
σ_o (MPa)	estimate as $S_{uts}/2$
S_{UTS} (MPa)	
% elongation	
K_{Ic} $\left(MPa\sqrt{m}\right)$	

2) For the following NOMINAL specimen dimensions, determine the corresponding predicted fracture forces



Un-notched (PMMA)
B=t=______mm
Yield: $P_m = \sigma_o A_W = \sigma_o WB =$ _________N
Ultimate: $P_m = S_{UTS} A_W = S_{UTS} WB =$ _________N

Un-notched (PC)
B=t=______mm
Yield: $P_m = \sigma_o A_W = \sigma_o WB =$ _________N
Ultimate: $P_m = S_{UTS} A_W = S_{UTS} WB =$ _________N

Notched (PMMA) [Net cross section]
Yield: $P_m = \sigma_o A_{W-a} = \sigma_o (W-a)B =$ _______N
Ultimate: $P_m = S_{UTS} A_{W-a} = S_{UTS}(W-a)B =$ ______N

Notched (PC) [Net cross section]
Yield: $P_m = \sigma_o A_{W-a} = \sigma_o (W-a)B =$ ______N
Ultimate: $P_m = S_{UTS} A_{W-a} = S_{UTS}(W-a)B =$ ___N

Fracture (PMMA)
a = _____m for K_{Ic} but a = ____mm for a/W
W = ____________ mm
$a/W = \alpha =$ ____________
B = ____________ mm
$K_{Ic} =$ ____________ $MPa\sqrt{m}$

$$F(\alpha) = 0.265(1-\alpha)^4 + \frac{0.857+0.265\alpha}{(1-\alpha)^{3/2}} = \_\_\_\_\_\_\_\_$$

for(h/W ≥ 1) where $\alpha = a/W$

$$P_f = \frac{K_{Ic}WB}{F(\alpha)\sqrt{\pi a}} = \_\_\_\_\_\_\_\_\_\_\_\_ N$$

Fracture (PC)
a = _____m for K_{Ic} but a = ____mm for a/W
W = ____________ mm
$a/W = \alpha =$ ____________
B = ____________ mm
$K_{Ic} =$ ____________ $MPa\sqrt{m}$

$$F(\alpha) = 0.265(1-\alpha)^4 + \frac{0.857+0.265\alpha}{(1-\alpha)^{3/2}} = \_\_\_\_\_\_\_\_$$

for(h/W ≥ 1) where $\alpha = a/W$

$$P_f = \frac{K_{Ic}WB}{F(\alpha)\sqrt{\pi a}} = \_\_\_\_\_\_\_\_\_\_\_\_ N$$

3) Determine the fracture initiation force, P_Q and the maximum force, P_{max} from the force vs displacement test results. Measure the actual width, W, actual thickness, B, and actual notch/crack length, a.

PMMA
Fracture Test Results

W (mm) (measured)	
B=t (mm) (measured)	
a (mm) (measured)	
$P_Q(N)$ (measured)	
$P_{max}(N)$ (measured)	

PC
Fracture Test Results

W (mm) (measured)	
B=t (mm) (measured)	
a (mm) (measured)	
$P_Q(N)$ (measured)	
$P_{max}(N)$ (measured)	

4) Compare the measured fracture initiation force, P_Q, , to the predicted forces, P_m and P_f, calculated above. Which approach (Un-notched or Notched (yield and ultimate) or Fracture) is closer to the measured fracture force? Is this what you expected? If so, why or why not?

Note: Do the 'fracture' tests meet the requirements of ASTM E399?

i) Valid specimen with pre-crack and known S.I.F., **ii)** $\frac{P_{max}}{P_Q} < 1.10$ and **iii)** $B > 2.5\left(\frac{K_{Ic}}{\sigma_o}\right)^2$

Based on these results, are cracks or crack-like notches important concerns to a designer? How would you design to account for these features?

5) Calculate a tentative plane strain fracture toughness value, K_Q, from the fracture force and compare this to the 'book' value of the plane strain fracture toughness, K_{Ic} .

PMMA

$$F(\alpha) = 0.265(1-\alpha)^4 + \frac{0.857+0.265\alpha}{(1-\alpha)^{3/2}} = \_\_\_\_\_\_\_\_$$

for(h/W ≥ 1) where $\alpha = a/W$

a = ____________ m

$$K_Q = F(\alpha)\frac{P_Q}{WB}\sqrt{\pi a} = \_\_\_\_\_\_\_ \left(MPa\sqrt{m}\right)$$

PC

$$F(\alpha) = 0.265(1-\alpha)^4 + \frac{0.857+0.265\alpha}{(1-\alpha)^{3/2}} = \_\_\_\_\_\_\_\_$$

for(h/W ≥ 1) where $\alpha = a/W$

a = ____________ m

$$K_Q = F(\alpha)\frac{P_Q}{WB}\sqrt{\pi a} = \_\_\_\_\_\_\_ \left(MPa\sqrt{m}\right)$$

PMMA
Fracture Test Results

K_Q $\left(MPa\sqrt{m}\right)$	
K_{Ic} $\left(MPa\sqrt{m}\right)$	

PC
Fracture Test Results

K_Q $\left(MPa\sqrt{m}\right)$	
K_{Ic} $\left(MPa\sqrt{m}\right)$	

Are K_Q and K_{Ic} similar? If not, what factors (e.g. simulated crack, ductility, test rate, material properties, etc.) might account for these differences? Are these valid fracture tests or more notch sensitivity tests? Do these tests indicate a susceptibility of components comprised of certain materials to brittle fracture from crack or crack-like notches, even though they normally display moderate ductility?

COMPRESSION AND BUCKLING

ME 354, MECHANICS OF MATERIALS LABORATORY

COMPRESSION AND BUCKLING

01 January 2000 / mgj

PURPOSE

The purpose of this exercise is to study the effects of end conditions, column length, and material properties on compressive behaviour and buckling in columns.

EQUIPMENT

- Solid rods of various lengths of aluminum and steel
- Universal test machine with grips, controller, and data acquisition system

PROCEDURE

Repeat the following steps for each specimen.

- Measure the diameter and lengths of each specimen to 0.02 mm.
- Zero the force output (balance).
- Activate force protect (~50 N) on the test machine to prevent overloading the specimen during installation.
- Install the top end of the test specimen in the top grip of the test machine while the test machine is in displacement control.
- Install the bottom end of the test specimen in the lower grip of the test machine.
- In displacement control adjust the actuator position of the test machine to achieve nearly zero force on the specimen.
- Deactivate force protect.
- Initiate the data acquisition and control program.
- Enter the correct file name and specimen information as required.
- Initiate the test sequence via the computer program.
- Continue the test until buckling or compressive failure of the test specimen occurs
- Examine the force versus displacement trace for each test. Note the force at the onset of buckling or compressive failure (i.e., significant deviation from linearity)

RESULTS

Structures and machines may fail in many ways depending on the materials, kinds of loads, and conditions of support. Many machine elements can be modeled as uniform members under uniaxial tension or compression. For tensile loading, these members tend to self-align and fail either by ductile deformation or brittle fracture depending on the material. In compression, the failure mode is complicated by the possibility of a geometric instability, called buckling, in addition to ductile deformation.

Columns are structural members which support compressive forces. Buckling occurs when the column has a tendency to deflect laterally, out of the line of action of the force. Once buckling initiates, the instability can lead to failure of the column because the eccentric force acts as a moment causing greater stresses and deflections due to the combination of the bending and axial forces.

The possibility of buckling increases for the following column conditions: 1) longer, "thinner" columns, 2) pinned, free, or non-fixed end conditions, 3) initial eccentricity of the force (e.g., bent columns) and/or 4) lower elastic modulus of the column material.

In this exercise, two materials and two column lengths will be studied. Anticipated buckling or compressive failure forces will first be calculated for various length specimens and materials.

For compressive failure, $P_o = \sigma_o A_o$

and (1).

For buckling, $P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$

where P_o is the compressive failure force (yield), σ_o is proportional limit stress (or yield strength), A_o is the initial area of the gage section, P_{cr} is the Euler critical buckling force, I is the least moment of inertia of the cross section, and L_e is the effective, unsupported length of the column.

The anticipated buckling or compressive failure forces will then be compared to the actual measured forces at the onset of instability. Observations will be made on the effects of end conditions, material type, and column length.

Show all work and answers on the Worksheet, turning this in as the In-class Laboratory report.

References:

"Mechanics of Materials," J.M. Gere and S.P. Timoshenko
"Mechanics of Materials," R.C. Hibbeler

ME 354, MECHANICS OF MATERIALS LABORATORY

COMPRESSION AND BUCKLING

01 January 2000 /mgj

WORK SHEET

NAME______________________________________**DATE**_____________

EQUIPMENT IDENTIFICATION______________________________________

1) Determine (look up) the following mechanical properties.

Table 1 Selected Properties for Test Materials

6061-T6 Aluminum
Selected Mechanical Properties (R.T.)

E (GPa)	
σ_o (MPa)	
S_{UTS} (MPa)	
% elongation	

1018 Steel (CD)
Selected Mechanical Properties (R.T.)

E (GPa)	
σ_o (MPa)	
S_{UTS} (MPa)	
% elongation	

2) Measure and record the following dimensions.

Table 2 Pertinent column dimensions

Column Dimensions for Aluminum

Diameter, d (mm)	
Length 1, L1 (mm)	
Length 2, L2 (mm)	

Column Dimensions for Steel

Diameter, d (mm)	
Length 1, L1 (mm)	
Length 2, L2 (mm)	

3) For each column, determine the following geometric quantities.

Aluminum

Moment of Inertia: $I = \frac{\pi d^4}{64}$ _________ mm^4

Cross sectional area: $A = \frac{\pi d^2}{4}$ _________ mm^2

Radius of gyration squared: $k^2 = \frac{I}{A}$ _______ mm^2

Radius of gyration: $k = \sqrt{k^2} = \sqrt{\frac{I}{A}}$ _______ mm

Steel

Moment of Inertia: $I = \frac{\pi d^4}{64}$ _________ mm^4

Cross sectional area: $A = \frac{\pi d^2}{4}$ _________ mm^2

Radius of gyration squared: $k^2 = \frac{I}{A}$ _______ mm^2

Radius of gyration: $k = \sqrt{k^2} = \sqrt{\frac{I}{A}}$ _______ mm

4) Buckling of columns with pinned ends is often called the fundamental case of buckling. However, many other conditions such as fixed ends, elastic supports, and free ends are encountered in practice. The critical forces for buckling for each of these end conditions can be determined by applying the appropriate boundary conditions and solving the differential equations. These solutions lead to the concept of an "effective length," L_e, appropriate for each end condition which is a multiple of the actual length, L, of the column as shown in Table 3 and Figure 1.

Table 3 Effective column length for various end conditions

Pinned/Pinned	Fixed/Free	Fixed/Fixed	Pinned/Fixed
$L_e = L$	$L_e = 2L$	$L_e = L/2$	$L_e = 0.7\ L$

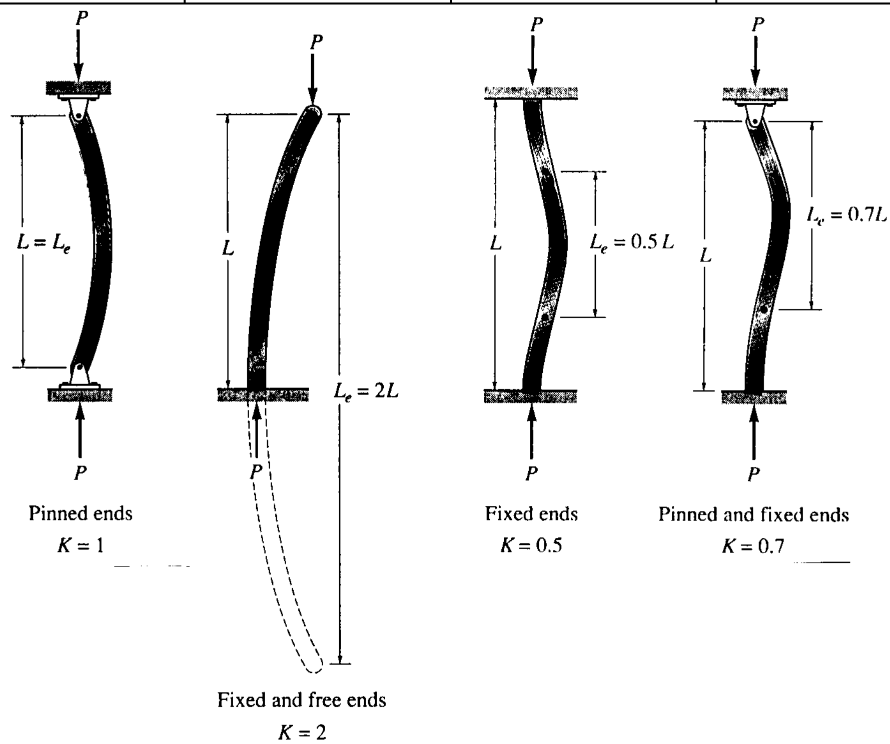


Figure 1 Illustration of end conditions for columns

5) In general, axially-loaded compression members may fail by one of three modes: crushing; a combination of crushing or buckling; or buckling alone. Columns can be placed into three groups:

1) Short columns - the failure mode is by crushing (simple compressive failure)
2) Intermediate columns - the failure mode depends on simple compressive and/or bending stress
3) Long columns - the failure mode is primarily a function of the bending stress (buckling).

A parameter which is employed to group these columns is the slenderness ratio, L_e/k.

The minimum slenderness ratio $\left.\frac{L_e}{k}\right|_{min}$ marks the nominal transition from crushing to buckling. If the axial stress, σ, is plotted as a function of slenderness ratio, then the minimum slenderness ratio is the nominal transition from the constant stress for crushing, $\sigma = \sigma_o$, to the stress as function of L_e/k for buckling, $\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$.

Aluminum

Elastic modulus: E =_________MPa

Proportional limit stress: σ_o =_________MPa

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =______

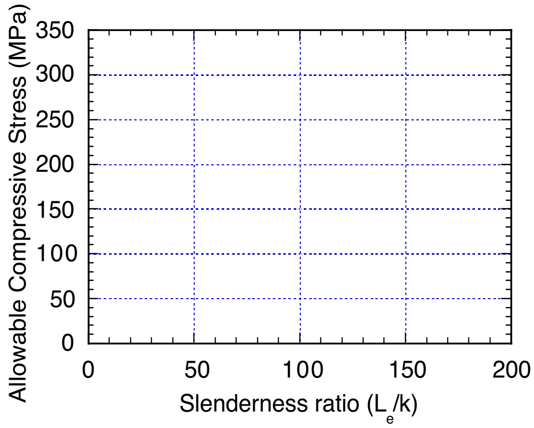
Steel

Elastic modulus: E =_________MPa

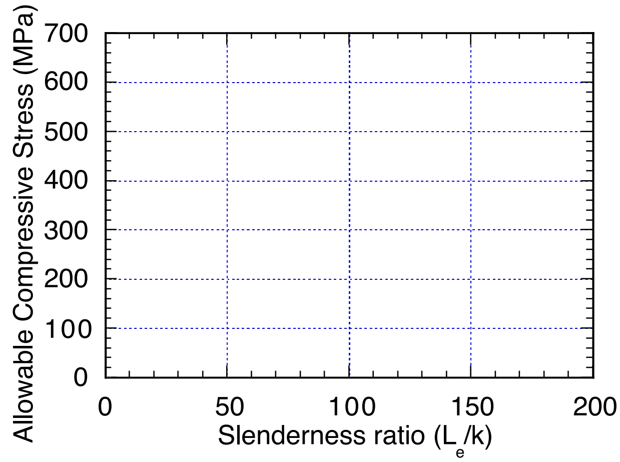
Proportional limit stress: σ_o =_________MPa

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

On the following graphs, plot $\sigma = \sigma_o$ for $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$ **and** $\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ for $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$.



a) Allowable compressive stress for aluminum



b) Allowable compressive stress for steel

Figure 1 Allowable compressive stress for aluminum and steel

6) Determine the following critical compressive forces for the experimental columns

Aluminum

For column length L1, the unsupported length if each grip end is ℓ= _____ mm long such that L=L1-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L1, slenderness ratio, L_e/k=________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$. _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$. _________MPa

Cross sectional area, A=___________mm^2

Use the smaller of the stresses calculated above.
For L1, critical force, $P_{cr}^{L1} = \sigma A$=___________N

Steel

For column length L1, the unsupported length if each grip end is ℓ= _____ mm long such that L=L1-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L1, slenderness ratio, L_e/k=________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$ _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e/k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$. _________MPa

Cross sectional area, A=___________mm^2

Use the smaller of the stresses calculated above.
For L1, critical force, $P_{cr}^{L1} = \sigma A$=___________N

For column length L2, the unsupported length if each grip end is ℓ= _____ mm long such that L=L2-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L2 slenderness ratio, L_e / k=________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$. _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e / k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$._________MPa

Cross sectional area, A_=__________mm^2

Use the smaller of the stresses calculated above.
For L2, critical force, $P_{cr}^{L2} = \sigma A$=___________N

For column length L2, the unsupported length if each grip end is ℓ= _____ mm long such that L=L2-(2* ℓ) = ________mm

Effective length, L_e using Table 3 for the Fixed/Fixed end condition_______mm

For L2, slenderness ratio, L_e / k=________

Minimum slenderness ratio: $\left.\frac{L_e}{k}\right|_{min} = \sqrt{\frac{E\pi^2}{\sigma_o}}$ =_____

$\sigma = \sigma_o$ if $\frac{L_e}{k} < \left.\frac{L_e}{k}\right|_{min}$ _________MPa

OR

$\sigma = \sigma_{cr} = \frac{\pi^2 E}{(L_e / k)^2}$ if $\frac{L_e}{k} > \left.\frac{L_e}{k}\right|_{min}$._________MPa

Cross sectional area, A=___________mm^2

Use the smaller of the stresses calculated above.
For L2, critical force, $P_{cr}^{L2} = \sigma A$=___________N

7) Measure the actual critical compressive forces for the experimental columns.

For L1, Aluminum
Measured critical compressive force, P_{L1}=________N
For L1, critical force, $P_{cr}^{L1} = \sigma A$=___________N
% diff ___________

For L1, Steel
Measured critical compressive force, P_{L1}=_______N
For L1, critical force, $P_{cr}^{L1} = \sigma A$=___________N
% diff ___________

For L2, Aluminum
Measured critical compressive force, P_{L2}=________N
For L2, critical force, $P_{cr}^{L2} = \sigma A$=___________N
% diff ___________

For L2, Steel
Measured critical compressive force, P_{L2}=_______N
For L2, critical force, $P_{cr}^{L2} = \sigma A$_=__________N
% diff ___________

8) Comment on how well the equations predicted the actual critical compression force. Were discrepancies reasonable? If not, what could possible sources of error be attributed to? (Recall that the assumptions for the buckling forces assume no initial eccentricity, perfectly straight columns, and no off-axis loading).

9) As a designer, what steps can be taken to reduce the tendency to buckle, geometrically? material-wise?

FATIGUE

ME 354, MECHANICS OF MATERIALS LABORATORY

TIME-DEPENDENT FAILURE: FATIGUE

01 Januarly 2000 / mgj

PURPOSE

The purposes of this exercise are to determine the effect of cyclic forces on the long-term behaviour of structures and to determine the fatigue lives (N_f) as functions of uniaxial tensile stress for an aluminum alloy.

Axial fatigue tests are used to obtain the fatigue strength of materials where the strains are predominately elastic both upon initial loading and throughout the test.

EQUIPMENT

- Reduced gage section tensile test specimens of 6061-T6 aluminum
- Tensile test machine with grips, controller, and data acquisition system

PROCEDURE

- Measure the diameter, d, of the gage section of the test specimen to 0.02 mm.
- Calculate the maximum, P_{max}, and minimum, P_{min}, forces for the test based on the desired maximum and minimum stresses (Note: $P = \sigma^*A = \sigma * (\pi d^2/4)$. Since, these tests are being conducted in tension only, the stress ratio, R, is chosen to be close to but not exactly zero such that R=0.1. Thus, $\sigma_{min}=R^* \sigma_{max}$ where s_{max} is the desired maximum stress.
- Calculate the mean force as $P_m=(P_{max} +P_{min})/2$.
- Calculate the force amplitude as $P_a=(P_{max} -P_{min})/2$.
- Zero the force output (balance).
- Set the maximum force limit at ~5 kN during the test specimen installation. Activate the limit detect for actuator off.
- Do not set the minimum force limit during specimen installation
- Activate force protect (~0.05 kN) on the test machine to prevent overloading the test specimen during installation.
- Install the top end of the tensile specimen in the top grip of the test machine while the test machine is in displacement control.
- Install the bottom end of the tensile specimen in the lower grip of the test machine.
- Set the maximum force limit at ~0.5 kN greater than P_{max} and activate the limit detect for actuator off.
- Set the minimum force limit at -0.2 kN and activate the limit detect for actuator off.
- Deactivate force protect.
- Activate force control by going to this control mode immediately,
- On the test machine, zero the cycle counter for the total count.
- In force control adjust the setpoint in increments of not greater than 1 kN to achieve the mean force, P_m.
- Select the waveform as sine wave and input an initial frequency of 1 Hz
- Input the force amplitude, P_a.
- Activate amplitude control to ensure that the loading envelope maintains its integrity during the course of the test.
- Initiate the data acquisition and control program (if desired).
- Enter the correct file name and test specimen information as required.
- Initiate the test sequence via the computer program otherwise activate the test via the front control panel.
- After the test has been running for 30-60 s, increase the frequency in 1 Hz increments up to a maximum of 15 to 25 Hz.
- Activate event detector 1 for break detect but no action.
- Continue the test until test specimen fracture (or the break detect).
- Record the number of cycles on the counter at the end of the test.

* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01
E466 Standard Practice for Conducting Constant Amplitude Axial Fatigue Tests of Metallic Specimens
E468 Standard Practice for Presentation of Constant Amplitude Fatigue Test Results for Metallic Specimens

RESULTS

Fatigue test results may be significantly influenced by the properties and history of the parent material, the operations performed during the preparation of the fatigue specimens, and the testing machine and test procedures used during the generation of the data. The presentation of the fatigue test results should include citation of the basic information on the material, the specimens, and testing to increase the utility of the results and to reduce to a minimum the possibility of misinterpretation or improper application of the results.

Enter your results in Table 1, comparing your results to the control data generated for this same aluminum under uniaxial tensile fatigue conditions.

Plot your test results as maximum stress, σ_{max} , versus log of cycles to failure, N_f in Figure 1. Note that a log scale is used for N_f so there is no need to compute log N_f.

Answer the following questions on the Worksheet, turning this in as the In-class Laboratory report.

ME 354, MECHANICS OF MATERIALS LABORATORY

TIME-DEPENDENT FAILURE: FATIGUE

01 January 2000 / mgj

WORK SHEET

NAME______________________________________**DATE**_____________

EQUIPMENT IDENTIFICATION______________________________________

1) Tabulate the following mechanical properties from your tensile test results.

6061-T6 Aluminum
Selected Mechanical Properties (R.T.)

E (GPa)	
σ_o (MPa)	
S_{UTS} (MPa)	
% elongation	

2) For the maximum stress assigned to your laboratory section determine the required test forces from the measured diameter of the test specimen.

Test specimen diameter, d (mm)	
Gage section area, $A=\pi d^2/4$ (mm^2)	
Stress ratio, R	0.1
Maximum stress, σ_{max} (MPa)	
Minimum stress, $\sigma_{min} = R * \sigma_{max}$ (MPa)	
Mean stress, $\sigma_m = (\sigma_{max} + \sigma_{min})/2$ (MPa)	
Stress amplitude, $\sigma_a = (\sigma_{max} - \sigma_{min})/2$ (MPa)	
Stress Range = $\Delta\sigma = \sigma_{max} - \sigma_{min}$ (MPa)	
Maximum load, $P_{max} = \sigma_{max} * A$ (N)	
Minimum load, $P_{min} = \sigma_{min} * A$ (N)	
Mean load, $P_m = \sigma_m * A$ (N)	
Load amplitude, $P_a = \sigma_a * A$ (N)	

3) Tabulate your test results and compare them to the control data for this material.

Table 1 Fatigue Test Results for 6061-T6 Aluminum at R.T.

R	σ_{max} (MPa)	σ_{min} (MPa)	σ_m (MPa)	σ_a (MPa)	**N_f**
0	S_{uts}=	N/A	N/A	N/A	<1
-1	345	-345	0	345	10^2
-1	276	-276	0	276	10^3
-1	248	-248	0	248	10^4
-1	200	-200	0	200	10^5
-1	166	-166	0	166	10^6
-1	117	-117	0	117	10^7
-1	100	-100	0	100	10^8
0.1	322	32	177	145	2
0.1	304	30	167	137	28 788
0.1 (replicate)	285	28	156	128	42,677
0.1 (replicate)	285	28	156	128	34,900
0.1 (replicate)	285	28	156	128	49,671
0.1 (replicate)	285	28	156	128	91,711
0.1 (replicate)	285	28	156	128	35,964
0.1 (replicate)	285	28	156	128	51,700
0.1 (replicate)	285	28	156	128	23,872
0.1	250	28	139	115	124,319
0.1	215	21	118	99	226,038
0.1	178	17	98	82	1,169,307
Test Result	**for this**	**Laboratory**	**Exercise**		
0.1					

4) Plot the all the test results for R=0.1 on the S-N curve shown in Figure 1. For this material, is there evidence of a well-defined fatigue (endurance) limit, σ_e? Is this what you expected?

5) Do your test results agree with the control (or previous test) results? If so, why? if not, why not? Would you expect fatigue failures to have little or much scatter? Does it seem reasonable to try to fit a single curve through the data?

6) Examine the fracture surface of the test specimen. Given that the maximum force in the fatigue test was less than the yield force for material (as determined from the monotonic tensile test), discuss how fatigue can occur given that the loading was in the elastic range. Where do the fatigue cracks initiate from? Is surface condition important? How would you design components to minimize fatigue failures?

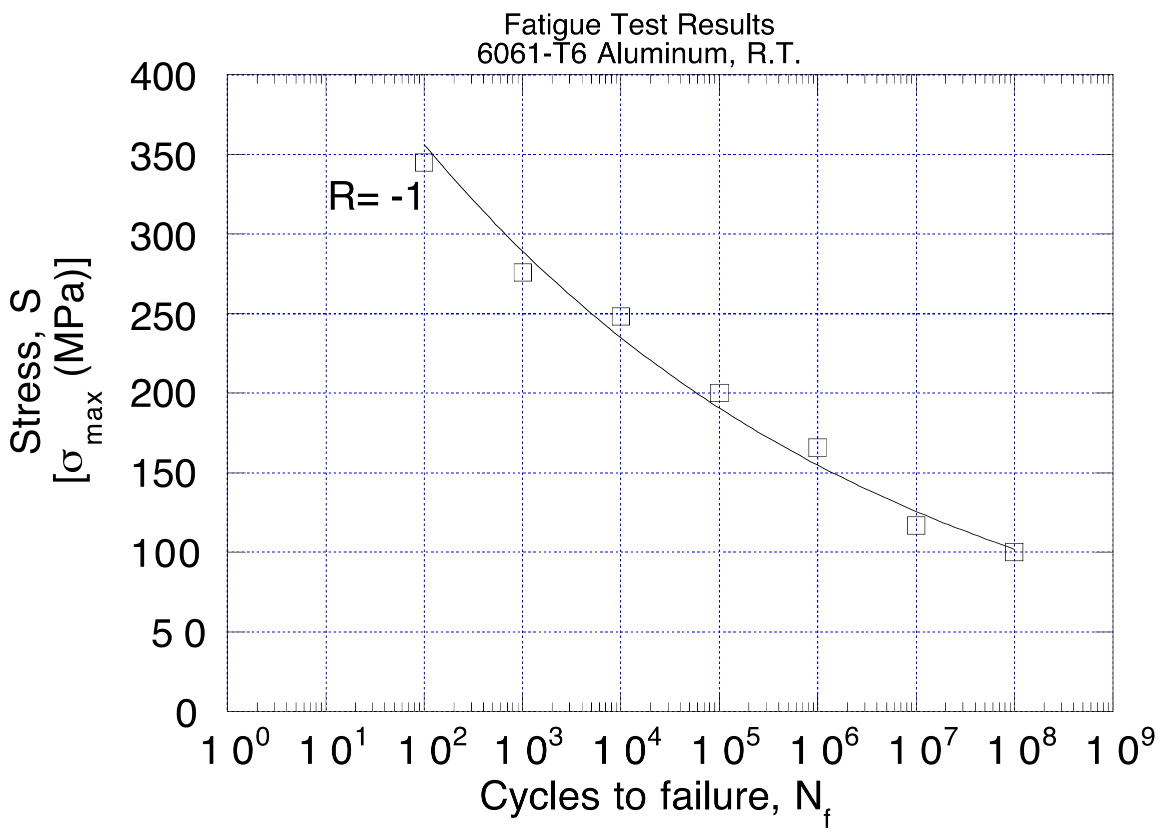


Figure 1 S-N curve for 60612-T6 aluminum at room temperature

6) (cont'd)

7) Fatigue can be analyzed from a fracture mechanics standpoint. If the stress intensity factor solution for this case can be approximated as $K_I = 1.75\sigma\sqrt{\pi a}$, determine the critical crack length at fracture such that $a_f = \frac{1}{\pi}\left(\frac{K_{Ic}}{1.75\sigma_{max}}\right)^2$ for your result (Note K_{Ic}=35 MPa√m). Compare calculated a_f to the actual a_f measured on the fracture surface. Are they similar? Why or why not? Finally, assuming a_i=0.1 mm and $da/dN = C(\Delta K)^m$ (Note: a has units of metres, σ_{max} and $\Delta\sigma$ have units of MPa, F=1.75, m=3.59 and C=1.6 X 10^{-11} with units to give da/dN in m/cycle), calculate the cycles to failure from tensile crack initiation to final fracture using the relation: $N_f = \frac{a_f^{(1-(m/2))} - a_i^{(1-(m/2))}}{C\left[F(\Delta\sigma)\sqrt{\pi}\right]^m\left[1-(m/2)\right]}$. Compare the N_f for crack propagation to the total N_f for the test. Is crack propagation a significant (i.e., large) part of the total fatigue life?

TIME-DEPENDENT DEFORMATION: CREEP

ME 354, MECHANICS OF MATERIALS LABORATORY

TIME-DEPENDENT DEFORMATION: CREEP

08 november 2000 / mgj

PURPOSE

The purposes of this exercise are to study the effect of loading on the time-dependent deformation (i.e., creep) and to characterize the room-temperature creep behaviour of a soft alloy under various forces. Specifically, short-term creep tests will be used to identify constants in the $\dot{\varepsilon}_{min} = B\sigma^n$ relation where $\dot{\varepsilon}_{min}$ is the minimum creep strain rate, σ is the engineering normal stress, B is the coefficient, and n is the creep stress exponent.. Predictions using these constants are compared to results measured from long-term creep tests of this same alloy.

EQUIPMENT

- Constant gage section diameter sections of a ~60% tin- ~40% lead alloy (solder).
- Extension-gage (dial indicator) for total elongation.
- "Dead-weight," lever arm creep test machine.
- Various "dead-weight" masses of 0.5, 1.0, 2.0 and 5.0 kg.
- Timing device.

PROCEDURE

- Measure out and cut to length (~150 mm) constant gage length test specimens.

- Measure the diameter, d, of the gage section each test specimen to 0.02 mm.

- Install the top end of each test specimen in the top grip of a creep test machine.

- Install the bottom end of the test specimen in the lower grip of the creep test machine and measure the initial gripped length, L_o, of the test specimen in mm.

- Apply "dead weight" masses of m_a=3.0, 4.0, 5.0, and 6.0 kg to the pan of the creep test machine for a total of four tests for four different untested test specimens, noting the mechanical advantage of the lever arm system of the creep test machine. (The actual force applied to the test specimen is two times the dead load). Record both the applied mass, m_a, and the mass, m_p, of the pan in kg.

- Record elongation readings (change in length=ΔL) in mm at time, t=10, 20, 30, 60, 90, 120, 180, 240, 360, 480, 600, 720 s, etc. (every 120 s) until 5% engineering strain is achieved.

* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01

E139 Standard Test Method for Conducting Creep, Creep-Rupture, and Stress-Rupture Tests of Metallic Materials

BACKGROUND AND ANALYSIS OF RESULTS

Creep in materials can be defined as time dependent deformation. Often in engineering materials creep becomes of concern at homologous temperatures equal to or greater than 0.3 to 0.6 (rule of thumb is 0.5) . Homologous temperature is defined as:

$$\frac{T(absolute)}{T_{mp}(absolute)} \qquad (1)$$

where T is the absolute temperature of the application and T_{mp} is the absolute temperature of the melting point of the material. For example, steels might be expected to creep at T≈600°C which is a homologous temperature of (600°C+273)K/(1500°C+273)K=0.49. Similarly, a lead-tin solder might be expected to creep at room because the homologous temperatures is (20°C+273)K/(200°C+273)K=0.62. Finally, polycrystalline hydrogen oxide (solid H_2O=ice) creeps at –40°C because the homologous temperatures is (-40°C+273)K/(0°C+273)K=0.82.

Thus, creep is not necessarily dependent on high temperature from a human perspective, but is dependent on high temperature from a material's "perspective."

From an engineering mechanics point of view strain measured during the time-dependent deformation of creep can be thought of as the macroscopic manifestation of the cumulative damage process under the action of temperature and stress. Therefore, predictive models of the creep deformation often include strain, strain rate, applied stress, the use temperature as well as various material-related constants such as activation energy for creep and a stress exponent.

In this laboratory exercise, on a single graph, total engineering creep strain ($\varepsilon=\Delta L/L_o$) is plotted versus time, t, (s) for the four short- term creep tests. The minimum creep strain rate, ($\dot{\varepsilon}_{min}$=dε/dt) (s^{-1}) can be determined for each short-term creep test by using a linear regression over the linear portion of each creep curve.

Next a linear plot of log $\dot{\varepsilon}_{\min}$ versus log engineering stress, σ, (σ=P/A_o where P=2*(m_a+m_p)* (g=9.816 m/s^2) and $A_o = \pi d^2/4$) for the short-term creep tests can be constructed. The coefficient, B, and the creep stress exponent, n, can be determined for the relation:

$$\dot{\varepsilon}_{\min} = B\sigma^n \qquad (1)$$

from a least squares linear regression of the linear plot of **<u>only the short term creep test results (i.e., log $\dot{\varepsilon}_{\min}$ versus log σ).</u>**

Next, a plot of total engineering creep strain ($\varepsilon=\Delta L/L_o$) versus time, t, (s) can be constructed for long-term creep tests (see Table 1 for test data). Note that the long-term creep test results are given in instantaneous length, L_i, versus time such that the change in length ΔL is $\Delta L=(L_i-L_o)$ where L_o is the initial instantaneous length at t=0.

Using similar methods as for the short terms tests, $\dot{\varepsilon}_{\min}$ (s^{-1}) can be determined for each long-term creep test by using a linear regression over the linear portion of each creep curve

On the same linear plot of log $\dot{\varepsilon}_{\min}$ versus log σ, results of the long-term creep tests can be plotted as identified points. Note that the masses, m_a, for the long-term creep tests were directly applied to the test specimens with no pan or lever arm advantage such that σ=P/A_o where P=(m_a)* (g=9.816 m/s^2 and A_o = $\pi d^{2/4}$. (Do not use these points in the curve fit of the short-term test results)

The relative error of measured creep strain rates for the long-term tests can be compared to creep strain rates calculated using B and n determined from the short-term creep tests. Do not curve fit the long term tests and try to compare B and n values determined from long and short tests.

Note that a rule of thumb is that the time for collecting material test data for creep should be on the order of 10% of the time required for the design.

Table 1 - Long-term tensile creep results for a lead-tin alloy (solder).

Mass on Test Specimen, ma = 1.07 kg		Mass on Test Specimen, ma = 1.47 kg	
Time, t (day)	Length, Li (mm)	Time, t (day)	Length, Li (mm)
0	504	0	502
2	513	1	514
3	513	2	529
6	528	3	542
7	531	4	555
9	541	5	571
10	542	6	586
11	544	7	604
14	557	8	620
15	562	9	640
16	568	10	680
17	571	12	712
18	574	13	753
19	586	15	893
20	592		
21	593		
22	609		

Initial diameters, d = 3.18 mm, Initial lengths, Lo at t=0
Mass directly applied (no pan or lever arm advantage creep test machine)

ME 354, MECHANICS OF MATERIALS LABORATORY

TIME-DEPENDENT DEFORMATION: CREEP

08 november 2000 / mgj

DATA SHEET

NAME______________________________________DATE_____________

LABORATORY PARTNER NAMES____

____

EQUIPMENT IDENTIFICATION______________________________________

____

Note: Be sure to note units of each quantity.

Added Mass, ma (kg)		Added Mass, ma (kg)		Added Mass, ma (kg)		Added Mass, ma (kg)	
Mass of Pan , mp(kg)		Mass of Pan , mp(kg)		Mass of Pan , mp(kg)		Mass of Pan , mp(kg)	
Initial Dia., d (mm)		Initial Dia., d (mm)		Initial Dia., d (mm)		Initial Dia., d (mm)	
Final Dia., df (mm)		Final Dia., d_f (mm)		Final Dia., d_f (mm)		Final Dia., d_f (mm)	
Initial Length, L_o (mm)		Initial Length, L_o (mm)		Initial Length, L_o (mm)		Initial Length, L_o (mm)	
Time, t(s)	ΔL, Length Change (mm)	Time, t(s)	ΔL, Length Change (mm)	Time, t(s)	ΔL, Length Change (mm)	Time, t(s)	ΔL, Length Change (mm)

ME 354, MECHANICS OF MATERIALS LABORATORY

TIME-DEPENDENT DEFORMATION: CREEP

08 november 2000 / mgj

WORKSHEET

NAME______________________________________**DATE**_____________

1) Determine the values of total creep strain vs time from the experimental data

Short term tests (from Data Sheet of measured elongation vs time). Note that for each case, applied stress is σ=P/A_o where P=2*(m_a+m_p)* (g=9.816 m/s^2) and A_o = $\pi d^2/4$ and the creep strain is $\varepsilon=\Delta L/L_o$ where m is kg and d, ΔL and L_o are mm.

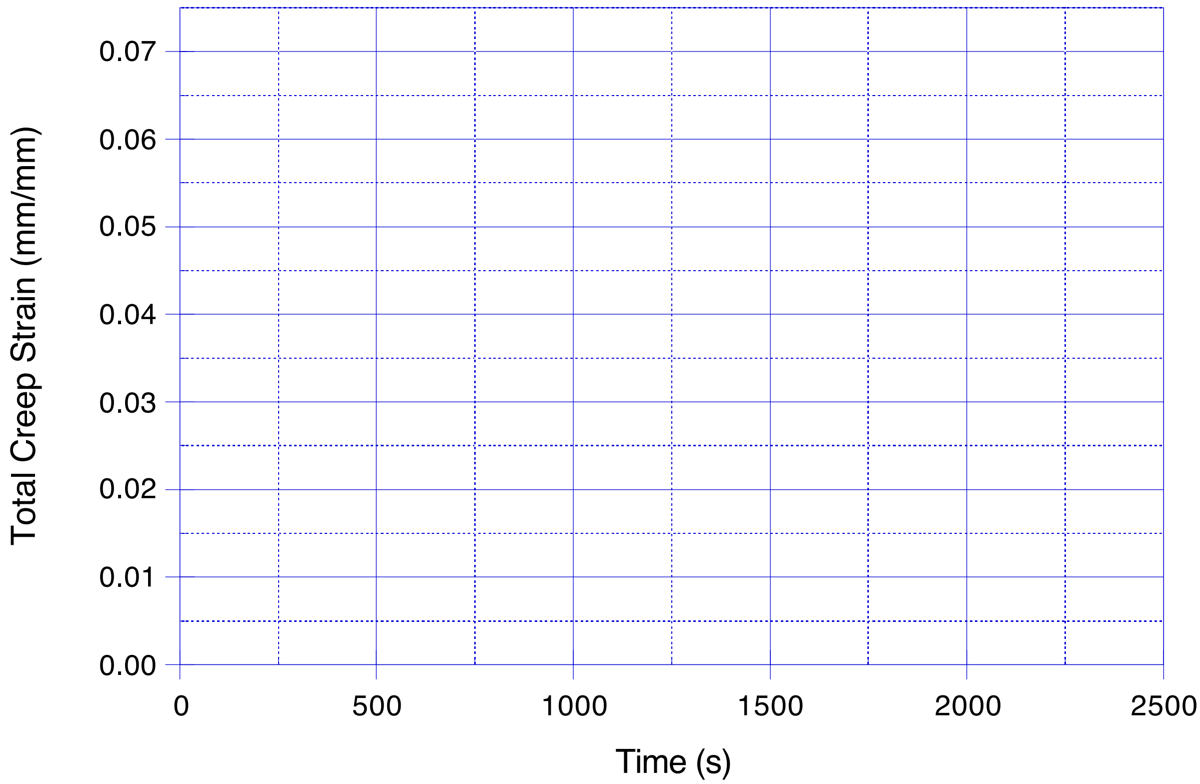
Applied Stress, σ = MPa		Applied Stress, σ = MPa		Applied Stress, σ = MPa		Applied Stress, σ = MPa	
Time, t(s)	Creep Strain, ε (mm/mm)	Time, t(s)	Creep Strain, ε (mm/mm)	Time, t(s)	Creep Strain, ε (mm/mm)	Time, t(s)	Creep Strain, ε (mm/mm)

<u>Long term tests</u> (from Table 1 of measured elongation vs time). Note that for each case, applied stress is σ=P/A_o where P=(m_a) x (g=9.816 m/s^2) and A_o = $\pi d^2/4$ and the creep strain is ε=$\Delta L/L_o$ where m is kg and d, ΔL and L_o are mm. Note also that t (s)= t (days) x 24 (h/day)x(60 min/h)x(60 s/min).

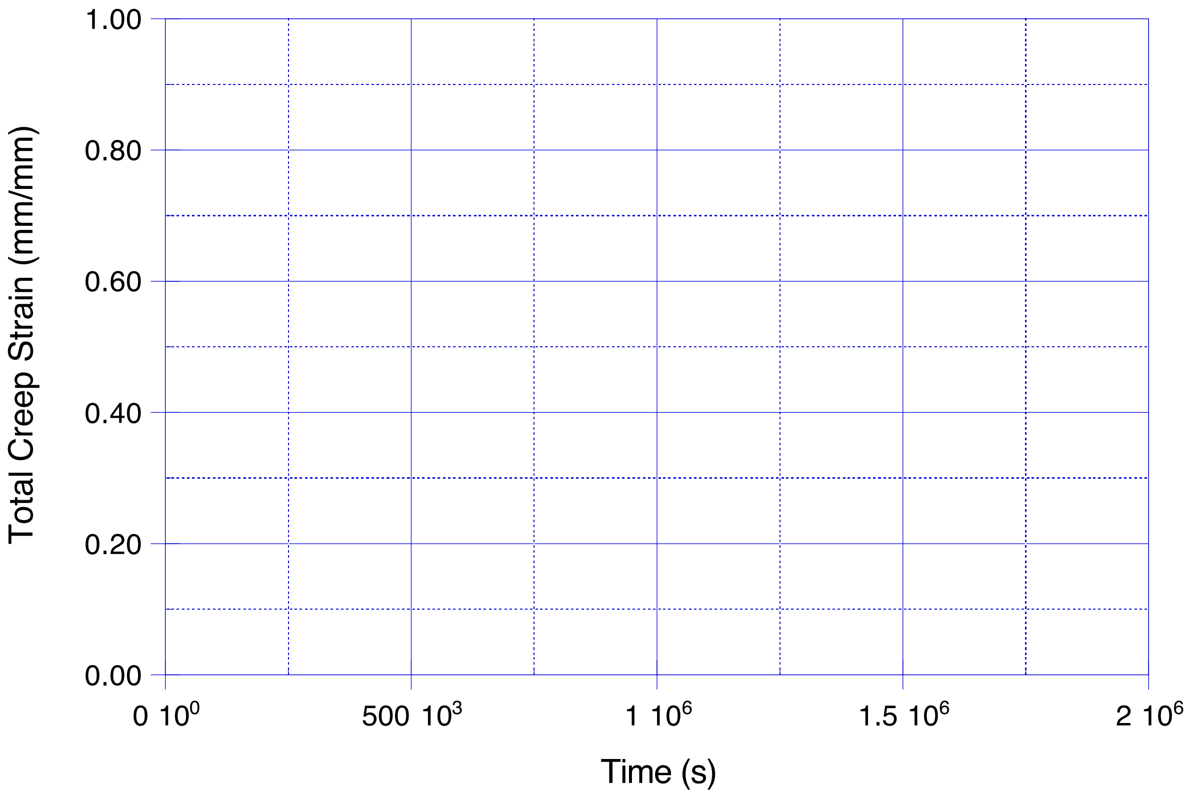
Applied Stress, σ = MPa		Applied Stress, σ = MPa	
Time, t(s)	Creep Strain, ε (mm/mm)	Time, t(s)	Creep Strain, ε (mm/mm)

2) Plot strain vs time to show the creep curves

Short term tests



Long term tests

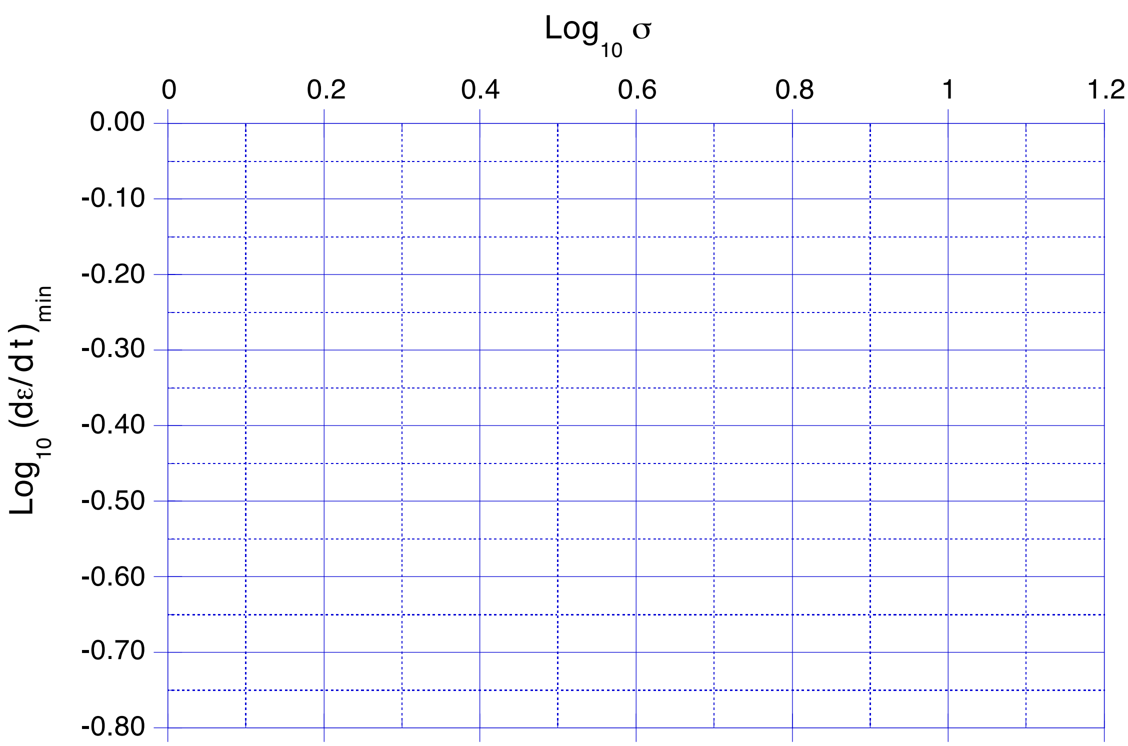


3) Determine the slopes of the linear (secondary) portions of the plots of creep strain vs time. Insert the results in the table and determine the logarithms of each value.

Short-term test	σ (MPa)	$\dot{\varepsilon}_{\min}$ (s^{-1})	$Log_{10}\ \sigma$	$Log_{10}\ \dot{\varepsilon}_{\min}$
Force #1				
Force #2				
Force #3				
Force #4				

Long-term test	σ (MPa)	$\dot{\varepsilon}_{\min}$ (s^{-1})	$Log_{10}\ \sigma$	$Log_{10}\ \dot{\varepsilon}_{\min}$
Force #1				
Force #2				

4) Plot $Log_{10}\ \dot{\varepsilon}_{\min}$ vs $Log_{10}\ \sigma$



5) Determine the slope, m, and intercept, b, of the linear curve fit of the results of the **SHORT TERM** tests **ONLY**. Draw the line on the plot and label the short and long term results. The slope is the stress exponent, n, such that m=n. The intercept, b, is the Log_{10} of the pre-exponential constant B such that $B=10^b$.

Short-term test results	Linear curve fit parameters of log-log plot
Slope of log-log curve fit, m	
Intercept of log-log curve fit, b	
	Parameters for $\dot{\varepsilon}_{\min} = B\sigma^n$
n=m	
B (MPa^{-n} /s)=10^b	

6) Summarize the results and calculate the prediction of minimum strain rates for the long-term stresses using B and n determined from the short term results. Calculate the differences between the predicted long term strain rates and the measured long terms strains.

Short-term test	$\dot{\varepsilon}_{min}$ (s^{-1})
Force #1, σ = MPa	
Force #2, σ = MPa	
Force #3, σ = MPa	
Force #4, σ = MPa	

Short-term test results	Parameters for $\dot{\varepsilon}_{min} = B\sigma^n$
B (MPa^{-n} /s)	
n	

Long term tests	$\dot{\varepsilon}_{min}$ (s^{-1})
σ = MPa, $\dot{\varepsilon}_{min}$ measured	
σ = MPa, $\dot{\varepsilon}_{min} = B\sigma^n$	
% difference	
σ = MPa, $\dot{\varepsilon}_{min}$ measured	
σ = MPa, $\dot{\varepsilon}_{min} = B\sigma^n$	
% difference	

7) If possible, compare the n and B values to book values for this solder alloy at room temperature. Discuss any differences. Discuss differences between measured and predicted minimum creep strain rates for the long-term tests discussions about limitations about predicting long-term creep behaviour from short term test results. Schematically sketch curves of creep strain vs. time and show how strain rate calculated at short times (i.e. short term tests) could be different from the strain rate determined at long times (i.e. long term tests).

STRUCTURES

ME 354, MECHANICS OF MATERIALS LABORATORY
STRUCTURES

01 January 2000 / mgj

PURPOSE

The purpose of this exercise is to the study the effects of various assumptions in analyzing the stresses and forces in an engineering structure using engineering mechanics, experimental mechanics, and numerical modeling.

EQUIPMENT

- Strain-gaged bicycle.
- Strain gage conditioning equipment and data acquisition system.
- "Dial indicators", holders and magnetic bases.

PROCEDURE

- Re-read the reference document "NOTES on Strain Gages."

- Carefully examine attached Figs. 1-3. Note that a total of 10 stacked rectangular rosettes have been applied at various locations on the bicycle frame. Each rosette has three strain gages such that 30 possible strain gage circuits are involved. Identify all strain gage circuits and strain gage channel numbers on both the figures as well as on the bicycle frame itself.

- Note which strain gage locations will be used in the analysis.
- Note the location of the dial indicator measurement.
- Note the type of input forces and reactions (axle connections) for the bicycle frame.

- If not already done so, set the gage factor to 2.08 and balance each strain gage circuit to zero or a reasonable minimum offset strain.

- Record this offset strain, if any, (starting value with no force applied) for each channel on the data sheet.
- Zero the "dial indicators". Note the location of the deflection measurements on the bicycle frame.

- Apply a modest concentrated force (approximately the weight of a bicyclist with equipment) to the bicycle frame.

- Record the reading for each strain gage channel on the data sheet.

- Record the reading of the dial indicator

- Remove the force from the bicycle frame.

BACKGROUND

Engineering structures may take many forms, from the simple shapes of square cross section beams to the complex and intricate shapes of trusses. Trusses are one of the major types of engineering structures, providing practical and economical solutions to many engineering situations. Trusses consist of straight members connected at joints (for example, see Figure 1). Note that truss members are connected at their extremities only: thus no truss members are continuous through a joint.

In general, truss members are slender and can support little lateral force. Therefore, major forces must be applied to the various joints and not the members themselves. Often the weights of truss members are assumed to be applied only at the joints (half the weight at each joint). In addition, even though the joints are actually rivets or welds, it is customary to assume that the truss members are pinned together (i.e., the force acting at the end of each truss member is a single force with no couple). Each truss member may then be treated as a two force member and the entire truss is treated as a group of pins and two force members.

A bicycle frame, on first inspection, appears to be an example of a truss. Each tube (truss member) is connected to the other at a joint, the principal forces are applied at joints (e.g., seat, steering head, and bottom bracket), and the reaction forces are carried at joints as well (e.g., front and rear axles). Although the joints are not pinned, a reasonable first approximation for analyzing forces, deflections, and stresses in the various tubes of the bicycle frame might be made using a simple truss analysis.

Forces in various truss members can be found using such analysis techniques as the method of joints or the method of sections. Deflections at any given joint may be found by using such analysis techniques as the unit force method of virtual work.

REFERENCES

ME354 NOTES on Strain Gages

ANALYSIS

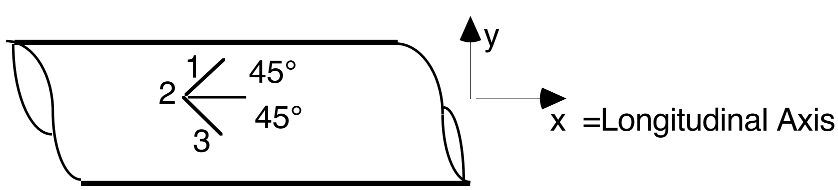
1) Draw a free body diagram of the truss, showing your assumptions for the reactions at the front and rear axles.

2) For the applied force P, use the assumed dimensions and angles of Fig. 3 along with a simple truss analysis to complete the table. Note: for truss forces, use "+" to indicate tensile force and "-" to indicate compressive force.

Applied Force, P (N)	
Reaction at Front Axle, Rf (N)	
Reaction at Rear Axle, Rr (N)	
Force in Top Tube Ftt (N)	
Force in Down Tube Fdt (N)	
Force in Seat Tube Fst (N)	
Force in Seat Stay Fss (N)	
Force in Chain Stay Fcs (N)	
Force in Head Tube/Fork Ff (N)	

3) Use the strain gage information to find the x-y-z coordinate strains in each member of interest, noting that the orientation of the individual strain gages in each rosette is as shown. Note: this may require using the three strains from the rosette to find the principal strains and then using the complete strain state to find the strains acting in the directions of interest of each member. Use local coordinates on each member to define the coordinate strains. In all cases define x as being along the longitudinal axis of the member, y being in the plane of the member's surface and z being normal to the surface.

Member	x- Strain (microstrain)	y - Strain (microstrain)	z-Strain (microstrain)
Top Tube (top SG)			
Top Tube (bottom SG)			
Down Tube			
Seat Tube			
Seat Stay			
Chain Stay			
Head Tube/Fork			



4) Use 3-D (i.e., Generalized) Hooke's law to find the stress acting in the x-direction (longitudinal direction). Note: $\sigma_x = \frac{E}{(1+\nu)}\varepsilon_x + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right)$ and E for the steel tubes is 200 GPa.

Member	X- Stress (MPa)
Top Tube (top SG)	
Top Tube (bottom SG)	
Down Tube	
Seat Tube	
Seat Stay	
Chain Stay	
Head Tube/Fork	

5) Use the longitudinal stress calculated in Part **4)** to estimate the longitudinal force in each member. Note: Note that this analysis requires the assumption that the stress is uniform across the cross section. The cross sectional dimensions for the members are as follows and the cross sectional area is $A=(\pi\,(OD^2-ID^2)/4)$:

Top Tube:OD=28.8 mm, ID=26.5 mm; Down Tube: OD=32.2 mm, ID=29.9 mm,
Seat Tube: OD=28.8 mm, ID=26.5 mm; Head Tube/Fork:OD=34 mm, ID=31.7 mm,
Seat and Chain Stays:OD=16.1 mm; ID=13.8 mm.

Member	X- Stress (MPa)	Cross Sectional Area (mm^2)	Longitudinal Force =stress *area (N)
Top Tube (top SG)			
Top Tube (bottom SG)			
Down Tube			
Seat Tube			
Seat Stay			
Chain Stay			
Head Tube/Fork			

6) Compare the measured longitudinal forces to the longitudinal forces calculated using the simple truss analysis. Explain any differences by the answering the questions:
i) What assumptions were made in the truss analysis?
ii) What assumptions were made in analyzing the strain gage results to find the forces?
iii) From the strain gage results for the top tube, is the stress distribution uniform across the cross section of the tube? If not, is the truss analysis of uniform axial forces valid?

Member	Longitudinal Force from Truss Analysis (N)	Longitudinal Force from Strain Gage Analysis (N)	% difference
Top Tube (top SG)			
Top Tube (bottom SG)			
Down Tube			
Seat Tube			
Seat Stay			
Chain Stay			
Head Tube			

7) Note that because of the choice of the locations (i.e., A and J) for obtaining strain information, it is possible to separate axial stresses (P/A) from uniaxial bending (Mc/I) at the center of the top tube. By taking the average of the total X-stress at A and total stress at J, the bending component cancels and the axial stress acting in the top tube is obtained.

$$(\text{X-stress}^{A} + \text{X stress}^{J})/2 = \text{axial stress}$$

The axial force can then be obtained by multiplying the axial stress times the cross sectional area.

$$\text{axial force} = \text{axial stress} \times \text{cross sectional area}$$

The principle of superposition allows the addition of the axial and bending stresses because they are the same type of stress (i.e., normal) acting in the same direction. (i.e., X-stress=axial stress + bending stress). Therefore, once the axial stress is found, the bending stress can be obtained by subtracting the axial stress from the total X-stress

$$\text{bending stress} = \text{X-stress} - \text{axial stress}$$

8) As it turns out, due to the variability of the loading scenarios, the stress state in a bicycle frame is more complex than can be analyzed using a simple truss analysis or the simple assumption of uniformly stressed tubes. Finite element analysis (FEA) lends itself to solving this complex stress state. Using the results of an FEA of a model of the bicycle frame for the applied force of this test, quantitatively and qualitatively compare the stresses at the various locations and tubes.

i) Are the stresses uniform across the cross sections?
ii) What are the effects of bending and torsion on the stress state?
iii) Are the axial, bending, and total stresses constant over the lengths of the tubes?
iv) Are there any stress concentrations (e.g., are the maximum stresses greater at the joints than in the middle?
v) Compare the axial (longitudinal) forces determined from the truss analysis to that determined from the strain gage analysis (from the axial stress after subtracting the bending stress) to the that determined from the FEA for the top tube. Does bending significantly affect the results?

Member	Longitudinal Force from Truss Analysis (N)	Longitudinal (Axial) Force from Strain Gage Analysis (N)	Axial Force from FEA (N)
Top Tube			
	Axial Stress from Truss Analysis (MPa)	Axial Stress (no bending) from Strain Gage Analysis (MPa)	Axial Stress (no bending) from FEA (MPa)
Top Tube			
	Total X-stress from Truss Analysis (MPa)	Total X-stress from Strain Gage Analysis (MPa)	Total X-stress from FEA (MPa)
Top Tube (top)			
Top Tube (bottom)			

9) Deflections in trusses can often be found using energy methods. Again, to simplify the analysis it is assumed that the axial force in each tube only acts at the joints and therefore the axial force is constant throughout the length of each member. The unit force method is used as follows in which the deflection at the point of interest is:

$$\Delta = \sum \frac{N_U N_L L}{EA}$$

where N_U and N_L are the forces in each member due to unit and actual forces (in this case use the forces found from the truss analysis, not the experimental measurement), respectively, L is the length of each member, E is the elastic modulus of each member and A is the cross sectional area of each member. In this case, the deflection of interest at the bottom bracket is in the same direction and at the same location as the applied force. Nonetheless, the unit force method can still be used by filling in the appropriate sections of the table where E=200,000 MPa for all members.

Member	L (mm)	A (mm^2)	N_L [due to P] (N)	N_U [due to unit P] (N)	$\frac{N_U N_L L}{EA}$
Top Tube					
Down Tube					
Seat Tube					
Seat Stay					
Chain Stay					
Head Tube/fork					

$$\Delta = \sum \frac{N_U N_L L}{EA} = \underline{\hspace{3cm}}$$

10) Compare the measured deflection at the bottom bracket to the deflection predicted from the unit force method due to axial forces only and the FEA model. Comment on any difference and the reasons (for example, assumptions of the unit force method for deflection or truss analysis for the axial forces). Suggest a other ways to predict the deflections at joints.

Measured Deflection (mm)	Deflection for Unit Force Analysis (mm)	% difference	Measured Deflection (mm)	Deflection from FEA Model (mm)	% difference

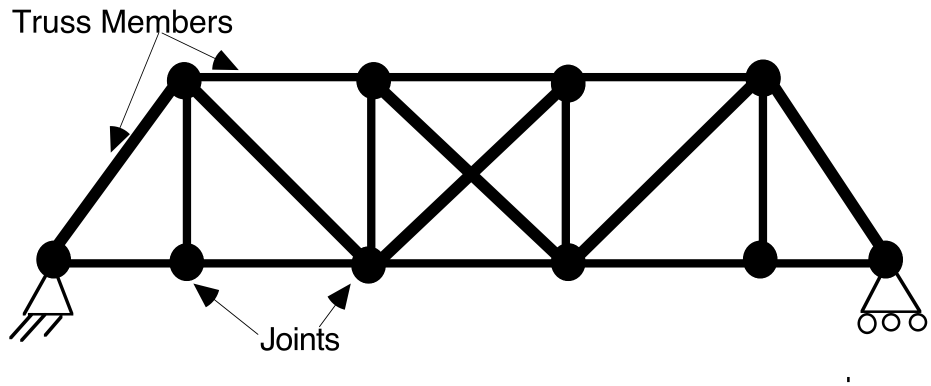


Figure 1 Example of a Simple Truss

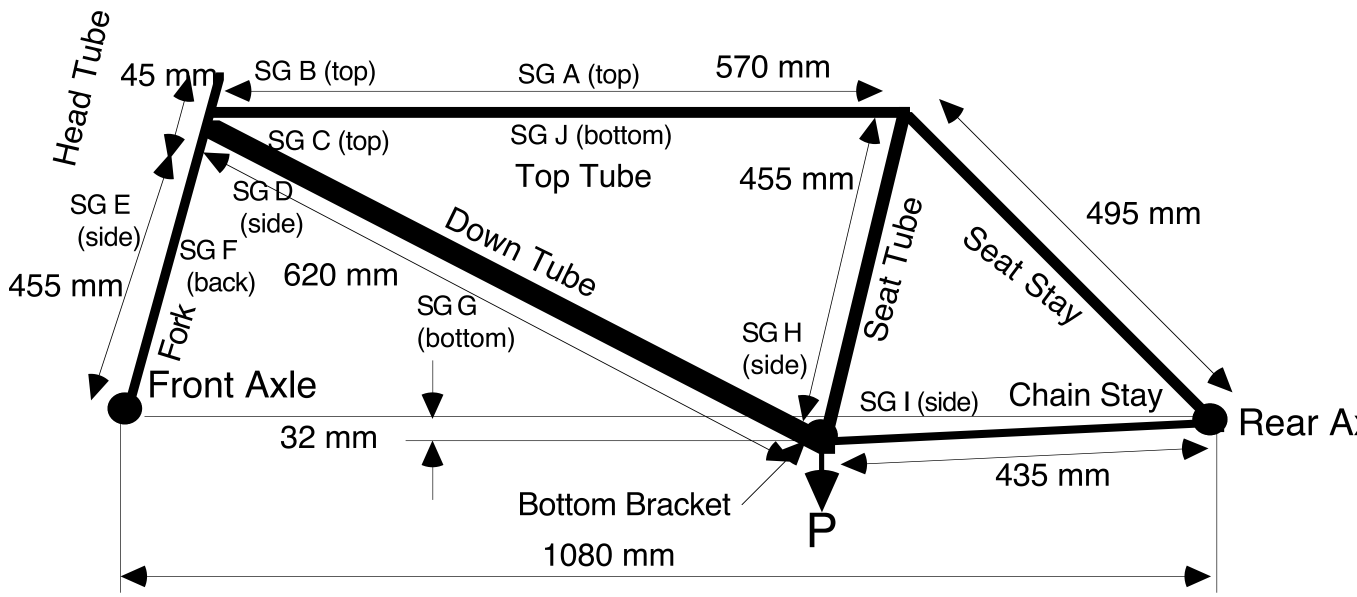
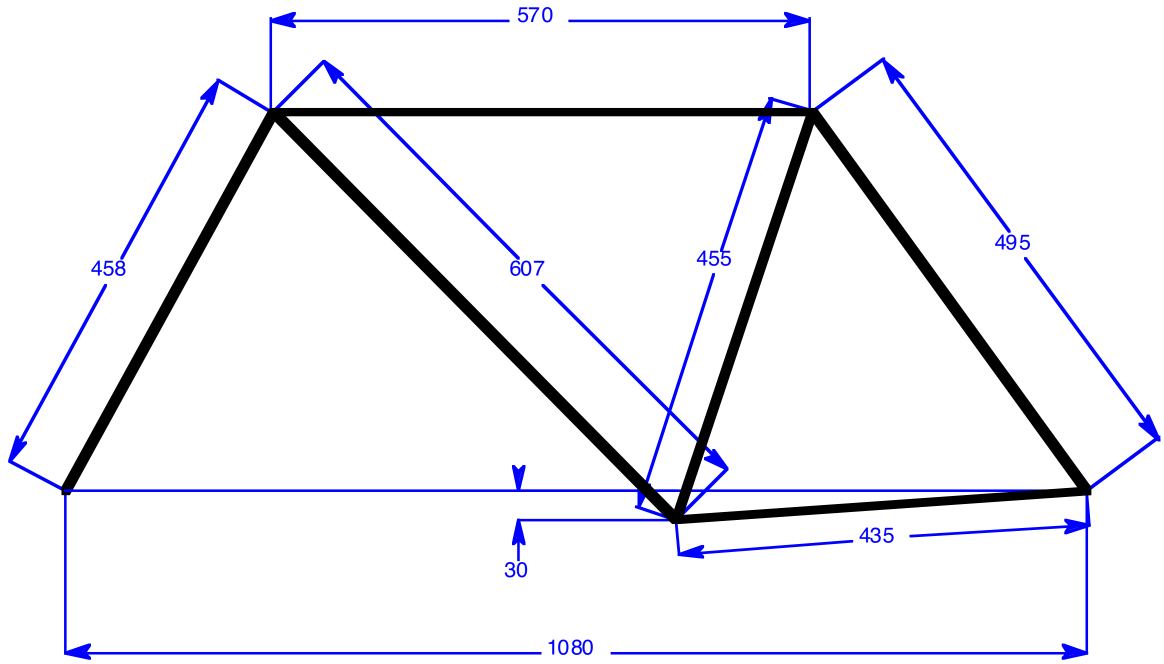
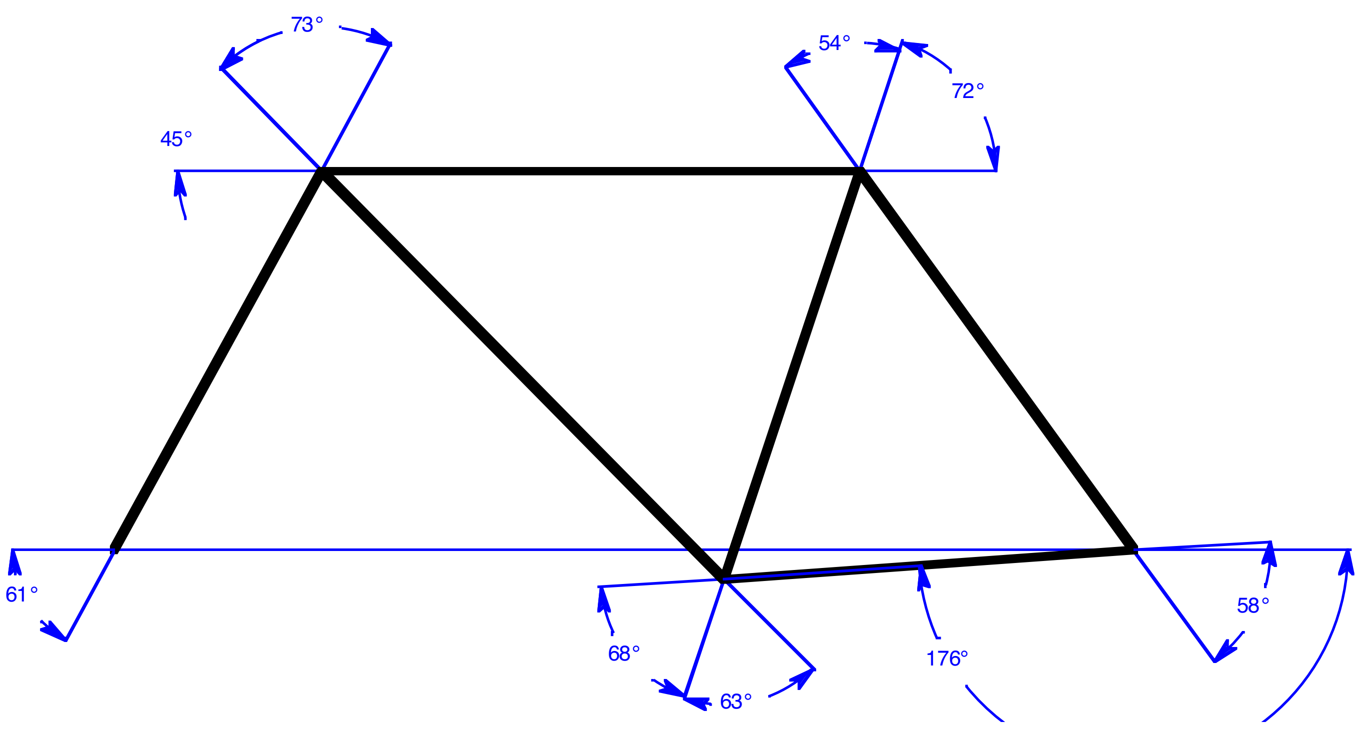


Figure 2 As Measured Dimensions, Nomenclature, and Strain Gage Locations on Bicycle Frame



a) assumed linear dimensions (mm)



b) assumed angular dimensions (degrees)

Figure 3 Assumed Dimensions and Angles for Simplified Truss Analysis

LABORATORY REPORT

1. As a minimum include the following information in the laboratory report.
   a) Free body diagram of the truss, showing your assumptions for the reactions at the front and rear axles.
   b) For forces in each member found from a simple truss analysis. Note: for truss forces, use "+" to indicate tensile force and "-" to indicate compressive force.

Applied Force, P (N)	
Reaction at Front Axle, Rf (N)	
Reaction at Rear Axle, Rr (N)	
Force in Top Tube Ftt (N)	
Force in Down Tube Fdt (N)	
Force in Seat Tube Fst (N)	
Force in Seat Stay Fss (N)	
Force in Chain Stay Fcs (N)	
Force in Head Tube/Fork Ff (N)	

   c) Comparison of the longitudinal forces in each member from the truss analysis and the experimental measurements.

Member	Longitudinal Force from Truss Analysis (N)	Longitudinal Force from Strain Gage Analysis (N)	% difference
Top Tube (top SG)			
Top Tube (bottom SG)			
Down Tube			
Seat Tube			
Seat Stay			
Chain Stay			
Head Tube			

   d) Comparison of the deflection at the bottom bracket found from the experimental measurements, energy methods, and FEA model.

Measured Deflection (mm)	Deflection for Unit Force Analysis (mm)	% difference	Measured Deflection (mm)	Deflection from FEA Model (mm)	% difference

2. As a minimum, discuss the following in the laboratory report.
   a) Answers to these questions (DO NOT simply answer the questions, but instead use the questions are starting points for explanations about the results:
      i) What assumptions were made in the truss analysis?

      ii) What assumptions were made in analyzing the strain gage results to find the forces?

      iii) From the strain gage results for the top tube, is the stress distribution uniform across the cross section of the tube? Can the axial stress be separated from any bending stress, if any? If the stress distribution is not uniform, is the truss analysis assuming uniform axial forces valid?

      iv) From the FEA model, are the stresses uniform across the cross sections?

      v) From the FEA model, what are the effects of bending and torsion on the stress state?

      vi) From the FEA model, are the stresses constant over the lengths of the tubes?

      vii) From the FEA model, are there any stress concentrations?

      viii) From the FEA model, how do the deflections compare?

   b) Error analysis in the measurements.

3. At a minimum, include the following information in the appendix of the laboratory report. THIS MAY NOT BE ALL THAT IS NECESSARY (i.e., don't limit yourself to this list.)
   a. Original data sheets and/or printouts
   b. All supporting calculations. Include sample calculations if using a spread sheet program.

ME 354, MECHANICS OF MATERIALS LABORATORY

STRUCTURES

01 January 2000 / mgj

DATA SHEET

NAME____________________________________**DATE**____________

EQUIPMENT IDENTIFICATION____________________________________

Applied Force, P (kg)	
Total Deflection at Bottom Bracket (mm)	
"Machine" Deflection at Reaction Point (mm)	

	Gage 1 (microstrain)		Gage 2 (microstrain)		Gage 3 (microstrain)	
	Initial	Final	Initial	Final	Initial	Final
Location A						
Location B						
Location C						
Location D						
Location E						
Location F						
Location G						
Location H						
Location I						
Location J						