ME 374, System Dynamics Analysis and Design
Homework 9

Distributed: 5/21/2012, Due: 6/1/2012

(There are 4 problems in this set.)

1. Figure 1 shows a micro-electro-mechanical system that consists of a silicon cantilever beam and a point mass. The point mass has inertia m. The cantilever beam, modeled as a linear spring, has spring constant k. The ambient air and fluid could present some squeeze damper effects resulting in damping coefficient c. At the root of the cantilever, there is a PZT film. PZT is a piezoelectric material that can transform electrical energy to mechanical deformation, and vice versa. Therefore, the design in Fig. 1 can serve as a sensor and as an actuator.

   (a) Let's consider the case when the cantilever beam serves as an actuator stirring the ambient fluid. In this case, $s(t) = 0$. Also, the driving force to the point mass is $pV(t)$, where p is a proportional constant depending on material properties and the dimensions of the cantilever beam. The equation of motion is

   $$m\ddot{x} + c\dot{x} + kx = pV(t) \tag{1}$$

   Derive the frequency response function and sketch its magnitude. What is the form of $V(t)$ in order to obtain the largest stirring amplitude $x(t)$? Why? What is the largest amplitude of $x(t)$ under this condition?

   (b) If the driving voltage is periodic with the following form

   $$V(t) = \sin\omega_0 t + 3\cos 3\omega_0 t \tag{2}$$

   where ω_0 is the fundamental frequency of $V(t)$. Moreover, $\omega_0 = 0.5\omega_n$, where ω_n is the natural frequency of the actuator. Consider the case when $m = 1$, $c = 1$, $k = 4$, and $p = 1$. Determine the steady state response of $x(t)$.

   (c) Now let's consider the case of using the cantilever beam as a sensor. The governing equation is

   $$m\ddot{x} + c\dot{x} + kx = -m\ddot{s} \tag{3}$$

   Also, the displacement $x(t)$ will result in deformation of the PZT film generating charge $q(t)$, which is proportional to $x(t)$. Based on (3), what is the frequency range in which the sensor can measure acceleration $\ddot{s}(t)$? What is the frequency range in which the sensor can measure displacement $s(t)$? Explain why.

   (d) The sensor is subjected to a composite shock that consists of a negative and a positive impulse described mathematically as

   $$\ddot{s}(t) = \begin{cases} 1, & -T < t < 0 \\ -1, & 0 < t < T \end{cases} \tag{4}$$

   where T is half the duration of the composite shock. Determine the Fourier transform of the composite shock and plot its amplitude. What happens when the natural frequency ω_n coincides with $2\pi/T$?

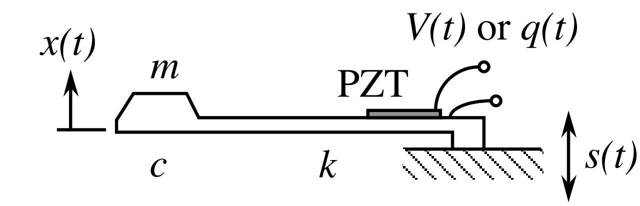

Figure 1: A silicon cantilever beam with a point mass

2. This is an old exam problem of Winter Quarter of 2000. An alien spacecraft landed in eastern Washington, and you were appointed to identify the frequency response function of the spacecraft. Because of security reasons, you were only allowed to send an exponentially decaying signal into the spacecraft as input. Mathematically, this signal is described by

$$u(t) = \begin{cases} ae^{-at}, a > 0, & t > 0 \\ 0, & t < 0 \end{cases} \tag{5}$$

where a is a positive parameter that you can choose. Answer the following questions.

   (a) Find the Fourier transform of $u(t)$.

   (b) Plot the magnitude of $\mathcal{F}\{u(t)\}$ as a function of ω. The magnitude plot should look like a low-pass filter. What is the asymptotic value of $\mathcal{F}\{u(t)\}$ when ω approaches 0? At what frequency will $\mathcal{F}\{u(t)\}$ roll off to 0.707 (or $1/\sqrt{2}$) of the asymptotic value at 0 frequency?

   (c) The alien spacecraft seemed to be very rigid. Your initial guess was that the first resonance would be at least 1000 Hz. What "a" value should you choose to excite this system? Explain why.

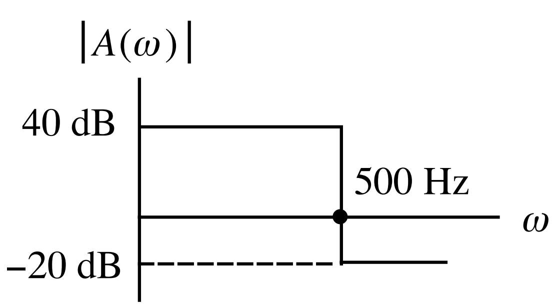

Figure 2: Input spectrum for operational test

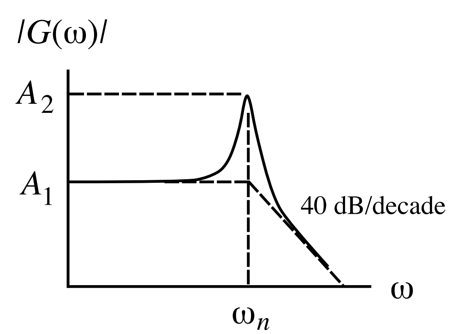

Figure 3: Frequency response function of a disk drive

3. Computer hard disk drives must pass operational vibration tests to be qualified. During the test, the disk drives are subjected to an input acceleration spectrum whose magnitude is

specified in Fig. 2. Moreover, vibration response $x(t)$ of the disk drives are related to the input acceleration $a(t)$ via $m\ddot{x}+c\dot{x}+kx=-ma(t)$, where m is the mass of the spinning part, c and k are damping and stiffness of the bearings. Therefore, the frequency response function from $a(t)$ to $x(t)$ is

$$G(\omega)=-\frac{1}{(\omega_n^2-\omega^2)+j\,(2\zeta\omega_n\omega)}, \qquad \omega\equiv\sqrt{\frac{k}{m}}, \qquad \zeta\equiv\frac{c}{2m\omega_n} \tag{6}$$

Moreover, the magnitude Bode plot of $G(\omega)$ is shown in Fig. 3 with

$$A_1=20\log_{10}\left(\frac{1}{\omega_n^2}\right), \qquad A_2=20\log_{10}\left(\frac{1}{2\zeta\omega_n^2}\right) \tag{7}$$

Answer the following questions.

(a) If $\omega_n=600$ Hz and $\zeta=0.02$, calculate A_1 and A_2 in dB.

(b) Predict the magnitude Bode plot of the output response spectrum.

(c) If the resonance in Fig. 3 is excited during the test, the disk drives will not pass. The bearing stiffness, however, drops 5% for every 10°C increase in temperature. How big the temperature increase can the disk drives tolerate without failing the test?

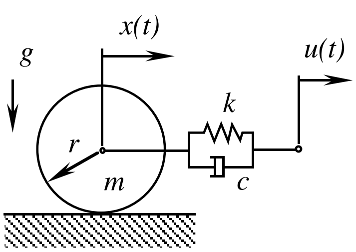

Figure 4: A rolling cylinder with displacement input $u(t)$

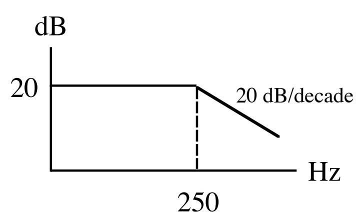

Figure 5: Bode plot of input spectrum

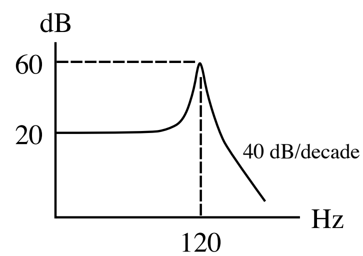

Figure 6: Bode plot of output spectrum

4. This is an old exam problem of Winter Quarter of 2001. A young engineer is trying to identify stiffness k and damping c of a cylinder suspension system shown in Fig. 4. First, the engineer uses displacement $u(t)$ as the input to excite the system. The frequency spectrum of $u(t)$ is given in Fig. 5 as a Bode plot (magnitude only). Then the displacement $x(t)$ of the cylinder is measured. The frequency spectrum of $x(t)$ is shown in Fig. 6 as a Bode plot (magnitude only).

(a) Draw the Bode plot (magnitude only) of the frequency response function of the cylinder system system.

(b) The young engineer develops a model for the cylinder suspension system and finds that $u(t)$ and $x(t)$ are related through

$$\frac{3}{2}m\ddot{x}+c\dot{x}+kx=c\dot{u}+ku \tag{8}$$

where m is the mass of the cylinder. If m is 2 kg, use the frequency response function you obtained in part (a) to identify c and k. (Hint: Look at the frequency response function at resonance.)