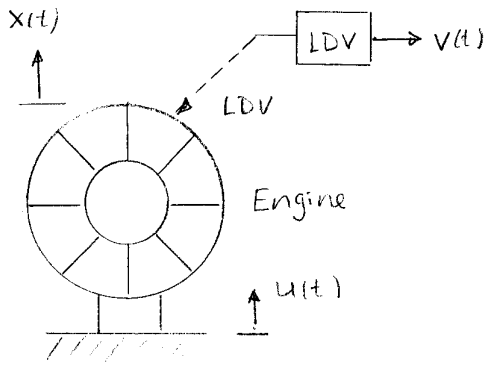
Exam Problem: Spring 2004



(a) $\frac{d^2x}{dt^2} + 4x = \frac{du}{dt} + u$

(b) $u(t) = \cos t = Re[e^{jt}]$

$= Re[F(s)e^{st}]$

$\Rightarrow F(s) = 1, \; s = j$

i.e., magnitude = 1

phase = $2t$

$$H(s) = \left.\frac{s+1}{s^2+4}\right|_{s=j} = \frac{j+1}{4+j^2} = \frac{1}{3}(1+j) = \frac{\sqrt{2}}{3}e^{j\pi/4}$$

$$|H(s)| = \frac{\sqrt{2}}{3}, \quad \angle H(s) = \frac{\pi}{4}$$

FOR X(t):

$$|X(t)| = |U(t)| \cdot |H(s)| = \frac{\sqrt{2}}{3}$$

$$\angle X(t) = \angle U(t) + \angle H(s) = t + \pi/4$$

(c) $$H(s) = \frac{s+1}{s^2+4} \times \frac{s}{s^2+a^2}$$

(d) Poles of $H_e(s) = \pm 2j$

Poles of $H(s) = \pm 2j, \pm aj$

Zeros of $H_e(s) = -1$

Zeros of $H(s) = 0$

(e) Engineer A is measuring the pole & zero of the LDV.