ME 374, System Dynamics Analysis and Design
Homework 6

Distributed: 4/30/2012, Due: 5/11/2012

(There are 4 problems in this set.)

1. This is an old exam problem that I gave in the Spring Quarter of 2004. Micro-electro-mechanical systems (MEMS) are emerging technology that uses semiconductor processes to fabricate tiny sensors and actuators. One type of MEMS actuator is thermal drives consisting of a silicon base beam and a metal electrode with large resistance; see Fig. 1(a). When the electric voltage $V(t)$ is applied to the electrode, it generates heat $q(t)$ increasing the relative temperature $T(t)$ of the beam to the ambient fluid. Since silicon and metal have different coefficients of thermal expansion, the temperature change causes the motion $x(t)$. Figure 1(b) shows the block diagram describing the dynamics, where R and k are both constants. Moreover, the relationship between the temperature $T(t)$ and and input heat flow $q(t)$ satisfies

$$\rho v c \frac{dT(t)}{dt} + hAT(t) = q(t) \tag{1}$$

where ρ is the density of the beam, v is the volume of the beam, c is the specific heat of the beam, h is the heat transfer coefficient of the beam, and A is the surface area of the beam. Answer the following questions.

   (a) Derive the frequency response function $G(\omega)$ from $q(t)$ to $T(t)$.

   (b) Plot the magnitude $G(\omega)$ as a function of ω. Is it a low-pass filter or a high-pass filter? What is the bandwidth of $G(\omega)$? What is the time constant of the system?

   (c) Two thermal drives A and B have identical materials. The size of A is twice as large as B in every dimension. Which thermal drive has a larger bandwidth and why?

   (d) Thermal drive C has the following frequency response function

$$G(\omega) = \frac{0.01}{1 + j\tau\omega} \tag{2}$$

   where τ is the time constant. In addition, the thermal drive has a corner frequency of 100 Hz. The background noise has a magnitude of 10^{-3}. When $|G(\omega)|$ is less than the background noise, one cannot measure the response of the thermal drive any more. Determine the frequency range where the response cannot be measured.

2. This is an old exam question from Spring Quarter of 2008. Figure 2 shows a mechanical filter consisting of a spring with stiffness k and a linear damper with damping coefficient c. The filter is subjected to an input displacement $x(t)$, and the corresponding force output is $f(t)$. The forces and displacement then satisfy

$$c\dot{f}(t) + kf(t) = ck\dot{x}(t) \tag{3}$$

Answer the following questions.

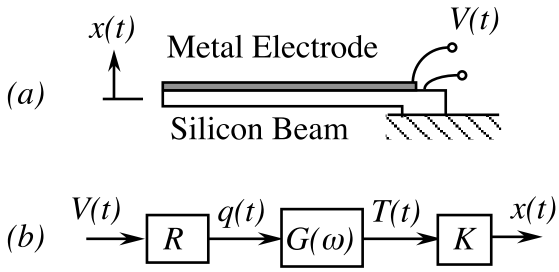

Figure 1: a thermal drive

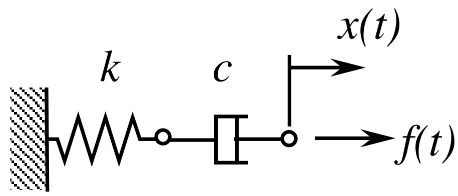

Figure 2: A mechanical filter

(a) Derive the frequency response function $G(\omega)$ with $x(t)$ being the input and $f(t)$ being the output. You can leave the answer in the form of a complex fraction.

(b) Derive the magnitude and phase of the frequency response function $G(\omega)$. What is the cut-off frequency of the filter? Is it a high-pass or a low-pass filter?

(c) Consider the case when $k = 10$ N/m and $c = 1$ Ns/m. If the driving displacement is $x(t) = \cos 10t$, derive the output force $f(t)$.

(d) Engineer X wants to use this filter as a force sensor. When a force $f(t)$ is applied, one can measure the displacement $x(t)$ and calculate the force through $G(\omega)$. What is the frequency range in which $G(\omega)$ is roughly constant? Also, what is the sensitivity of the sensor (i.e., displacement per unit force) in this frequency range?

3. This is a real problem that we encounter in our hard disk drive (HDD) vibration research. In the past, HDD used ball bearings generating a lot of noise and vibration. Currently, HDD on the market adopt hydrodynamic bearings. Figure 3 shows a simplified model. The inner circle in Fig. 3 represents the shaft carrying all the disks, the outer circle in Fig. 3 represents the bearing sleeve (which is fixed in space), and the shaded area is the radial hydrodynamic bearing. Let's define a coordinate system xy with its origin attached to the center of the bearing sleeve. The motion of the shaft is then described by the coordinates x and y of the shaft center. The vibration of HDD spindles with hydrodynamic bearings is very complicated, but a simplified version of the equations of motion takes the following form.

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f_x(t) \\ f_y(t) \end{pmatrix} \tag{4}$$

In (4), m is the mass of the spindle (including the disks) and c is the damping coefficient of the bearing. In addition, k_1 and k_2 are called in-line stiffness and cross stiffness of a hydrodynamic bearing. f_x and f_y are the forces acting on the spindle (e.g., vibration from your laptop computer). As shown in Fig. 3, the system has two degrees of freedom requiring a matrix formulation in (4). With the following complex notation

$$z(t) \equiv x(t) + jy(t); \quad f(t) \equiv f_x(t) + jf_y(t); \quad j \equiv \sqrt{-1} \tag{5}$$

one can rewrite (4) as a second-order ordinary differential equation with complex coefficients

$$m\ddot{z}(t) + c\dot{z}(t) + (k_1 - jk_2)\, z(t) = f(t) \tag{6}$$

Based on the equation of motion (6), please answer the following questions.

(a) Derive the frequency response function $G(\omega)$ from (6). You can assume that $f(t) \equiv f_0 e^{j\omega t}$ and $z(t) \equiv z_0 e^{j\omega t}$. Then $G(\omega) \equiv z_0/f_0$.

(b) Consider the case when $\omega \ll \sqrt{\frac{k_1}{m}}$. Show that the frequency response function can be approximated as

$$G(\omega) \approx \frac{1}{k_1 + j\,(c\omega - k_2)} \tag{7}$$

(c) Derive the magnitude and phase of $G(\omega)$ from (7). Plot the magnitude and phase as a function of ω for both positive and negative ω. Do $|G(\omega)| = |G(-\omega)|$ and $\angle G(\omega) = -\angle G(-\omega)$ as we discuss in the class? Why?

(d) According to magnitude of $G(\omega)$, determine when the resonance occur. What is the magnitude of the resonance? What happens when k_1 is extremely small? When k_1 is large, how could you use the phase of $G(\omega)$ to find the resonance?

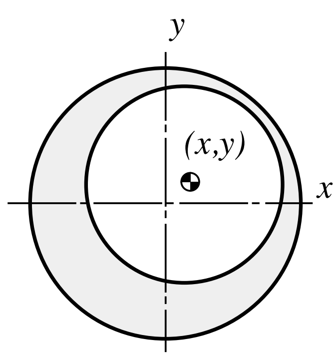

Figure 3: Whirling of a spindle with hydrodynamic bearing

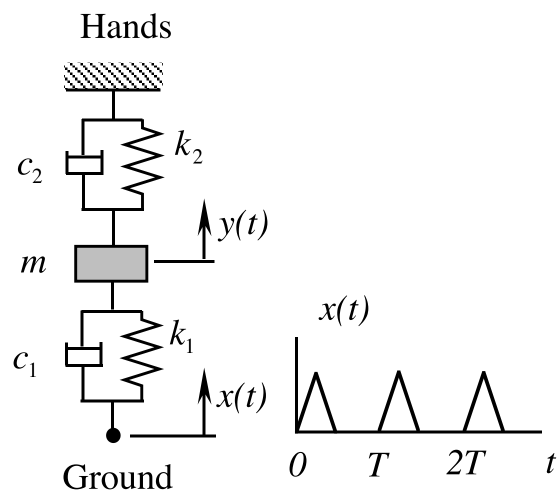

Figure 4: A model of a pneumatic hammer

4. This is an old exam problem that I gave in Spring Quarter of 1999. Figure 4 shows a simplified model describing a pneumatic hammer used in road construction. The mass of the hammer is m, the stiffness and damping between the hammer and the ground are k_1 and c_1, and the stiffness and damping between the hammer and the operator's hands are k_2 and c_2. The impact between the hammer and the ground is modeled as a prescribed ground motion $x(t)$ in the form of periodic triangular pulses with period T. The vibration of the pnumatic hammer is described by $y(t)$. The hands are considered fixed in space.

(a) Consider the ground motion $x(t)$ as input and the vibration $y(t)$ of the hammer as output. The differential equation is given by

$$m\ddot{y} + (c_1 + c_2)\dot{y} + (k_1 + k_2)y = k_1 x + c_1 \dot{x} \tag{8}$$

Derive the frequency response function from $x(t)$ to $y(t)$ using equation (8). What is the natural frequency and viscous damping factor ζ of the pneumatic hammer?

(b) The frequency response function $G_1(\omega)$ from the ground motion $x(t)$ to the force $f(t)$ transmitted to the operator's hands is given by

$$G_1(\omega) = \frac{(k_2 + jc_2\omega)(k_1 + jc_1\omega)}{-m\omega^2 + j(c_1 + c_2)\omega + (k_1 + k_2)} \tag{9}$$

In general, c_1 and c_2 are designed to be small. Determine the magnitude of $G_1(\omega)$ when $\omega \to 0$ and $\omega \to \infty$. Sketch the magnitude of $G_1(\omega)$. Assume that the damping c_1 and c_2 are small. How would you design the natural frequency of the hammer so that minimum forces are transmitted to the operator's hands given the periodic ground motion of period T? Why?