ME 374, System Dynamics Analysis and Design
Homework 4
Distributed: 4/16/2012, Due: 4/27/2012
(There are 4 problems in this set.)

1. This is an exam problem that I gave in the Winter Quarter of 2001. Consider the fluid system shown in Fig. 1. The state variables are the pressure p_C of the tank and the flow rate Q_I in the long pipe. In addition, the input is the pump pressure $P(t)$. Moreover, the state equation of the system is given by

$$\frac{d}{dt}\begin{pmatrix} p_C \\ Q_I \end{pmatrix} = \begin{bmatrix} -2 & 1 \\ -5 & -4 \end{bmatrix}\begin{pmatrix} p_C \\ Q_I \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} P(t) \tag{1}$$

Answer the following questions.

(a) Derive the transfer function $H_C(s)$ from $P(t)$ to $p_C(t)$. Hint: A useful formula is the inversion of 2×2 matrix as follows.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{2}$$

(b) If the transfer function $H_I(s)$ from $P(t)$ to $Q_I(t)$ is

$$H_I(s) = \frac{4(s+k)}{s^2 + 6s + 13} \tag{3}$$

where $k > 0$ is a constant parameter. Determine the poles and zeros. Is the system stable? Why?

(c) If $P(t)$ is a constant pressure, and you want to maximize the flow rate Q_I, will you increase k or decrease k? Why?

(d) Assume that $k = 2$. If $P(t) = e^{-t}$, determine $Q_I(t)$ through the transfer function approach.

(e) Assume that $k = 2$. If $P(t) = \sin 2t$, determine the magnitude and phase of $Q_I(t)$.

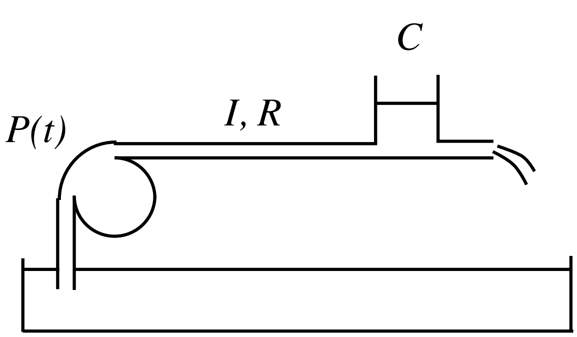

Figure 1: A fluid system

2. This is an exam problem of Spring 2008. Figure 2 shows the setup of a load cell consisting of a mass and a cantilever spring. A force $f(t)$ applied at the mass deflects the beam resulting in a normal strain $\epsilon(t)$. By measuring the strain $\epsilon(t)$, one can determine the force $f(t)$ applied. The strain $\epsilon(t)$ can be measured in two ways. One is through a strain gage, and the other is through a piezoelectric film. Figure 3 shows a block diagram of the load cell. The mass and the cantilever spring form a transfer function

$$H(s) = \frac{274}{s^2 + 100} \tag{4}$$

with $f(t)$ being the input and $\epsilon(t)$ being the output. If one uses the strain gage, its transfer function from the strain ϵ to the output voltage $v_s(t)$ is

$$H_1(s) = \frac{1}{s + 6} \tag{5}$$

If one uses the piezoelectric film, its transfer function from the strain ϵ to the output voltage $v_p(t)$ is

$$H_2(s) = \frac{s}{s + 20} \tag{6}$$

Answer the following questions.

(a) Derive the transfer functions from the force $f(t)$ to the output voltage $v_s(t)$ and $v_p(t)$ of the strain gage and the piezoelectric film, respectively.

(b) If the force $f(t)$ to be measured is a constant force, will you choose the strain gage or the piezoelectric film for the load cell? Please justify your answer.

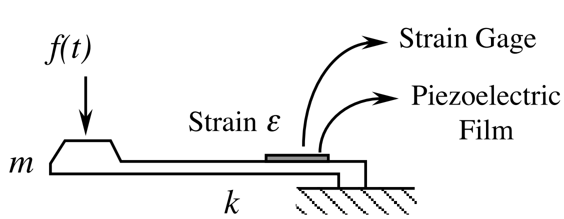

Figure 2: A load cell to measure force $f(t)$

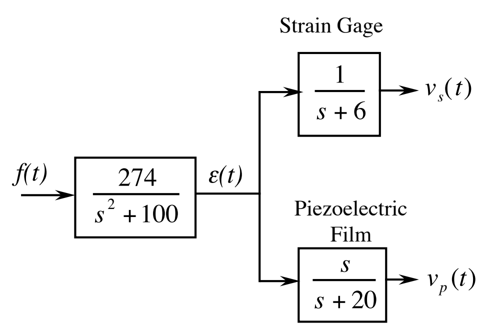

Figure 3: Block diagram of the load cell

3. Figure 4 shows the block diagram of a spindle motor along with its controller. The purpose of the controller is to make sure that the motor spins at constant speed. The motor, with input voltage v_s and output angular velocity Ω, has the following transfer function.

$$H(s) = \frac{50}{s^2 + 21s + 20} \tag{7}$$

The tachometer, with a gain of $K_1 = 0.05$ V/rad, converts the spin speed to the output voltage v_o. The output voltage is fed back to compared with the desired voltage (i.e., desired spin speed) v_d. The difference is then amplified with a gain $K_2 = 100$ to drive the motor. As one can see, if $v_o < v_d$, v_s will be positive increasing the motor speed. In contrast, if $v_o > v_d$, v_s will be negative decreasing the motor speed. Therefore, the feedback control tends to maintain the motor spinning at a constant speed. Answer the following questions.

(a) What is the differential equation representing the motor?

(b) Find the transfer function from v_d to Ω, and determine the poles and zeros of the transfer function.

(c) Consider two sinusoidal inputs $v_d^{(1)}$ and $v_d^{(2)}$ with equal magnitude. The frequency of $v_d^{(1)}$ and $v_d^{(2)}$ is 4 Hz and 10 Hz, respectively. Calculate how much each input will be amplified by the closed-loop controlled motor. Which one will have a larger output magnitude? Why? Be careful about the unit. The unit of poles and zero is rad/s.

(d) After 5 years of intensive use, the motor is wearing out and its transfer function changes to

$$H(s) = \frac{50}{s^2 + 2s + 20} \tag{8}$$

Where are the poles and zeros of the worn motor from v_d to Ω?

(e) Judging from the location of the poles and zeros of the worn motor, can you explain the physical implication of the wear to the motor performance? What danger could the wear cause to the motor?

(f) If you are an engineer trying to identify a worn-out component inside the motor. Which one will you guess? Should it be most likely a component providing mass, stiffness, or damping? Why?

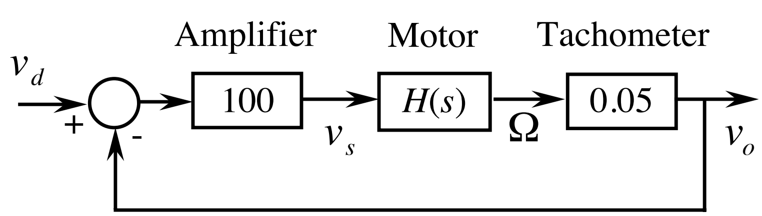

Figure 4: A spindle motor with controller

4. This is an exam problem from the Spring Quarter of 2004. Figure 5 shows an engine under a vibration test. The engine is subjected to a given base excitation $u(t)$, and the displacement of the engine at the top is $x(t)$. A laser Doppler vibrometer picks up the displacement and output a voltage $V(t)$. During the design phase, engineers have designed the engine so that its transfer function from $u(t)$ to $x(t)$ is

$$H_e(s) = \frac{s+1}{s^2+4} \tag{9}$$

Given by its manufacturer, the transfer function of the laser Doppler vibrometer from $x(t)$ to $V(t)$ is

$$H_{LDV}(s) = \frac{s}{s^2+a^2} \tag{10}$$

where $a > 0$ is a constant and can be changed from the front panel of the laser Doppler vibrometer. Figure 6 shows the block diagrams relating $u(t)$, $x(t)$, and $V(t)$. Answer the following questions.

(a) What is the ordinary differential equation relating $x(t)$ and $u(t)$?

(b) If $u(t) = \cos t$, find the magnitude and phase of $H_e(s)$. What are the magnitude and phase of $x(t)$?

(c) Let $H(s)$ be the transfer function from $u(t)$ to $V(t)$? Find $H(s)$.

(d) Determine the poles and zero of $H_e(s)$ and $H(s)$. Are they the same?

(e) In the testing, only the composite transfer function $H(s)$ can be measured. Engineer A conducts a test with $a = 100$. When the input is $\sin 99t$, Engineer A measures an extremely large voltage reading from the laser Doppler vibrometer. What does Engineer A measure? Is it a pole (or a zero) of the transfer function associated with the engine (or the laser Doppler vibrometer)? Why? When the input is $\cos 0.02t$, Engineer A measures a very small voltage from the laser Doppler vibrometer. What does Engineer A measure? Is it a pole (or a zero) of the transfer function associated with the engine (or the laser Doppler vibrometer)? why?

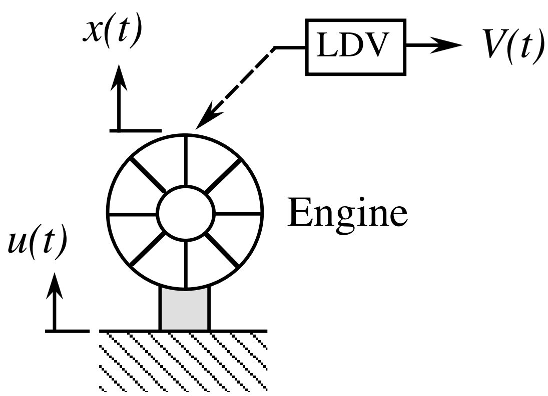

Figure 5: An engine under vibration test

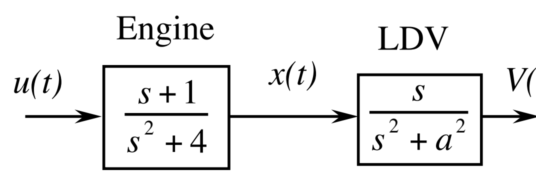

Figure 6: The block diagram