(a) $\underline{A}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

$$|\underline{A}_1 - \lambda \underline{I}| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 3-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix} = (1-\lambda)\left\{(3-\lambda)^2 - 1\right\} = (1-\lambda)\left[\lambda^2 - 6\lambda + 8\right] = 0$$

$\lambda_1 = 1, \quad \lambda_2 = 2, \quad \lambda_3 = 4$

FOR $\lambda_1 = 1$,

$$\left[\underline{A}_1 - \lambda_1 \underline{I}\right] \underset{\sim}{u}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix} \underset{\sim}{u}_1 = 0, \quad \underset{\sim}{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

FOR $\lambda_2 = 2$

$$\left[\underline{A}_1 - \lambda_2 \underline{I}\right] \underset{\sim}{u}_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \underset{\sim}{u}_2 = 0, \quad \underset{\sim}{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

FOR $\lambda_3 = 4$

$$\left[\underline{A}_1 - \lambda_3 \underline{I}\right] \underset{\sim}{u}_3 = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \underset{\sim}{u}_3 = 0, \quad \underset{\sim}{u}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\underline{M} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}, \quad \underline{M}^{-1} \underline{A}\, \underline{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\underline{A}_2 \equiv \begin{bmatrix} 1 & 4 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|\underline{A}_2 - \lambda \underline{I}| = \begin{vmatrix} 1-\lambda & 4 & 0 \\ 1 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 - 4(1-\lambda)$$

$$= (1-\lambda)\left[\lambda^2 - 2\lambda + 1 - 4\right] = 0$$

$$\lambda_1 = 1, \quad \lambda_2 = -1, \quad \lambda_3 = 3$$

$$(\underline{A}_2 - \lambda_1 \underline{I})\underset{\sim}{u}_1 = \begin{bmatrix} 0 & 4 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \underset{\sim}{u}_1 = 0 \qquad \underset{\sim}{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$(\underline{A}_2 - \lambda_2 \underline{I})\underset{\sim}{u}_2 = \begin{bmatrix} 2 & 4 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \underset{\sim}{u}_2 = 0, \qquad \underset{\sim}{u}_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$(\underline{A}_2 - \lambda_3 \underline{I})\underset{\sim}{u}_3 = \begin{bmatrix} -2 & 4 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \underset{\sim}{u}_3 = 0, \qquad \underset{\sim}{u}_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{M} \equiv \begin{bmatrix} 1 & 2 & 2 \\ 0 & -1 & 1 \\ -1 & 0 & 0 \end{bmatrix}, \quad \underline{M}^{-1} \underline{A} \underline{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

<u>No Jordan Form</u>

$$\underline{A}_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\left| \underline{A}_3 - \lambda \underline{\underline{I}} \right| = \begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & 2 & 3-\lambda \end{vmatrix} = (1-\lambda)^2 (3-\lambda) = 0$$

$$\lambda_1 = \lambda_2 = 1, \quad \lambda_3 = 3$$

$$\left(\underline{A}_3 - \lambda_1 \underline{\underline{I}} \right) \underset{\sim}{u}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 2 \end{bmatrix} \underset{\sim}{u}_1 = 0, \qquad \underset{\sim}{u}_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$\left(\underline{A}_3 - \lambda_2 \underline{\underline{I}} \right) \underset{\sim}{u}_2 = \underset{\sim}{u}_1$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 2 \end{bmatrix} \underset{\sim}{u}_2 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \qquad \underset{\sim}{u}_2 = \begin{bmatrix} -5 \\ 2 \\ 0 \end{bmatrix}$$

$$\left(\underline{A}_3 - \lambda_3 \underline{\underline{I}} \right) \underset{\sim}{u}_3 = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 1 & 2 & 0 \end{bmatrix} \underset{\sim}{u}_3 = \underset{\sim}{0}, \qquad \underset{\sim}{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{M} \equiv \begin{bmatrix} 2 & -5 & 0 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \qquad \underline{M}^{-1} \underline{A} \underline{M} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Jordan Form