ME 374, System Dynamics Analysis and Design
Homework 8
Distributed: 5/14/2012, Due: 5/25/2012
(There are 5 problems in this set.)

1. This is an old exam problem for Spring Quarter of 2008. Figure 1 shows a periodic function with period T governed by

$$f(t) = e^{-t/T}, \quad 0 < t < T, \quad f(t+T) = f(t) \tag{1}$$

Answer the following questions.

(a) Expand $f(t)$ in (1) into a complex Fourier series.

(b) Derive the magnitude and phase of the spectrum of periodic function $f(t)$ in (1). Also plot the spectrum for the first three harmonics.

(c) If the function $f(t)$ in (1) is padded with zero outside the first period, the function $f(t)$ becomes

$$f(t) = \begin{cases} e^{-t/T}, & 0 < t < T \\ 0, & \text{Otherwise} \end{cases} \tag{2}$$

as shown in Fig. 2. Note that the function is no longer periodic. Derive the Fourier transform of $f(t)$ in (2).

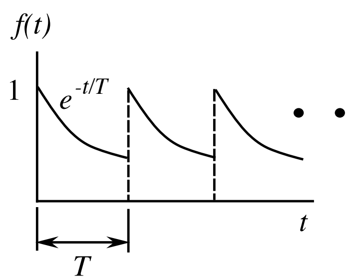


Figure 1: A periodic function with exponential decay

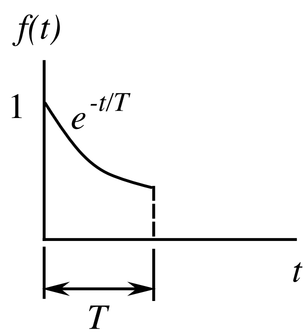


Figure 2: Aperiodic function by padding with zero

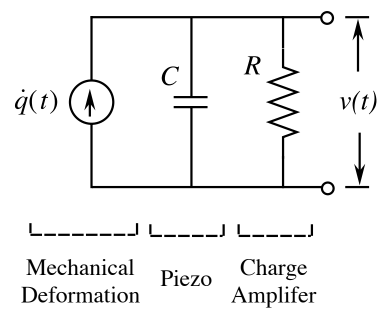


Figure 3: A simple model for piezoelectric sensor

2. This is an old exam problem for Spring Quarter of 2008. Figure 3 is a simple circuit modeling a piezoelectric sensor. Mechanical deformation of the sensor induces charge $q(t)$, which is first modeled as a current source. The capacitance C models the piezoelectric material, and the resistance R represents impedance of a charge amplifier with output voltage $v(t)$. Usually, C is fixed for a sensor, but R is adjustable from the amplifier. The frequency response function from $q(t)$ to $v(t)$ is

$$G(\omega) = \frac{jR\omega}{1 + jRC\omega} \tag{3}$$

Answer the following questions.

(a) Consider the case when $C = 1$ Farad and $R = 0.5$ Ohm. (The numbers are chosen to ease calculations. In practical applications, C is much smaller and R is much bigger.) If the input deformation induces electric charge

$$q(t) = \cos t + 2 \sin 3t \tag{4}$$

predict the voltage output $v(t)$ in time domain.

(b) For the output voltage $v(t)$ in part (a), calculate and plot its frequency spectrum.

(c) For the same sensor with $C = 1$ Farad, what should be the resistance R to ensure that the output voltage $v(t)$ is proportional to the induced charge $q(t)$ in (4)?

3. This is an old exam problem of Winter Quarter 2000. Answer the following questions.

(a) A dental drill is making a periodic noise as shown in Fig. 4, which is described mathematically by

$$u(t) = \begin{cases} 1, & 0 < t < T/2 \\ -1, & T/2 < t < T \end{cases} \tag{5}$$

The input noise is pressure and its unit is Pa. Express $u(t)$ as a complex Fourier series.

(b) Assume that the dental drill is producing a periodic noise whose spectrum (i.e., coefficients c_n from the Fourier series) is known. Moreover, the magnitude of the spectrum is shown in Figure 5. Now, we want to use a microphone to measure the noise from this dental drill. The microphone measures the pressure (in Pa) and gives voltage output (in V). The frequency response function of the microphone (in dBV) is shown in Fig. 6 for reference. Consider the case when $\omega_0 = 50$ Hz. Determine and draw the magnitude of the output voltage spectrum from the microphone in V. (Recall that dBV = 20 $\log_{10}$ V and V = $10^{(\text{dBV}/20)}$.) Will the microphone distort the input noise? Explain why.

(c) Consider part (b) with $\omega_0 = 5000$ Hz. Determine and draw the magnitude of the output voltage spectrum from the microphone in V. Will the microphone distort the input noise? Explain why.

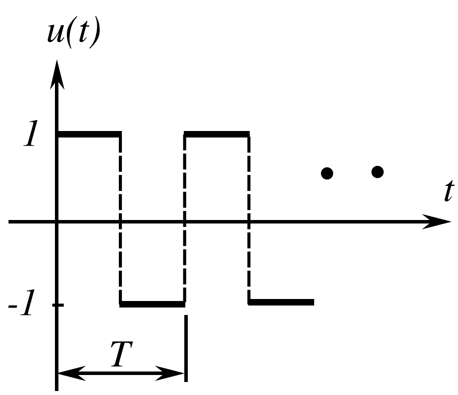


Figure 4: A periodic noise

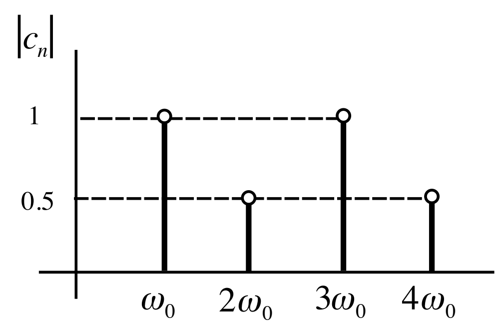
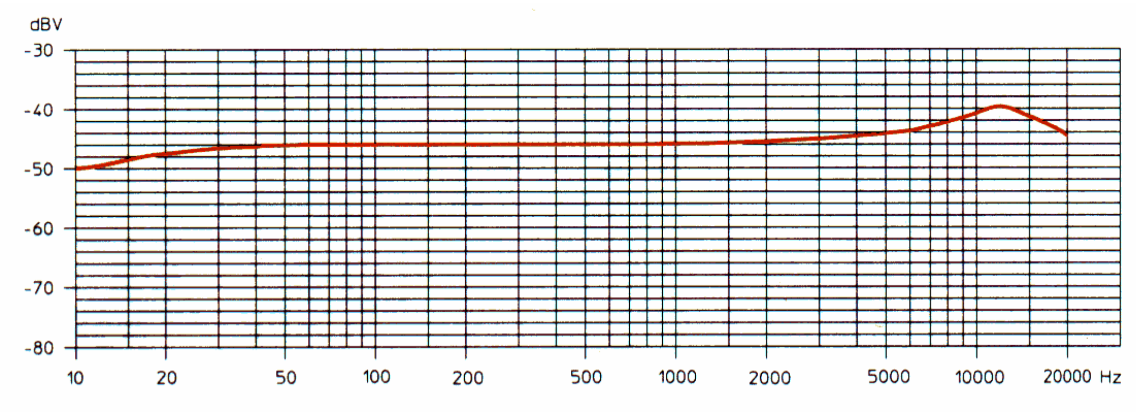


Figure 5: Spectrum of input noise



microphone.doc

Figure 6: Frequency response function of the microphone

4. This is an old exam problem of the Winter Quarter of 2001. Consider a periodic input signal whose complex Fourier seris is given by

$$f_{\text{in}}(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \tag{6}$$

where

$$c_n = \begin{cases} 1, & n = 0 \\ \frac{1}{n\pi}(1+j), & n > 0 \end{cases} \tag{7}$$

and c_{-n} is the complex conjugate of c_n in (7).

(a) Plot the spectrum of $f_{\text{in}}(t)$. Need to work out both phase and magnitude.

(b) The signal $f_{\text{in}}(t)$ is fed into a phase shifter, who frequency response function is shown in Fig. 7, where ω_c is the cutoff frequency of the phase shifter. Let $f_{\text{out}}(t)$ be the output from the phase shifter. Determine the spectrum of $f_{\text{out}}(t)$ if $\omega_c = 1.5\omega_0$. Need to work out both phase and magnitude.

(c) Will $f_{\text{out}}(t)$ be proportional to $f_{\text{in}}(t)$ if $\omega_c = 1.5\omega_0$? Why? When will $f_{\text{out}}(t)$ be proportional to $f_{\text{in}}(t)$? Why?

5. This is an old exam problem from the Spring Quarter of 2004. Figure 8 shows the schematic diagram of a computer hard disk drive. Data are recorded on the rotating magnetic disk as concentric circles called data tracks. A rotary actuator moves the read/write head radially to access different data tracks. The rotational motion is achieved through use of a voice coil motor. As shown in the block diagram of Fig. 8, the input current of the voice coil motor $i(t)$ transforms into the torque $T(t)$ through the torque constant k_T of the voice coil motor. The torque $T(t)$ then results in angular velocity $\dot{\theta}(t)$ of the suspension and its carrier. More specifically, the frequency response function from $i(t)$ to $\dot{\theta}(t)$ is

$$G(\omega) = \frac{k_T}{j\omega I} \tag{8}$$

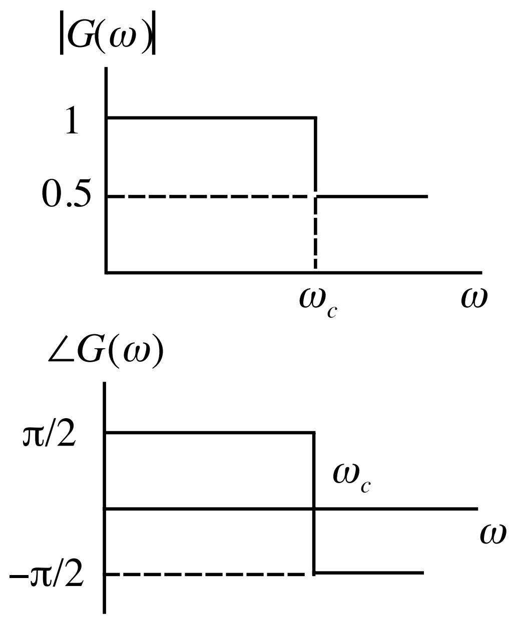


Figure 7: Frequency response function of a phase shifter

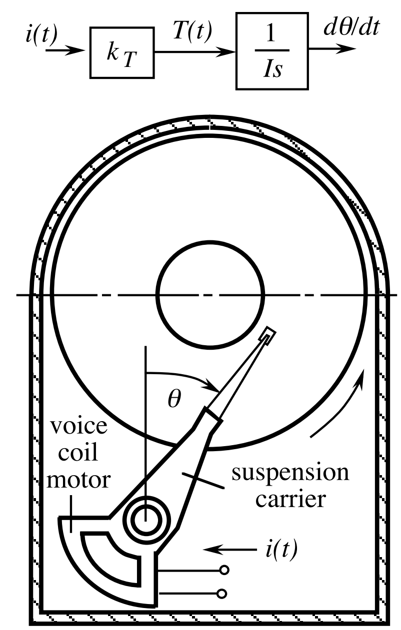


Figure 8: Suspension in computer hard disk drives

where I is the mass moment of inertia of the suspension carrier with respect to the pivot point. Let's consider the special case $k_T/I = 10$, so that the algebra becomes easy.

(a) Draw the Bode plot of $|G(\omega)|$ with respect to $\log_{10}\omega$ in dB scale.

(b) Consider an input current given by

$$i(t) = 2\sin 10t + \cos\left(20t + \frac{\pi}{4}\right) \tag{9}$$

Determine and plot the complex spectrum of $i(t)$. Note that the complex spectrum has both magnitude and phase.

(c) For the input current shown in (9), determine the corresponding output angular velocity.

(d) Will the output angular velocity $\dot{\theta}(t)$ resemble the input current $i(t)$ in the time domain? Why?