ME 374, System Dynamics Analysis and Design
Homework 5
Distributed: 4/23/2012, Due: 5/4/2012
(There are 6 problems in this set.)

1. Determine the impedance of the rotational system shown in Fig. 1.

2. This is an old exam problem that I gave in Winter Quarter of 2001. Consider the simplified model we discussed in class about a car on a bumpy road. The car has a mass m supported by stiffness k and damping c. The road gives a displacement excitation $R(t)$ to the car. The transfer function from $R(t)$ to the car displacement $y(t)$ is

$$H_y(s) = \frac{Y(s)}{R(s)} = \frac{cs+k}{ms^2+cs+k} \tag{1}$$

   (a) Determine the driving point impedance $Z_R(s)$, which is the ratio of the velocity $\dot{R}(t)$ to the force acting on the wheel from the road.

   (b) Engineer X is testing the car in the Vehicle Dynamics Lab of a start-up company GF.com. Because of insufficient cash flow, Engineer X only has the equipment to measure $Z_R(s)$, and identify the zeros and poles of $Z_R(s)$. But eigineer X wants to find the poles of $H_y(s)$. To do so, should Engineer X use the zeros or poles of $Z_R(s)$ instead? Explain why?

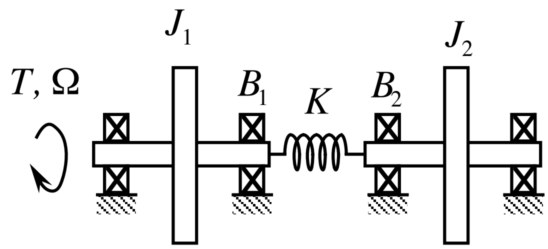

Figure 1: A rotational system

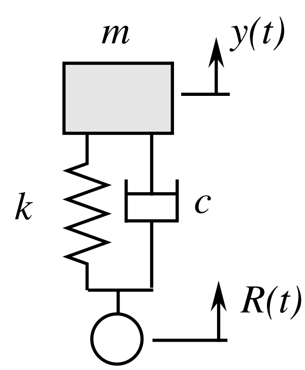

Figure 2: A simple model of a car on a bumpy road

3. Piezoelectric materials can convert mechanical energy to electrical energy, and vice versa. Therefore, piezoelectric materials are widely used in various sensors (e.g., microphones) and actuators (e.g., metronome). Figure 3 shows a piezoelectric accelerometer measuring vibration of a floor. The floor moves with velocity $V(t)$. The accelerometer consist of a mass m and a piezoelectric block with stiffness k. The output current $i(t)$ of the piezoelectric block drives a resistor with resistance R. Figure 4 shows the corresponding linear graph, where $f_e(t)$ is the force needed to generate the current $i(t)$, c is the capacitance of the piezoelectric block,

and the gyrator represents the piezoelectric effect converting mechanical energy to electrical energy. For ideal piezoelectricity, $i(t) = pv_1(t)$, where p is a constant. Since there is no energy loss in ideal piezoelectricy, $f_e(t) = pv_2(t)$. Derive the driving point impedance $Z(s)$ at the floor.

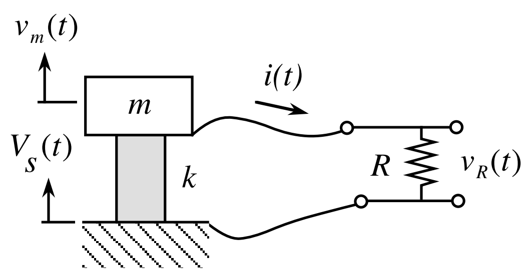

Figure 3: A piezoelectric accelerometer

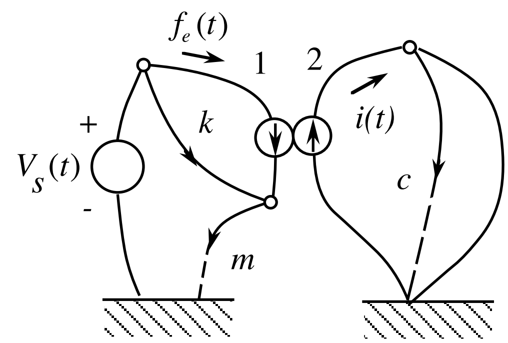

Figure 4: Corresponding linear graph

4. Consider the system shown in Fig. 5, where the cylinder of radius r and mass m is pulled through a massless spring with spring constant k and a massless dashpot with damping coefficient c. Assume that the cylinder rotates freely about its axis and that the input displacement $u(t)$ is known. The transfer function from $u(t)$ to $x(t)$ is

$$H(s) = \frac{cs + k}{\frac{3}{2}ms^2 + cs + k} \tag{2}$$

Answer the following questions.

(a) Derive the driving point impedance $Z(s)$.

(b) How would you find the poles of $H(s)$ from the impedance $Z(s)$? Why?

5. Determine the impedance of the fluid system shown in Fig. 6.

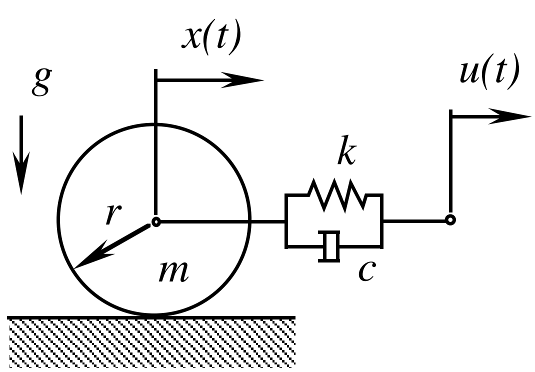

Figure 5: Rolling cylinder excited by $u(t)$

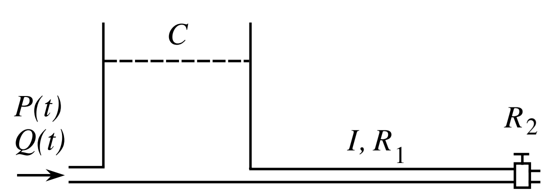

Figure 6: A fluid system

6. Consider the hydraulic system shown in Fig. 7, which consists of a hydraulic cylinder, a tank and a valve. The hydraulic cylinder is a gyrator governed by $v = Q/A$ and $f = Ap$, where $v(t)$ and $f(t)$ are the velocity and the force of the piston, A is the area of the cylinder, and $p(t)$ and $Q(t)$ are the pressure and flow rate at the inlet of the tank. Also, the tank has fluid capacitance C and the valve has fluid resistance R. Moreover, Fig. 8 shows the linear graph of the hydraulic system. Answer the following questions.

   (a) Determine the driving point impedance $Z(s)$.

   (b) Define $H(s)$ as the transfer function with the piston velocity $v(t)$ as the input and the inlet pressure $p(t)$ as the output. Derive the relationship between the transfer function $H(s)$ and the driving point impedance $Z(s)$.

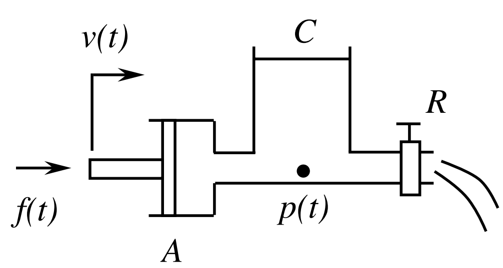

Figure 7: A hydraulic cylinder driving a tank and a valve

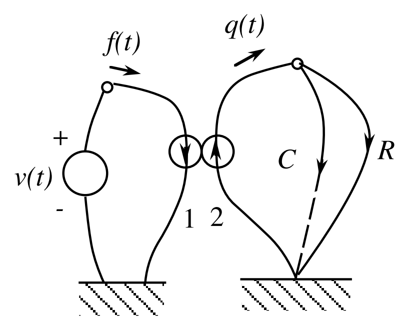

Figure 8: System linear graph