



$$\frac{1}{Z_3} = \frac{1}{1/sJ} + \frac{1}{S/K_2} = sJ + \frac{K_2}{S} = \frac{s^2 J + K_2}{S}$$

$$Z_3(s) = \frac{S}{s^2 J + K_2}$$

(a)

$$\text{Impedance } Z(s) = \frac{\Omega_s}{T_s} = \frac{\Omega_s}{\frac{S}{K_1}} = \frac{S}{K_1} + \frac{S}{s^2 J + K_2} = \frac{S(s^2 J + K_2) + SK_1}{K_1(s^2 J + K_2)}$$

(b) Transfer function from  $\Omega_s$  to  $\Omega_J$ 

$$H(s) = \frac{\Omega_J}{\Omega_s} : \quad \Omega_J = \Omega_s - \frac{S}{K_1} T_s = \Omega_s - \frac{S}{K_1} T_s$$

$$\text{Hence } H(s) = 1 - \frac{S}{K_1} \frac{T_s}{\Omega_s} = 1 - \frac{S}{K_1} \frac{1}{Z(s)}$$

$$(c) \quad H(s) = 1 - \frac{S}{K_1} \cdot \frac{K_1(s^2 J + K_2)}{S(s^2 J + K_2) + SK_1} = \frac{SK_1}{S(s^2 J + K_2) + SK_1}$$

Pole-zero cancellation with  $s=0$ 

Physical meaning:

- Pole @  $s=0$  implies  $\Omega_s(t) = \text{constant}$ , response of part of the system (e.g., the spring force  $f_{K_2}$ ) goes to infinity.
- Zero @  $s=0$  implies response  $\Omega_J$  (angular velocity) remains finite (and constant), because it cancels with the pole.
- (d) The poles of  $H(s)$  are zeros of  $Z(s)$ . So find zeros of  $Z(s)$  instead.