$$\underline{A} = \begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix}$$

(a)

$$|\underline{A} - \lambda \underline{I}| = \begin{vmatrix} -\lambda & 1 \\ -9 & 6-\lambda \end{vmatrix} = \lambda(\lambda-6)+9 = \lambda^2 - 6\lambda + 9$$

$$= (\lambda-3)^2 = 0, \quad \lambda = 3, 3$$

when $\lambda = 3$

$$(\underline{A} - \lambda \underline{I})\underset{\sim}{x} = \begin{bmatrix} 0-3 & 1 \\ -9 & 6-3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underset{\sim}{0}$$

$$\begin{cases} -3x_1 + x_2 = 0 \\ -9x_1 + 3x_2 = 0 \end{cases} \Rightarrow \quad x_1 = c_1, \quad x_2 = 3c_1$$

$$\underset{\sim}{u}_1 = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow$$ 2 Eigenvalues, only 1 eigenvector

(b) Since there is only 1 eigenvector, we need to build $\underset{\sim}{u}_2$ so that

$$(\underline{A} - \lambda \underline{I})\underset{\sim}{u}_2 = \underset{\sim}{u}_1$$

OR

$$\begin{bmatrix} 0-3 & 1 \\ -9 & 6-3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -3x_1 + x_2 = 1 \\ -9x_1 + 3x_2 = 3 \end{cases} \Rightarrow \quad \text{Choose } x_1 = 0, \; x_2 = 1$$

$$\underset{\sim}{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Note: x_1 can be chosen arbitrarily. I chose $x_1 = 0$ to simplify calculation.

Hence

$$\underline{M} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \quad , \quad \underline{M}^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

Similarity Transformation :

$$\underline{M}^{-1}\underline{A}\,\underline{M} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

This is a Jordan form, because $\underline{A}$ is NOT symmetric but has repeated eigenvalues.