ME 374, System Dynamics Analysis and Design
Homework 3

Distributed: 4/9/2012 Due: 4/23/2012
(There are 4 problems in this set.)

1. This is an old exam problem from Spring Quarter of 2008. Engineer X is analyzing a first-order system whose dynamics is governed by

$$\tau \frac{dx(t)}{dt} + x(t) = Af(t) \tag{1}$$

where A and τ are unknown parameters to be determined. Answer the following questions.

   (a) For this system, derive the transfer function from $f(t)$ to $x(t)$ based on (1).

   (b) Consider an input force

$$f(t) = \cos 3t + \sin 3t = \mathrm{Re}\left[(1+j)e^{-3jt}\right] \tag{2}$$

   Determine the magnitude and phase of the input force $f(t)$.

   (c) With the input given in (2), Engineer X found that the transfer function has a magnitude of $2\sqrt{2}$ and a phase $\frac{\pi}{4}$. What is the output response $x(t)$ of the system corresponding to the excitation in (2)?

   (d) With the transfer function given in part (c), determine the parameter A and τ. Hint: Use the phase of the transfer function to find τ first.

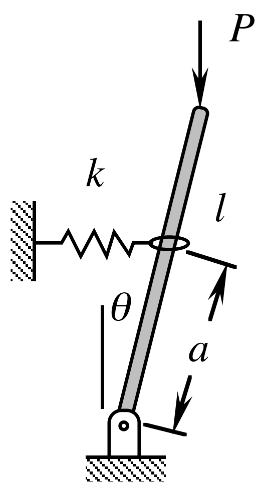

Figure 1: Spring-loaded rigid bar with P

2. Figure 1 shows a rigid bar of length l, hinged at one end, free at the other, and supported through a frictionless ring connected to a spring that can move only horizontally. The free end is loaded with a force P whose direction remains vertical during the motion. Let's not consider the gravity in this problem.

(a) If the vertical wall to which the the spring is attached moves with a prescribed horizontal displacement $u(t)$, show that the equation of motion of the system, under the assumption of small angular position θ, is

$$I\ddot{\theta} + \left(ka^2 - Pl\right)\theta = kau(t) \tag{3}$$

where I is the moment of inertia of the rigid bar about the pivot point.

(b) Find the transfer function relating the input $u(t)$ to the output $\theta(t)$.

(c) Determine the poles and zeros of the system. Discuss the cases when $ka^2 > Pl$ and $ka^2 < Pl$.

(d) Plot the poles and zeros on the complex s plane for the two cases. How do the stability relate to the location of the poles?

3. This is a midterm exam problem of Spring Quarter of 1998. Consider a system governed by the following differential equation

$$\ddot{x} + 2\dot{x} + 5x = \dot{u}(t) \tag{4}$$

where $u(t)$ is the input and $x(t)$ is the output.

(a) Determine the transfer function $H(s)$.

(b) If $u(t) = \cos 2t$, what is the magnitude and phase of $H(s)$? Determine $x(t)$ through use of the transfer function $H(s)$.

(c) Find the poles and zeros of the system. Plot them on the complex s plane.

(d) Consider the following two inputs: $f_1(t) = \sin 2t$ and $f_2(t) = \sin 4t$. Judging from the pole-zero plot, determine which input will give the larger output amplitude. Why?

4. Here is an exam problem of Spring Quarter of 1999. Consider a simple AM radio consisting of a tunable L-C circuit. The input is the voltage received by the antenna $v_s(t)$, and the output is the voltage $v_o(t)$ to an amplifier. The ODE governing the circuit is

$$4C\frac{d^2v_o}{dt^2} + 2\times10^{-4}\frac{dv_o}{dt} + 2\times10^4 v_o = 2\times10^{-4}\frac{dv_s}{dt} + 1\times10^{-2}v_s \tag{5}$$

where C is capacitance of the variable capacitor to select stations.

(a) Determine the transfer function from $v_s(t)$ to $v_o(t)$.

(b) When $C = 50\times10^{-12}$ F, determine the poles and the zeros of the system.

(c) Input A is sinusoidal with frequency of 3×10^6 rad/s. Input B is sinusoidal with frequency of 1×10^7 rad/s. When $C = 50\times10^{-12}$ F, if both inputs have the same amplitude, which input will lead to a smaller output? Why? (Hint: Don't need to calculate anything.)

(d) What is the physical meaning of the magnitude and phase of the transfer function?