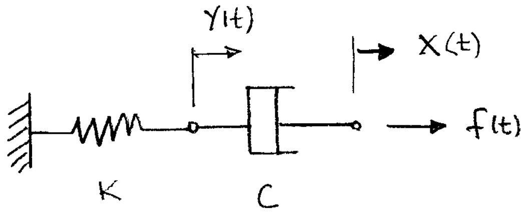
Mechanical Filter



$$f = c(\dot{x} - \dot{y})$$

$$f = ky$$

$$\Rightarrow f = c\left(\dot{x} - \frac{\dot{f}}{k}\right)$$

$$\Rightarrow \quad c\dot{f} + kf = ck\dot{x}$$

(a) $x(t)$ = input , $f(t)$ = output

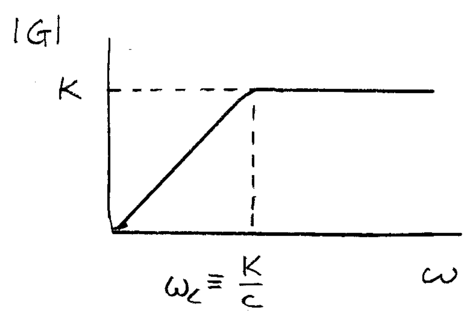
$$H(s) = \frac{cks}{cs+k} \quad ; \quad G(\omega) = \frac{jck\omega}{k + jc\omega}$$

(b) Magnitude

$$|G(\omega)| = \frac{ck\omega}{\sqrt{k^2 + c^2\omega^2}}$$

Phase

$$\measuredangle G(\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{c\omega}{k}\right)$$



cutoff frequency = $\frac{k}{c}$

high-pass filter

(c) If $k = 10$, $c = 1$, $x(t) = \cos 10t$,

$$|G(10)| = \frac{1 \cdot 10 \cdot 10}{\sqrt{10^2 + 1^2\, 10^2}} = \frac{10}{\sqrt{2}}$$

$$\measuredangle G(10) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1 \cdot 10}{10}\right) = \frac{\pi}{4}$$

$$f(t) = \frac{10}{\sqrt{2}} \cos\left(10t + \frac{\pi}{4}\right)$$

(d) Use as a force sensor when

$$\omega \gg \frac{K}{c}$$

The sensitivity of the force sensor is

$$\frac{X(t)}{f(t)} \propto \frac{1}{K} \Rightarrow \text{Want the stiffness to be small}$$

The sensitivity is independent of damping.