Principal Stresses

$$\underline{A} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

Pure Shear

(a) $$\underline{A} = \begin{bmatrix} 0 & 50 \\ 50 & 0 \end{bmatrix}$$

$$\left| \underline{A} - \lambda \underline{I} \right| = \begin{vmatrix} -\lambda & 50 \\ 50 & -\lambda \end{vmatrix} = \lambda^2 - 2500 = 0$$

$$\lambda_1 = 50 \,, \quad \lambda_2 = -50$$

$$\lambda_1 = 50, \quad \left[\underline{A} - \lambda_1 \underline{I} \right] \underset{\sim}{u}_1 = \begin{bmatrix} -50 & 50 \\ 50 & -50 \end{bmatrix} \underset{\sim}{u}_1 = \underset{\sim}{0}$$

$$\underset{\sim}{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -50, \quad \left[\underline{A} - \lambda_2 \underline{I} \right] \underset{\sim}{u}_2 = \begin{bmatrix} 50 & 50 \\ 50 & 50 \end{bmatrix} \underset{\sim}{u}_2 = 0$$

$$\underset{\sim}{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(b) Symmetric matrices imply that

- principal stresses are real
- Principal directions are real