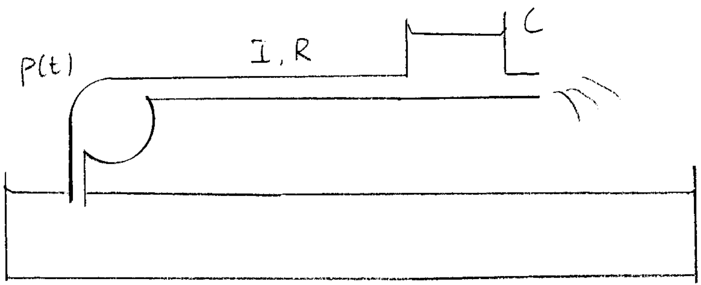
Old Exam Problem: Winter 2001



(a)

$$\begin{bmatrix} H_C \\ H_I \end{bmatrix} = \begin{bmatrix} s+2 & -1 \\ 5 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$= \frac{1}{(s+2)(s+4)+5} \begin{bmatrix} s+4 & 1 \\ -5 & s+2 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Hence $$H_c(s) = \frac{4}{s^2+6s+13}$$

(b) Poles: $s^2+6s+13=0$, $p_{1,2} = -3 \pm 2j$

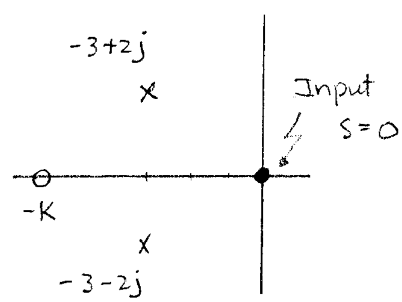
It's stable because $Re[p_{1,2}] < 0$

Zero: $s+K=0$, $z_1 = -K$

(c) If $p(t) =$ constant, $s=0$.

You want to increase K so that the zero is far away from the input

If zero is near the input, the output will be small.



(d) If $k=2$, $p(t)=e^{-t}$, $s=-1$

$$H_I(s) = \frac{4(-1+2)}{(-1)^2+6(-1)+13} = \frac{4}{8} = \frac{1}{2}$$

Hence $\quad x(t) = \frac{1}{2}e^{-t}$

(e) If $k=2$, $p(t)=\sin 2t$, $s=2j$, $\measuredangle[p(t)] = 2t$

$$H_I(s) = \frac{4(2j+2)}{(2j)^2+6(2j)+13} = \frac{8(1+j)}{9+12j} = \frac{8\sqrt{2}}{15}e^{j(-0.1396)}$$

Hence

$$|Q_I(t)| = \frac{8\sqrt{2}}{15} = 0.754$$

$$\measuredangle Q_I(t) = 2t - 0.1396$$