$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) Stability

$$\begin{vmatrix} 0-\lambda & 1 \\ -8 & -6-\lambda \end{vmatrix} = \lambda(\lambda+6)+8 = \lambda^2+6\lambda+8=0$$

$$\lambda_1 = -2 \, , \quad \lambda_2 = -4$$

Both are real and negative $\Rightarrow$ Stable, Not oscillating.

(b) $\Phi(t)$:

$$(\underset{\sim}{A}-\lambda_1 \underline{I})\,\underset{\sim}{u}_1 = \begin{bmatrix} +2 & 1 \\ -8 & -4 \end{bmatrix} \underset{\sim}{u}_1 = \underset{\sim}{0} . \qquad \underset{\sim}{u}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$(\underline{A}-\lambda_2 \underline{I})\,\underset{\sim}{u}_2 = \begin{bmatrix} 4 & 1 \\ -8 & -2 \end{bmatrix} \underset{\sim}{u}_2 = 0 . \qquad \underset{\sim}{u}_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\underline{M} = \begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix} , \qquad \det \underline{M} = -4+2 = -2$$

$$\underline{M}^{-1} = -\frac{1}{2}\begin{bmatrix} -4 & -1 \\ +2 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & 1 \\ -2 & -1 \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-4t} \end{bmatrix} \cdot \frac{1}{2}\begin{bmatrix} 4 & 1 \\ -2 & -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} 4e^{-2t} & e^{-2t} \\ -2e^{-4t} & -e^{-4t} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-2t}-e^{-4t} & \frac{1}{2}e^{-2t}-\frac{1}{2}e^{-4t} \\ -4e^{2t}+e^{-4t} & -e^{-2t}+2e^{-4t} \end{bmatrix}$$

(c)

$$\underset{\sim}{x}(t) = \underset{\sim}{\Phi}(t) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t} \\ -e^{-2t} + 2e^{-4t} \end{bmatrix}$$