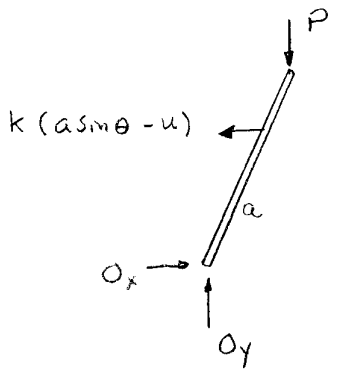
Inverted Pendulum



$$\Sigma M_O = I\alpha$$

$$-k(a\sin\theta - u)a\cos\theta + Pl\sin\theta = I\ddot{\theta}$$

For small angle, $\sin\theta \approx \theta$, $\cos\theta \approx 1$

$$I\ddot{\theta} + (ka^2 - Pl)\theta = ka\,u(t)$$

(a) Transfer function

$$u(t) = U(s)e^{st}, \quad \theta(t) = \Theta(s)e^{st}$$

$$\left[Is^2 + (ka^2 - Pl)\right]\Theta(s)e^{st} = ka\,U(s)e^{st}$$

Hence

$$H(s) = \frac{\Theta(s)}{U(s)} = \frac{ka}{Is^2 + (ka^2 - Pl)}$$

(b) Poles:

$$Is^2 + (ka^2 - Pl) = 0$$

$$s = \pm\sqrt{\frac{Pl - ka^2}{I}} \quad \text{if} \quad Pl > ka^2$$

$$s = \pm i\sqrt{\frac{ka^2 - Pl}{I}} \quad \text{if} \quad Pl < ka^2$$

Zeros: No zero.

(c)