



$$\frac{d}{dt} \begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} = \begin{bmatrix} -\frac{C}{m} & \frac{1}{m} \\ -K_1 & 0 \end{bmatrix} \begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} + \begin{bmatrix} \frac{C}{m} & \frac{1}{m} \\ K_1 & 0 \end{bmatrix} \begin{bmatrix} V_s(t) \\ F_s(t) \end{bmatrix}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{K_1} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{K_2} \end{bmatrix} \begin{bmatrix} V_s(t) \\ F_s(t) \end{bmatrix}$$

When $C = 0.1$, $m = 0.1$, $K_1 = K_2 = 1$

$$\frac{d}{dt} \begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 10 \\ -1 & 0 \end{bmatrix}}_{\underline{A}} \begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 10 \\ 1 & 0 \end{bmatrix}}_{\underline{B}} \begin{bmatrix} V_s(t) \\ F_s(t) \end{bmatrix}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\underline{C}} \begin{bmatrix} v_m \\ f_{k_1} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\underline{D}} \begin{bmatrix} V_s(t) \\ F_s(t) \end{bmatrix}$$

Transfer Matrix

$$\underline{H}(s) = \underline{C}(s\underline{I} - \underline{A})^{-1}\underline{B} + \underline{D},$$

$$(s\underline{I} - \underline{A})^{-1} = \begin{bmatrix} s+1 & -10 \\ 1 & s \end{bmatrix}^{-1} = \frac{1}{s^2 + s + 10} \begin{bmatrix} s & 10 \\ -1 & s+1 \end{bmatrix}$$

$$\begin{aligned}
 \underline{H}(s) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \frac{1}{s^2+s+10} \begin{bmatrix} s & 10 \\ -1 & s+1 \end{bmatrix} \begin{bmatrix} 1 & 10 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \frac{1}{s^2+s+10} \begin{bmatrix} -1 & s+1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 10 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \frac{1}{s^2+s+10} \begin{bmatrix} s & -10 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}
 \end{aligned}$$

- Transfer function from $V_s(t)$ to δ_1 is $H_{11}(s)$

$$\Rightarrow H_{11}(s) = \frac{s}{s^2+s+10}$$

- Zero occurs when $s=0$, i.e., $V_s(t) = \text{constant}$
- Transfer function from $V_s(t)$ to δ_2 is $H_{21}(s)$

$$H_{21}(s) = 0 \Rightarrow \text{No } \delta_2 \text{ for any } V_s(t).$$

- Poles are located at

$$\begin{aligned}
 s^2+s+10 &= 0, \quad p_{1,2} = \frac{1}{2} \left[-1 \pm \sqrt{1-40} \right] \\
 &= \frac{1}{2} (-1 \pm \sqrt{39} j)
 \end{aligned}$$