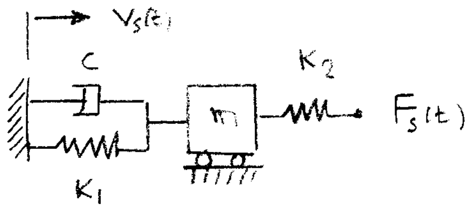

$m = 0.1 \qquad C = 0.1$

State Equation

$$\frac{d}{dt}\begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} = \begin{bmatrix} -1 & 10 \\ -k_1 & 0 \end{bmatrix}\begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} + \begin{bmatrix} 1 & 10 \\ k_1 & 0 \end{bmatrix}\begin{Bmatrix} V_s(t) \\ F_s(t) \end{Bmatrix}$$

1. Homogeneous Equation

$$\frac{d}{dt}\begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} = \begin{bmatrix} -1 & 10 \\ -k_1 & 0 \end{bmatrix}\begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix}$$

2. Asymptotically Stable with oscillation

Eigenvalues satisfy

$$\begin{vmatrix} -1-\lambda & 10 \\ -k_1 & -\lambda \end{vmatrix} = \lambda(\lambda+1) + 10k_1 = 0$$

OR

$$\lambda^2 + \lambda + 10k_1 = 0$$

Asymptotically stable with oscillatory response occurs

$$\Delta = 1 - 4(10)k_1 < 0 \Rightarrow k_1 > \frac{1}{40} = 0.025$$

3. State Transition matrix when $k_1 = 1$

$$\underline{A} = \begin{bmatrix} -1 & 10 \\ -1 & 0 \end{bmatrix}, \quad \underline{A}^2 = \begin{bmatrix} -1 & 10 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 10 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -9 & -10 \\ 1 & -10 \end{bmatrix}$$

$$\underline{A}^3 = \begin{bmatrix} -9 & -10 \\ 1 & -10 \end{bmatrix}\begin{bmatrix} -1 & 10 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 19 & -90 \\ 9 & 10 \end{bmatrix}$$

Hence

$$\Phi(t) = \underline{I} + \underline{A}t + \frac{\underline{A}^2 t^2}{2!} + \frac{\underline{A}^3 t^3}{3!} + \cdots$$

$$= \begin{bmatrix} 1 - t - \frac{9t^2}{2} + \frac{19}{3!}t^3 \cdots & 10t - \frac{10}{2!}t^2 - \frac{90}{3!}t^3 \cdots \\ -t + \frac{t^2}{2} + \frac{9}{3!}t^3 \cdots & 1 - \frac{10}{2!}t^2 + \frac{10}{3!}t^3 \cdots \end{bmatrix}$$

4. If initial conditions are (for $k_1 = 1$)

$$\begin{cases} x_m(0) = 1 \\ v_m(0) = 0 \end{cases} \Rightarrow \begin{array}{l} f_{k_1}(0) = k_1 x_m(0) = 1 \\ v_m(0) = 0 \end{array}$$

$$v_m = \begin{bmatrix} 1 - t - \frac{9t^2}{2} + \frac{19}{3!}t^3 \cdots , & 10t - \frac{10}{2!}t^2 - \frac{90}{3!}t^3 \cdots \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= 10t - \frac{10}{2!}t^2 - \frac{90}{3!}t^3 + \cdots$$