Exam Problem Spring 1998

$$\ddot{x} + 2\dot{x} + 5x = \dot{u}(t)$$

$u(t)$ = input, $x(t)$ = output

(a) $$H(s) = \frac{s}{s^2+2s+5}$$

(b) $u(t) = \cos 2t = Re[e^{j2t}]$; $U(s) = 1$, $s = 2j$

$$H(s) = \frac{2j}{(2j)^2 + 2(2j) + 5} = \frac{2j}{1+4j}$$

$$= \frac{2j}{17}(1-4j) = \frac{1}{17}(8+2j) = 0.4851 \angle 14.036^\circ$$

$$X(s) = H(s)U(s) = \frac{1}{17}(8+2j)$$

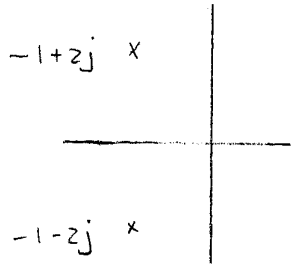
$$x(t) = Re\left[X(s)e^{st}\right]_{s=2j} = Re\left[\frac{1}{17}(8+2j)\,e^{j2t}\right]$$

$$= \frac{1}{17}(8\cos 2t - 2\sin 2t)$$

(c) Zero: 0

Poles: $s^2+2s+5 = 0$

$s = -1 \pm 2j$



(d) $f_1(t) = \sin 2t$, $s_1 = 2j$

$f_2(t) = \sin 4t$, $s_2 = 4j$

$f_1(t)$ will have Larger response, because s_1 is closer to poles.