ME 374, System Dynamics Analysis and Design
Homework 7

Distributed: 5/7/2012, Due: 5/18/2012
(There are 6 problems in this set.)

1. Consider the simplified model we discussed in class about a car on a bumpy road; see Fig. 1. The car has a mass m supported by stiffness k and damping c. The road gives a displacement excitation $R(t)$ to the car. The transfer function from $R(t)$ to the car displacement $y(t)$ is

$$H_y(s) = \frac{Y(s)}{R(s)} = \frac{cs + k}{ms^2 + cs + k} \tag{1}$$

Let the acceleration of the car be $a(t) \equiv \ddot{y}(t)$. Determine the frequency response function $G_a(\omega)$ from $R(t)$ to $a(t)$. Plot $|G_a(\omega)|$ as a function ω. Hint: Analyze $|G_a(\omega)|$ for $\omega << \omega_n$, $\omega \approx \omega_n$, and $\omega >> \omega_n$.

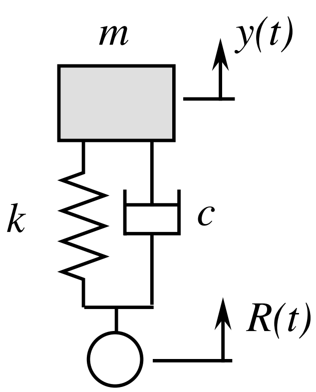

Figure 1: A simple model of a car on a bumpy road

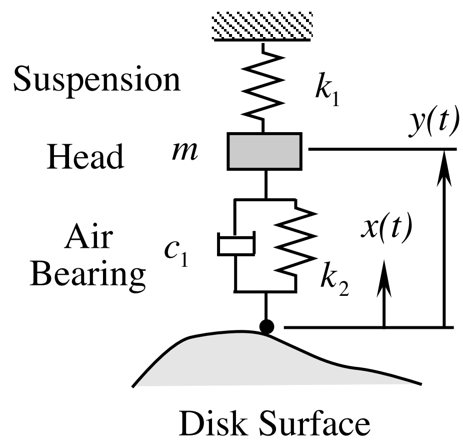

Figure 2: Suspension in computer hard disk drives

2. Figure 2 shows a simplified model to simulate a recording head flying over a rough disk surface in computer hard disk drives. The head has mass m and is supported by a suspension with stiffness k_1. Moreover, the moving disk surface will generate an air bearing lifting the head slightly above the disk surface (e.g., in the order of 20 nm). The air bearing is simplied as a linear spring with stiffness k_2 and damping coefficient c. Let $x(t)$ be the roughness of the disk surface and serve as the input excitation to the head/suspension system. Moreover, $y(t)$ is the relative displacement of the head to the disk. In real hard disk drive applications, we want to keep $y(t)$ almost constant, so that the head can follow the disk surface to perform read/write operations.

   (a) Show that the equation of motion is

$$m\ddot{y} + c\dot{y} + (k_1 + k_2)\, y = -m\ddot{x} - k_1 x \tag{2}$$

(b) Derive the frequency response function. Plot the magnitude and phase of the frequency response function. In plotting the frequency response function, let's define

$$\omega_1 = \sqrt{\frac{k_1}{m}}, \quad \omega_2 = \sqrt{\frac{k_1 + k_2}{m}} \tag{3}$$

Describe the motion of the head in the following three frequency ranges: $0 < \omega < \omega_1$, $\omega_1 < \omega < \omega_2$, and $\omega_2 < \omega$.

(c) If we want to design the disk drive so that the head can follow the disk surface for a wide frequency range, how should we choose k_1, k_2, and m?

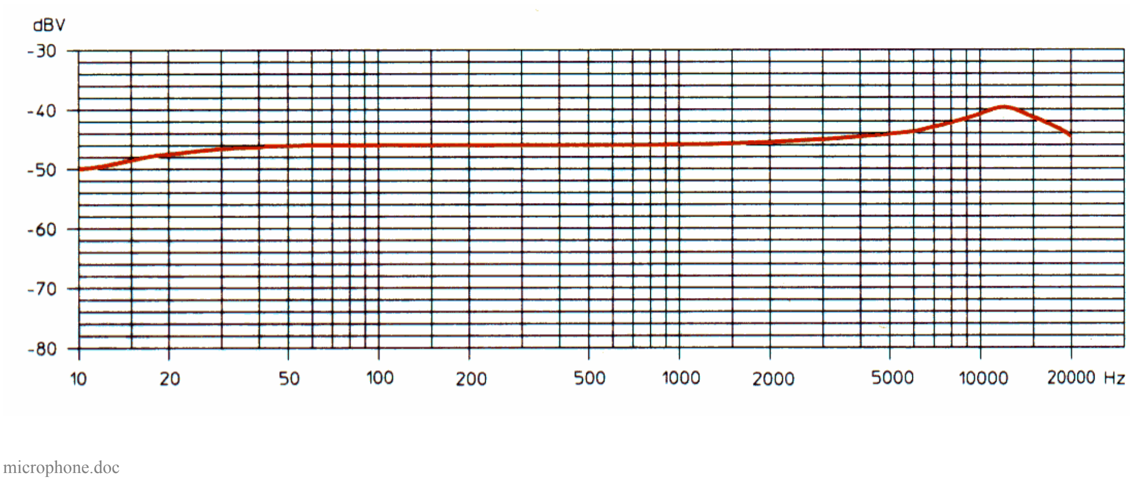

Figure 3: Frequency response function of the microphone

3. This is an old exam problem that I gave in the Winter Quarter of 2000. Figure 3 shows the Bode plot of a microphone that we recently purchased for the lab. The input the pressure (in Pa), and the output is voltage (in V). Answering the following questions by looking up the Bode plot. Also, explain how you come up with your answer.

   (a) If the input sound pressure has a frequency of 500 Hz and an amplitude of 1 Pa, What is the output frequency (in Hz) and amplitude (in V)?

   (b) For the frequency range from 10 Hz to 500 Hz, do you see a low-pass filter or a high-pass filter?

   (c) Does the system have a resonance? If yes, what is the resonance frequency?

   (d) What is the operating frequency range (or bandwith) of this microphone? Why? The bandwith can be estimated by a deviation of ± 3dB.

4. This is an old exam problem that I gave in Spring Quarter of 2008. Engineer Y is designing a piezoelectric accelerometer; see Fig. 4. The accelerometer is a spring-mass-damper system, where m is a proof mass and the spring and damper represent piezoelectric materials. When the rigid housing moves with an input displacement $u(t)$, the piezoelectric material deforms

with elongation $x(t)$. The elongation $x(t)$ generates electric charge that is measured by a charge amplifier leading to the output voltage $V(t)$. Figure 5 shows the block diagram, where $H_1(s)$ and $H_2(s)$ are transfer functions from input acceleration $a(t)$ to the elongation $x(t)$ and from the elongation $x(t)$ to the output voltage $V(t)$, respectively. In particular

$$H_1(s) = -\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad H_2(s) = \frac{\mu s}{1 + \tau s} \tag{4}$$

where ζ and ω_n are the viscous damping factor and natural frequency of the accelerometer, whereas μ and τ are the gain and time constant of the charge amplifier.

(a) Construct the **magnitude** of the Bode plot from acceleration $a(t)$ to voltage $V(t)$.

(b) For this sensor, the frequency response function from $a(t)$ to $V(t)$ must have a constant magnitude from 10 Hz to 10,000 Hz. How should Engineer Y choose the time constant τ and natural frequency ω_n to meet this design criterion?

(c) In addition, the constant magnitude in part (b) defines the sensitivity of the accelerometer. If Engineer Y chooses $\tau = 0.01$ s and $\omega_n = 10^5$ rad/s, what is the amplifier gain μ to achieve -60 dB sensitivity?

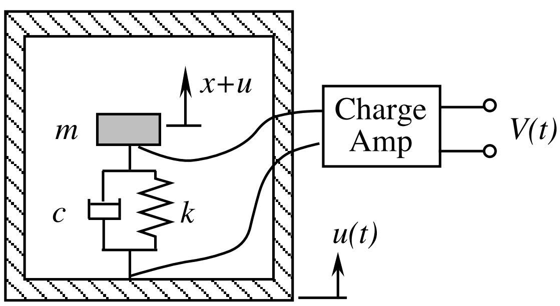

Figure 4: Piezoelectric accelerometer

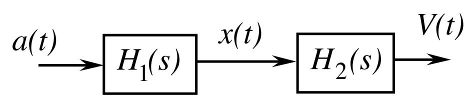

Figure 5: Block diagram showing the piezoelectric accelerometer

5. This is an old exam that I gave in Spring Quarter of 2004. Engineer X starts a new business building and selling vibration isolation tables. In principle, isolation tables can be modeled as a spring-mass-damper system as shown in Fig. 6, where m, c, and k are the mass, damping coefficient, and stiffness, respectively. Also, $u(t)$ is the motion of the floor, $x(t)$ is the displacement of the isolation table, and $f(t)$ is an external force acting on the table. For the isolation table to function well, we would like to minimize $x(t)$ as much as possible. Answer the following questions.

(a) When $f(t)$ is not present, the equation of motion of the isolation table is

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = c\frac{du(t)}{dt} + ku(t) \tag{5}$$

Derive the magnitude of the frequency response function $G(\omega)$ from $u(t)$ to $x(t)$ in terms of natural frequency ω_n and viscous damping factor $\zeta \equiv c/(2m\omega_n)$.

(b) Figure 7 shows the frequency response function of an isolation table made by Engineer X's competitor. If the mass of the isolation table is 1000 kg, determine the stiffness k and the viscous damping factor ζ needed to match the **vertical** performance in Fig. 7.

(c) How many dB does $|G(\omega)|$ roll off in Fig. 7 from 2 Hz to 10 Hz?

(d) To measure the frequency response function $G(\omega)$, Engineer X needs to use a large shaker to generate $u(t)$. This is very expensive for a start-up company. Therefore, engineer X measures the driving point impedance $Z(s)$ with an input force $f(t)$ using a hammer. Derives the driving point impedance when $u(t) = 0$. What useful information do you get from the impedance $Z(s)$?

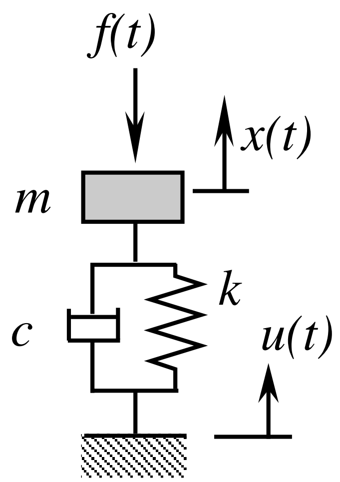

Figure 6: A simple model for isolation tables

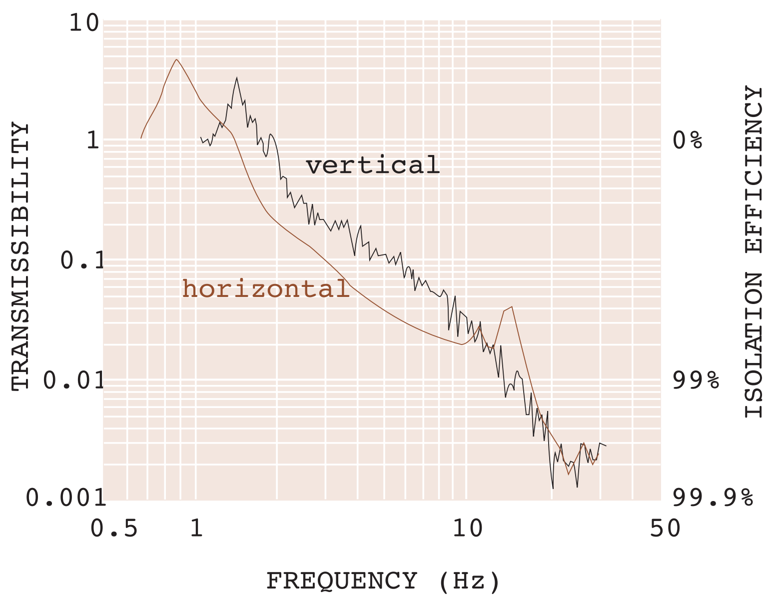

Figure 7: Frequency response function of isolation table

6. This is an old exam problem of Spring Quarter of 2004 Commercial piezoelectric actuators can be modeled as a spring-mass-damper system as shown in Fig. 8, where m, c, and k, are the mass, damping, and stiffness of the actuator. When a voltage is applied to the piezoelectric actuator, it is equivalent to applying a force $f(t)$. The actuator also has proof mass M to calibrate the actuator and to changes the natural frequency of the actuator. Answer the following questions.

   (a) Consider piezoactuator X, whoes natural frequency is 900 Hz when the proof mass is 100 grams. In addition, the natural frequency is 800 Hz when the proof mass is 150 grams. What is the natural frequency of the actuator, when the proof mass is absent? (Hint: the natural frequency is $\sqrt{k/(m+M)}$ when the proof mass is present.)

   (b) Figure 9 shows the frequency response function of a piezoelectric actuator that we made in our lab. The frequency response function roughly consists of a flat portion and a roll-off portion. In addition, the roll-off portion has several resonance peaks. Answer the following questions using Fig. 9.

i. Roughly estimate the bandwidth of this actuator. No calculation is required.

ii. What is the dB level at 1 Hz?

iii. How fast does the frequency response function roll off above 1000 Hz? Use dB/decade to answer the question.

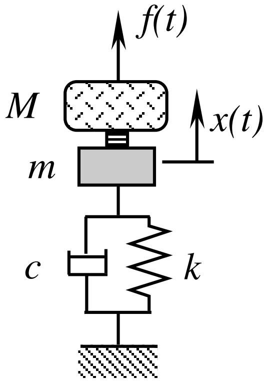

Figure 8: Actuator with a proof mass

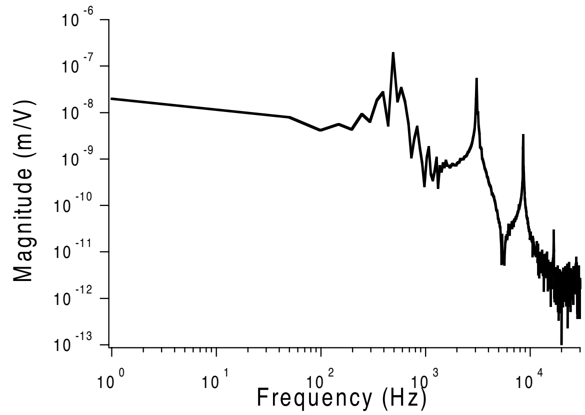

Figure 9: Frequency response of a piezoactuator