ME 374, System Dynamics Analysis and Design
Homework 1

Distributed: 3/26/2012, Due: 4/6/2012

(There are 4 problems in this set.)

1. Consider the following square matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix} \tag{1}$$

   (a) Determine the eigenvalues and eigenvector(s) of $\mathbf{A}$.

   (b) Find the modal matrix $\mathbf{M}$ and diagonalize $\mathbf{A}$ through similarity transformation $\mathbf{M}^{-1}\mathbf{A}\mathbf{M}$. do we get a Jordan form or not? Explain why?

2. Consider the following matrix

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 1 & 4 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix} \tag{2}$$

   (a) Find the eigenvalues and eigenvectors of each matrix in (2).

   (b) Find the modal matrix $\mathbf{M}$ and diagonal each matrix (2) through similarity transformation. Which matrix in (2) will lead to the Jordan form?

3. In mechanics of materials, the stress state at a point (Fig. 1) can be described through a square matrix

$$\mathbf{A} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \tag{3}$$

   In addition, the shear stresses are equal ($\sigma_{xy} = \sigma_{yx}$) in order to maintain moment equilibrium.

   (a) In fact, the principal stresses and principal directions that we learn in mechanics of materials are eigenvalues and eigenvectors of the stress state $\mathbf{A}$. Find the principal stresses and principal directions of a pure stress state

$$\sigma_{xx} = \sigma_{yy} = 0, \quad \sigma_{xy} = \sigma_{yx} = 50\text{MPa} \tag{4}$$

   (b) For symmetric matrices, all eigenvalues and eigenvectors are real. What is the physical meaning for the case of two-dimensional stress state in Fig. 1.

4. This problem shows that eigenvalues and eigenvectors are often the buckling loads and buckling shapes of a structure. Consider the three-link structure shown in Fig. 2(a). The length of each link is l, and the middle nodes are supported by springs with stiffness k. In addition, the structure is subjected to a pair of compressive end loads P. Let's consider a static analysis. Assume that the structure is buckled infinitesimally to the configuration shown in Fig. 2(b). The buckled shape is described by the angles θ_1 and θ_2. Since the buckling is infinitesimal, one can assume that

$$\sin\theta_1 \approx \theta_1, \quad \sin\theta_2 \approx \theta_2 \tag{5}$$

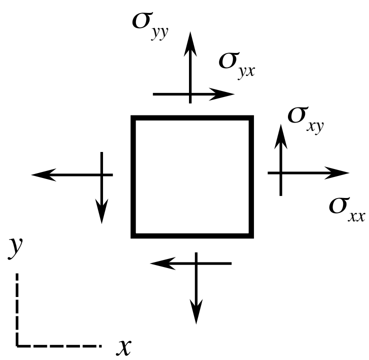

Figure 1: Two-dimensional stress state

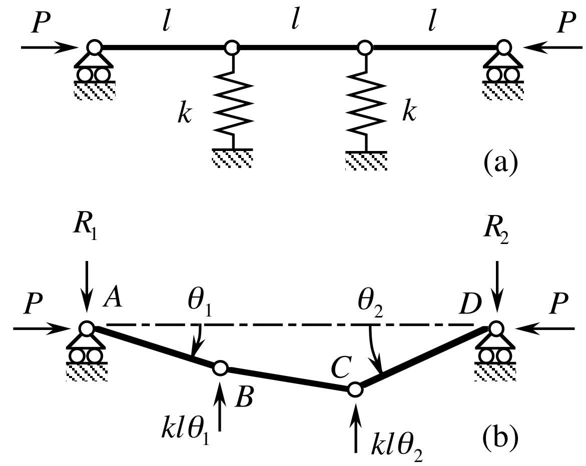

Figure 2: Buckling of a three-link structure

(a) Use the following procedure to derive the equation governing the infinitesimal buckling. First, apply moment equilibrium and force equilibrium to the whole structure to determine the reaction forces R_1 and R_2 at the end support. Then draw a free-body diagram of link AB and take moment about node B to derive the first equation. Similarly, draw a free-body diagram of link CD and take moment about node C to derive the second equation. The governing equation should be

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \lambda \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \tag{6}$$

where the eigenvalue λ is related to the applied load P through

$$\lambda = \frac{3P}{kl} \tag{7}$$

(b) Notice that the matrix in (6) is symmetric. What can you conclude from this observation about the buckling load? Does it make sense physically?

(c) Determine the eigenvalues and eigenvector to determine the buckling load and buckling shapes. Plot the buckling shapes.